This is a digital copy of a book that was preserved for generations on library shelves before it was carefully scanned by Google as part of a project to make the world's books discoverable online.

It has survived long enough for the copyright to expire and the book to enter the public domain. A public domain book is one that was never subject to copyright or whose legal copyright term has expired. Whether a book is in the public domain may vary country to country. Public domain books are our gateways to the past, representing a wealth of history, culture and knowledge that's often difficult to discover.

Marks, notations and other marginalia present in the original volume will appear in this file - a reminder of this book's long journey from the publisher to a library and finally to you.

Usage guidelines

Google is proud to partner with libraries to digitize public domain materials and make them widely accessible. Public domain books belong to the public and we are merely their custodians. Nevertheless, this work is expensive, so in order to keep providing this resource, we have taken steps to prevent abuse by commercial parties, including placing technical restrictions on automated querying.

We also ask that you:

+ *Make non-commercial use of the files* We designed Google Book Search for use by individuals, and we request that you use these files for personal, non-commercial purposes.

+ *Refrain from automated querying* Do not send automated queries of any sort to Google's system: If you are conducting research on machine translation, optical character recognition or other areas where access to a large amount of text is helpful, please contact us. We encourage the use of public domain materials for these purposes and may be able to help.

+ *Maintain attribution* The Google "watermark" you see on each file is essential for informing people about this project and helping them find additional materials through Google Book Search. Please do not remove it.

+ *Keep it legal* Whatever your use, remember that you are responsible for ensuring that what you are doing is legal. Do not assume that just because we believe a book is in the public domain for users in the United States, that the work is also in the public domain for users in other countries. Whether a book is still in copyright varies from country to country, and we can't offer guidance on whether any specific use of any specific book is allowed. Please do not assume that a book's appearance in Google Book Search means it can be used in any manner anywhere in the world. Copyright infringement liability can be quite severe.

About Google Book Search

Google's mission is to organize the world's information and to make it universally accessible and useful. Google Book Search helps readers discover the world's books while helping authors and publishers reach new audiences. You can search through the full text of this book on the web at `http://books.google.com/`

NYPL RESEARCH LIBRARIES
3 3433 06637490 5

Internati
[illegible]

A TEXTBOOK

ON

ELECTRIC LIGHTING AND RAILWAYS

INTERNATIONAL CORRESPONDENCE SCHOOLS
SCRANTON, PA.

1

ARITHMETIC
MENSURATION AND USE OF LETTERS IN FORMULAS
PRINCIPLES OF MECHANICS
MACHINE ELEMENTS
ELEMENTS OF ELECTRICITY AND MAGNETISM
DYNAMOS AND MOTORS

5650

SCRANTON
INTERNATIONAL TEXTBOOK COMPANY
B–2

THE NEW YORK
PUBLIC LIBRARY
72160[illegible]
ASTOR, LENOX AND
TILDEN FOUNDATIONS
R 1916 L

Copyright, 1897, by THE COLLIERY ENGINEER COMPANY.
Copyright, 1901, by INTERNATIONAL TEXTBOOK COMPANY.

Entered at Stationers' Hall, London.

Arithmetic: Copyright, 1893, 1894, 1895, 1896, 1897, 1898, 1899, by THE COLLIERY ENGINEER COMPANY. Copyright, 1901, by INTERNATIONAL TEXTBOOK COMPANY.

Mensuration and Use of Letters in Formulas: Copyright, 1894, 1895, 1897, 1898, by THE COLLIERY ENGINEER COMPANY.

Principles of Mechanics: Copyright, 1901, by INTERNATIONAL TEXTBOOK COMPANY. Entered at Stationers' Hall, London.

Machine Elements: Copyright, 1901, by INTERNATIONAL TEXTBOOK COMPANY. Entered at Stationers' Hall, London.

Elements of Electricity and Magnetism: Copyright, 1894, 1897, by THE COLLIERY ENGINEER COMPANY. Copyright, 1901, by INTERNATIONAL TEXTBOOK COMPANY.

Dynamos and Motors: Copyright, 1894, 1897, by THE COLLIERY ENGINEER COMPANY. Copyright, 1901, by INTERNATIONAL TEXTBOOK COMPANY.

Arithmetic, Key: Copyright, 1893, 1894, 1896, 1897, 1898, by THE COLLIERY ENGINEER COMPANY.

Mensuration and Use of Letters in Formulas, Key: Copyright, 1894, 1895, by THE COLLIERY ENGINEER COMPANY.

Elements of Electricity and Magnetism, Key: Copyright, 1894, 1897, by THE COLLIERY ENGINEER COMPANY.

Dynamos and Motors, Key: Copyright, 1894, 1897, by THE COLLIERY ENGINEER COMPANY.

All rights reserved.

CI

BURR PRINTING HOUSE,
FRANKFORT AND JACOB STREETS,
NEW YORK.

PREFACE.

These textbooks on Electric Lighting and Railways have been prepared to replace those formerly issued under the title of "A Treatise on Electric Power and Lighting." In order to keep pace with the advances and improvements of the past few years, it has been found necessary to rewrite our former textbooks and add a large amount of new material. This has resulted in practically a new Course throughout. In addition to covering the field of Electric Lighting and Railways in a most thorough manner, these volumes also contain all the necessary instruction on Alternating Currents, Electric Power Transmission, and allied subjects that the student requires in order to understand thoroughly the various methods of power distribution. The introduction of the enclosed-arc lamp has brought about many changes in the field of electric lighting, and this has been given the consideration it deserves. The subject of Interior Wiring is closely allied to Electric Lighting and Railways, and has been added to the Course, three sections having been specially prepared for this branch alone. The sections on Electric Railways have been made so complete that the student cannot help but obtain a good knowledge of all the component parts of a street-railway equipment. Controllers have been treated in particular detail in order that the student may become well posted on the methods and connections used for car control. In addition to the above, a section on the practical operation of dynamos and motors has also been added.

In preparing the descriptions of various types of apparatus, the aim has been to take typical examples that illustrate the underlying principles and to so illustrate and explain these that the student will have no difficulty in understanding and operating any similar devices. No expense has been spared in the preparation, and cuts have been used freely wherever they would aid the text. The Course has been prepared for those men who are called upon to install and operate electrical apparatus, rather than for designers; hence, the instruction throughout has been made as practical and simple as possible. Only those methods that are in common use have been described, and special and experimental methods have been entirely avoided or given very brief mention. While descriptions of new methods and appliances have been inserted, care has been taken not to leave out those older methods that are still in extensive use and with which the station engineer must be familiar if he is to be successful. It is believed, therefore, that these textbooks will be found to contain just that information which is needed for the successful operation of a light or power plant. While this Course is considerably larger than the one which it replaces, it will be found that the amount of study necessary is increased little, if any; whereas, the textbooks, as works of reference, are very much more valuable.

The method of numbering the pages, cuts, articles, etc. is such that each paper and part is complete in itself; hence, in order to make the indexes intelligible, it was necessary to give each paper and part a number. This number is placed at the top of each page, on the headline, opposite the page number; and to distinguish it from the page number, it is preceded by the printer's section mark (§). Consequently, a reference such as § 20, page 35, would readily be found by looking along the headlines until § 20 is found, and then through § 20 until page 35 is found.

The questions corresponding to each subject are grouped together near the end of the volume and follow the text-matter to which they refer. The answers to the questions are also grouped together and placed at the end of the volume following the questions.

INTERNATIONAL CORRESPONDENCE SCHOOLS.

CONTENTS.

ARITHMETIC.

	Section.	*Page.*
Definitions	1	1
Notation and Numeration	1	1
Addition	1	4
Subtraction	1	9
Multiplication	1	11
Division	1	16
Cancelation	1	20
Fractions	1	23
Reduction of Fractions	1	25
Addition of Fractions	1	28
Multiplication of Fractions	1	31
Division of Fractions	1	33
Decimals	1	37
Addition of Decimals	1	39
Subtraction of Decimals	1	41
Multiplication of Decimals	1	42
Division of Decimals	1	44
Signs of Aggregation	1	52
Percentage	2	1
Calculations Involving Percentage	2	4
Denominate Numbers	2	9
Measures	2	10
Reduction of Denominate Numbers	2	13
Addition of Denominate Numbers	2	16
Subtraction of Denominate Numbers	2	18
Multiplication of Denominate Numbers	2	20
Division of Denominate Numbers	2	21

ARITHMETIC—*Continued.*

	Section.	*Page.*
Involution	2	25
Evolution	2	27
Square Root	2	28
Cube Root	2	35
Ratio	2	40
Proportion	2	43
Unit Method	2	48

MENSURATION AND USE OF LETTERS IN FORMULAS.

Formulas	3	1
Mensuration	3	13
Quadrilaterals	3	17
The Triangle	3	20
Polygons	3	24
The Prism and Cylinder	3	32
The Pyramid and Cone	3	36
The Frustum of a Pyramid or Cone	3	37
The Sphere and Cylindrical Ring	3	39

PRINCIPLES OF MECHANICS.

Matter and Its Properties	4	1
Motion and Velocity	4	5
Force	4	10
Gravitation and Weight	4	15
Work, Power, and Energy	4	20
Composition and Resolution of Forces	4	26
Friction	4	33
Center of Gravity	4	39
Centrifugal Force	4	43
Equilibrium	4	45

MACHINE ELEMENTS.

The Lever, Wheel, and Axle	5	1
Pulleys.	5	7
Belts	5	12
Wheel Work	5	25
Gear Calculations	5	34

MACHINE ELEMENTS—*Continued.* *Section.* *Page.*

Fixed and Movable Pulleys 5 42
The Inclined Plane 5 47
The Screw 5 53
Velocity Ratio and Efficiency 5 59

ELEMENTS OF ELECTRICITY AND MAGNETISM.

Introduction 8 1
Static Charges 8 3
Conductors and Non-Conductors . . . 8 5
Electrodynamics 8 6
Magnetism 8 14
Electrical Units 8 26
Applications of Ohm's Law 8 50
Electrical Quantity 8 60
Electrical Work 8 61
Electrical Power 8 63

DYNAMOS AND MOTORS.

Electromagnetic Induction 9 1
Physical Theory of the Dynamo . . . 9 9
Armature Reactions 9 34
Field Magnets 9 42
Types of Dynamos 9 55
Constant-Potential Dynamos and Generators 9 56
Efficiency of Constant-Potential Dynamos 9 69
The Output of Constant-Potential Dynamos 9 81
Direct-Current Motors 9 87

EXAMINATION QUESTIONS. *Section.*

Arithmetic, Part 1 1
Arithmetic, Part 2 1
Arithmetic, Part 3 1
Arithmetic, Part 4 2
Arithmetic, Part 5 2
Mensuration and Use of Letters in Formulas 3

EXAMINATION QUESTIONS—*Continued.* *Section.*
Principles of Mechanics 4
Machine Elements 5
Elements of Electricity and Magnetism 8
Dynamos and Motors 9

ANSWERS TO QUESTIONS.
Arithmetic, Part 1 1
Arithmetic, Part 2 1
Arithmetic, Part 3 1
Arithmetic, Part 4 2
Arithmetic, Part 5 2
Mensuration and Use of Letters in Formulas 3
Principles of Mechanics 4
Machine Elements 5
Elements of Electricity and Magnetism . 8
Dynamos and Motors 9

ARITHMETIC.

(PART 1.)

DEFINITIONS.

1. **Arithmetic** is the art of reckoning, or the study of numbers.

2. A **unit** is *one*, or a single thing, as *one*, *one* bolt, *one* pulley, *one* dozen.

3. A **number** is a unit, or a collection of units, as *one*, *three* engines, *five* boilers.

4. The **unit of a number** is one of the collection of units which constitutes the number. Thus, the unit of *twelve* is *one*, of *twenty* dollars is *one* dollar, of *one hundred* bolts is *one* bolt.

5. A **concrete number** is a number applied to some particular kind of object or quantity, as three *grate bars*, five *dollars*, ten *pounds*.

6. An **abstract number** is a number that is not applied to any object or quantity, as *three*, *five*, *ten*.

7. **Like numbers** are numbers which express units of the *same kind*, as six *days* and ten *days*, two *feet* and five *feet*.

8. **Unlike numbers** are numbers which express units of *different kinds*, as ten *months* and eight *miles*, seven *wrenches* and five *bolts*.

NOTATION AND NUMERATION.

9. Numbers are expressed in three ways: (1) by words; (2) by figures; (3) by letters.

10. **Notation** is the art of expressing numbers by figures or letters.

11. **Numeration** is the art of reading the numbers which have been expressed by figures or letters.

For notice of copyright, see page immediately following the title page.

12. The **Arabic notation** is the method of expressing numbers by figures. This method employs ten different **figures** to represent numbers, viz.:

Figures	0	1	2	3	4	5	6	7	8	9
Names	*naught, cipher, or zero.*	*one*	*two*	*three*	*four*	*five*	*six*	*seven*	*eight*	*nine*

The first character (0) is called **naught, cipher,** or **zero,** and when standing alone has no value.

The other nine figures are called **digits,** and each has a value of its own.

Any whole number is called an **integer.**

13. As there are only ten *figures* used in expressing numbers, each figure must have a different *value* at different times.

14. The value of a figure depends upon its *position* in relation to other figures.

15. Figures have **simple** values, and **local,** or **relative,** values.

16. The **simple** value of a figure is the value it expresses when standing alone.

17. The **local,** or **relative,** value of a figure is the *increased* value it expresses by having other figures placed on its right.

For instance, if we see the figure 6 standing alone, thus 6

we consider it as *six units*, or simply **six.**

Place another 6 to the *left* of it; thus 66

The original figure is still *six units*, but the second figure is *ten times* 6, or 6 **tens.**

If a third 6 be now placed still one place further to the *left*, it is increased in value *ten times* more, thus making it 6 **hundreds** 666

A fourth 6 would be 6 **thousands** 6666

A fifth 6 would be 6 **tens of thousands,** or **sixty thousand** 66666

A sixth 6 would be 6 **hundreds of thousands** . . 666666

A seventh 6 would be 6 **millions** 6666666

The entire line of seven figures is read *six millions six hundred sixty-six thousands six hundred sixty-six*.

18. The **increased value** of each of these figures is its *local*, or *relative*, value. Each figure is *ten times* greater in value than the one immediately on its *right*.

19. The **cipher** (0) has no value in itself, but it is useful in determining the place of other figures. To represent the number *four hundred five*, two digits only are necessary, one to represent *four hundred* and the other to represent *five units;* but if these two digits are placed together, as 45, the 4 (being in the second place) will mean 4 *tens*. To mean 4 *hundreds*, the 4 should have two figures on its right, and a *cipher* is therefore inserted in the place usually given to *tens*, to show that the number is composed of *hundreds* and *units* only, and that there are no *tens*. *Four hundred five* is therefore expressed as 405. If the number were *four thousand and five*, two ciphers would be inserted; thus, 4005. If it were *four hundred fifty*, it would have the *cipher* at the right-hand side to show that there were no *units*, and only *hundreds and tens;* thus, 450. *Four thousand and fifty* would be expressed 4050.

20. In *reading* figures, it is usual to *point off* the number into groups of three figures each, beginning at the right-hand, or **units**, column, a comma (,) being used to point off these groups.

Billions.			*Millions.*			*Thousands.*			*Units.*		
Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds of Units.	Tens of Units.	Units.
4	3	2 ,	1	9	8 ,	7	6	5 ,	4	3	2

In *pointing off* these figures, begin at the right-hand figure and count—*units*, *tens of units*, *hundreds of units;* the next group of three figures is *thousands;* therefore, we insert a comma (,) before beginning with them. Beginning at the figure 5, we say *thousands*, *tens of thousands*, *hundreds of thousands*, and insert another comma. We next read *millions*, *tens of millions*, *hundreds of millions* (insert another comma), *billions*, *tens of billions*, *hundreds of billions.*

The entire line of figures would be read: *Four hundred thirty-two billions one hundred ninety-eight millions seven hundred sixty-five thousands four hundred thirty-two.* When we thus *read* a line of figures, it is called *numeration*, and if the numeration be changed back to *figures*, it is called *notation.*

For instance, the writing of the following figures,

72,584,623,

would be the notation, and the numeration would be *seventy-two millions five hundred eighty-four thousands six hundred twenty-three.*

21. Note.—It is customary to leave the "s" off the words millions, thousands, etc. in cases like the above, both in speaking and writing; hence, the above would usually be expressed seventy-two million five hundred eighty-four thousand six hundred twenty-three.

22. The four fundamental processes of arithmetic are addition, subtraction, multiplication, and division. They are called fundamental processes because all operations in arithmetic are based upon them.

ADDITION.

23. **Addition** is the *process of finding the sum of two or more numbers.* The sign of addition is +. It is read **plus,** and means *more.* Thus, 5 + 6 is read *5 plus 6*, and means that 5 and 6 are to be added.

24. The sign of equality is =. It is read **equals** or **is equal to.** Thus, 5 + 6 = 11 may be read *5 plus 6 equals 11*, or *5 plus 6 is equal to 11.*

25. *Like numbers* can be added; *unlike numbers* cannot be added. 6 dollars can be added to 7 dollars, and the sum will be 13 dollars; but 6 dollars cannot be added to 7 feet.

26. The following table gives the sum of any two numbers from 1 to 12; it should be carefully committed to memory:

1 and 1 is 2	2 and 1 is 3	3 and 1 is 4	4 and 1 is 5
1 and 2 is 3	2 and 2 is 4	3 and 2 is 5	4 and 2 is 6
1 and 3 is 4	2 and 3 is 5	3 and 3 is 6	4 and 3 is 7
1 and 4 is 5	2 and 4 is 6	3 and 4 is 7	4 and 4 is 8
1 and 5 is 6	2 and 5 is 7	3 and 5 is 8	4 and 5 is 9
1 and 6 is 7	2 and 6 is 8	3 and 6 is 9	4 and 6 is 10
1 and 7 is 8	2 and 7 is 9	3 and 7 is 10	4 and 7 is 11
1 and 8 is 9	2 and 8 is 10	3 and 8 is 11	4 and 8 is 12
1 and 9 is 10	2 and 9 is 11	3 and 9 is 12	4 and 9 is 13
1 and 10 is 11	2 and 10 is 12	3 and 10 is 13	4 and 10 is 14
1 and 11 is 12	2 and 11 is 13	3 and 11 is 14	4 and 11 is 15
1 and 12 is 13	2 and 12 is 14	3 and 12 is 15	4 and 12 is 16
5 and 1 is 6	6 and 1 is 7	7 and 1 is 8	8 and 1 is 9
5 and 2 is 7	6 and 2 is 8	7 and 2 is 9	8 and 2 is 10
5 and 3 is 8	6 and 3 is 9	7 and 3 is 10	8 and 3 is 11
5 and 4 is 9	6 and 4 is 10	7 and 4 is 11	8 and 4 is 12
5 and 5 is 10	6 and 5 is 11	7 and 5 is 12	8 and 5 is 13
5 and 6 is 11	6 and 6 is 12	7 and 6 is 13	8 and 6 is 14
5 and 7 is 12	6 and 7 is 13	7 and 7 is 14	8 and 7 is 15
5 and 8 is 13	6 and 8 is 14	7 and 8 is 15	8 and 8 is 16
5 and 9 is 14	6 and 9 is 15	7 and 9 is 16	8 and 9 is 17
5 and 10 is 15	6 and 10 is 16	7 and 10 is 17	8 and 10 is 18
5 and 11 is 16	6 and 11 is 17	7 and 11 is 18	8 and 11 is 19
5 and 12 is 17	6 and 12 is 18	7 and 12 is 19	8 and 12 is 20
9 and 1 is 10	10 and 1 is 11	11 and 1 is 12	12 and 1 is 13
9 and 2 is 11	10 and 2 is 12	11 and 2 is 13	12 and 2 is 14
9 and 3 is 12	10 and 3 is 13	11 and 3 is 14	12 and 3 is 15
9 and 4 is 13	10 and 4 is 14	11 and 4 is 15	12 and 4 is 16
9 and 5 is 14	10 and 5 is 15	11 and 5 is 16	12 and 5 is 17
9 and 6 is 15	10 and 6 is 16	11 and 6 is 17	12 and 6 is 18
9 and 7 is 16	10 and 7 is 17	11 and 7 is 18	12 and 7 is 19
9 and 8 is 17	10 and 8 is 18	11 and 8 is 19	12 and 8 is 20
9 and 9 is 18	10 and 9 is 19	11 and 9 is 20	12 and 9 is 21
9 and 10 is 19	10 and 10 is 20	11 and 10 is 21	12 and 10 is 22
9 and 11 is 20	10 and 11 is 21	11 and 11 is 22	12 and 11 is 23
9 and 12 is 21	10 and 12 is 22	11 and 12 is 23	12 and 12 is 24

27. For *addition*, place the numbers to be added directly under one another, taking care to place *units* under *units*, *tens* under *tens*, *hundreds* under *hundreds*, and so on.

When the numbers are thus written, the *right-hand figure of one number is placed directly under the right-hand figure of the one above it, thus bringing units under units, tens under tens, etc.* Proceed as in the following examples:

28. Example.—What is the sum of 131, 222, 21, 2, and 413?

Solution.—

$$\begin{array}{rrl} & 131 & \\ & 222 & \\ & 21 & \\ & 2 & \\ & 413 & \\ \cline{2-2} \textit{sum} & 789 & \text{Ans.} \end{array}$$

Explanation.—After placing the numbers in proper order, begin at the bottom of the right-hand, or units, column and add; thus, $3 + 2 + 1 + 2 + 1 = 9$, the sum of the numbers in units column. Place the 9 directly beneath as the first, or units, figure in the sum.

The sum of the numbers in the next, or tens, column equals 8 tens, which is the second, or tens, figure in the sum.

The sum of the numbers in the next, or hundreds, column equals 7 hundreds, which is the third, or hundreds, figure in the sum.

The sum, or answer, is 789.

29. Example.—What is the sum of 425, 36, 9,215, 4, and 907?

Solution.—

$$\begin{array}{rrl} & 425 & \\ & 36 & \\ & 9215 & \\ & 4 & \\ & 907 & \\ \cline{2-2} & 27 & \\ & 60 & \\ & 1500 & \\ & 9000 & \\ \cline{2-2} \textit{sum} & 10587 & \text{Ans.} \end{array}$$

Explanation.—The sum of the numbers in the first, or units, column is 27 units; i. e., 2 tens and 7 units. Write 27 as shown. The sum of the numbers in the second, or tens, column is 6 tens, or 60. Write 60 underneath 27 as shown. The sum of the numbers in the third, or hundreds, column is 15 hundreds, or 1,500. Write 1,500 under the two preceding results as shown. There is only one number in the fourth, or thousands, column, 9, which represents 9,000.

Write 9,000 under the three preceding results. Adding these four results, the sum is 10,587, which is the sum of 425, 36, 9,215, 4, and 907.

30. The addition may also be performed as follows:

```
      4 2 5
        3 6
    9 2 1 5
          4
      9 0 7
  ---------
sum 1 0 5 8 7  Ans.
```

EXPLANATION.—The sum of the numbers in units column = 27 units, or 2 tens and 7 units. Write the 7 units as the first, or right-hand, figure in the sum. Reserve the 2 tens and add them to the figures in tens column. The sum of the figures in the tens column plus the 2 tens reserved and carried from the units column is 8, which is written down as the second figure in the sum. There is nothing to carry to the next column, because 8 is less than 10. The sum of the numbers in the next column is 15 hundreds, or 1 thousand and 5 hundreds. Write down the 5 as the third, or hundreds, figure in the sum and carry the 1 to the next column. 1 + 9 = 10, which is written down at the left of the other figures.

The second method saves space and figures, but the first is to be preferred when adding a long column.

31. EXAMPLE.—Add the numbers in the column below:

SOLUTION.—

```
    [illegible] 9 0
      [illegible] 2
      9 [illegible]
          3 9 3
    2 [illegible] [illegible]
  [illegible] [illegible]
    7 7 [illegible]
      [illegible] 3
    4 [illegible] 2
  [illegible] [illegible]
    3 [illegible] 3
      [illegible] 4
  [illegible]

sum [illegible] Ans.
```

Explanation.—The sum of the digits in the first column equals 19 units, or 1 ten and 9 units. Write the 9 and carry 1 to the next column. The sum of the digits in the second column + the 1 carried from the first column is 109 tens, or 10 hundreds and 9 tens. Write down the 9 and carry the 10 to the next column. The sum of the digits in this column plus the 10 carried from the second column is 38.

The entire sum is 3,899.

32. Rule.—I. *Begin at the right, add each column separately, and write the sum, if it be only one figure, under the column added.*

II. *If the sum of any column consists of two or more figures, put the right-hand figure of the sum under that column and add the remaining figure or figures to the next column.*

33. Proof.—*To prove addition, add each column from top to bottom. If you obtain the same result as by adding from bottom to top, the work is probably correct.*

EXAMPLES FOR PRACTICE.

34. Find the sum of:

(*a*)	104 + 203 + 613 + 214.	Ans. (*a*)	1,134.
(*b*)	1,875 + 3,143 + 5,826 + 10,832.	(*b*)	21,676.
(*c*)	4,865 + 2,145 + 8,173 + 40,084.	(*c*)	55,267.
(*d*)	14,204 + 8,173 + 1,065 + 10,042.	(*d*)	33,484.
(*e*)	10,832 + 4,145 + 3,133 + 5,872.	(*e*)	23,982.
(*f*)	214 + 1,231 + 141 + 5,000.	(*f*)	6,586.
(*g*)	123 + 104 + 425 + 126 + 327.	(*g*)	1,105.
(*h*)	6,354 + 2,145 + 2,042 + 1,111 + 3,333.	(*h*)	14,985.

1. A week's record of coal burned in an engine room is as follows: Monday, 1,800 pounds; Tuesday, 1,655 pounds; Wednesday, 1,725 pounds; Thursday, 1,690 pounds; Friday, 1,648 pounds; Saturday, 1,020 pounds. How much coal was burned during the week?
Ans. 9,538 lb.

2. A steam pump, in one hour, pumps out of a cistern 4,200 gallons; in the next hour, 5,420 gallons; and in 45 minutes more an additional 3,600 gallons, when the cistern becomes empty. How many gallons were in the cistern at first? Ans. 13,220 gal.

3. What is the total cost of a steam plant, the several items of expense being as follows: steam engine, $900; boiler, $775; fittings and connections, $225; erecting the plant, $125; engine house, $650? Ans. $2,675.

SUBTRACTION.

35. In arithmetic, **subtraction** is the process of finding how much greater one number is than another.

The greater of the two numbers is called the **minuend.**

The smaller of the two numbers is called the **subtrahend.**

The number left after subtracting the subtrahend from the minuend is called the **difference,** or **remainder.**

36. The sign of subtraction is —. It is read **minus,** and means *less.* Thus, 12 — 7 is read *12 minus 7*, and means that 7 is to be taken from 12.

37. Example.—From 7,568 take 3,425.

Solution.— *minuend* 7 5 6 8
subtrahend 3 4 2 5
remainder 4 1 4 3 Ans.

Explanation.—Begin at the right-hand, or units, column and subtract in succession each figure in the subtrahend from the one directly above it in the minuend, and write the remainders below the line. The result is the entire remainder.

38. When there are more figures in the minuend than in the subtrahend, and when some figures in the minuend are *less* than the figures directly under them in the subtrahend, proceed as in the following example:

Example.—From 8,453 take 844.

Solution.— *minuend* 8 4 5 3
subtrahend 8 4 4
remainder 7 6 0 9 Ans.

Explanation.—Begin at the right-hand, or units, column to subtract. We cannot take 4 from 3, and must, therefore, borrow 1 from 5 in tens column and annex it to the 3 in units column. The 1 ten = 10 units, which added to the 3 in units column = 13 units. 4 from 13 = 9, the first, or units, figure in the remainder.

Since we borrowed 1 from the 5, only 4 remains; 4 from 4 = 0, the second, or tens, figure. We cannot take 8 from 4, so borrow 1 thousand, or 10 hundreds, from 8; 10 hundreds + 4 hundreds = 14 hundreds, and 8 from 14 = 6, the third, or hundreds, figure in the remainder.

Since we borrowed 1 from 8, only 7 remains, from which there is nothing to subtract; therefore, 7 is the next figure in the remainder, or answer.

The operation of borrowing is placing 1 before the figure following the one from which it is borrowed. In the above example the 1 borrowed from 5 is placed before 3, making it 13, from which we subtract 4. The 1 borrowed from 8 is placed before 4, making 14, from which 8 is taken.

39. EXAMPLE.—Find the difference between 10,000 and 8,763.

SOLUTION.—

minuend	10000	
subtrahend	8763	
remainder	1237	Ans.

EXPLANATION.—In the above example we borrow 1 from the second column and place it before 0, making 10; 3 from 10 = 7. In the same way we borrow 1 and place it before the next cipher, making 10; but as we have borrowed 1 from this column and have taken it to the units column, only 9 remains from which to subtract 6; 6 from 9 = 3. For the same reason we subtract 7 from 9 and 8 from 9 for the next two figures, and obtain a total remainder of 1,237.

40. Rule.—*Place the subtrahend (or smaller) number under the minuend (or larger) number, in the same manner as for addition, and proceed as in Arts.* **37, 38,** *and* **39.**

41. Proof.—*To prove an example in subtraction, add the subtrahend and the remainder. The sum should equal the minuend. If it does not, a mistake has been made, and the work should be done over.*

Proof of the above example:

subtrahend	8768
remainder	1237
minuend	10000

EXAMPLES FOR PRACTICE.

42. From:

			Ans.
(*a*)	94,278 take 62,574.	(*a*)	31,704.
(*b*)	53,714 take 25,824.	(*b*)	27,890.
(*c*)	71,832 take 58,109.	(*c*)	13,723.
(*d*)	20,804 take 10,408.	(*d*)	10,396.
(*e*)	310,465 take 102,141.	(*e*)	208,324.
(*f*)	(81,043 + 1,041) take 14,831.	(*f*)	67,253.
(*g*)	(20,482 + 18,216) take 21,214.	(*g*)	17,484.
(*h*)	(2,040 + 1,213 + 542) take 3,791.	(*h*)	4.

1. A cistern is fed by two pipes which supply 1,200 and 2,250 gallons per hour, respectively, and is being emptied by a pump which delivers 5,800 gallons per hour. Starting with 8,000 gallons in the cistern, how much water does it contain at the end of an hour? Ans. 5,650 gal.

2. A train in running from New York to Buffalo travels 38 miles the first hour, 42 the second, 39 the third, 56 the fourth, 52 the fifth, and 48 the sixth hour. How many miles remain to be traveled at the end of the sixth hour, the distance between the two places being 410 miles? Ans. 135 mi.

3. On Monday morning a bank had on hand $2,862. During the day $1,831 were deposited and $2,172 drawn out; on Tuesday, $3,126 were deposited and $1,954 drawn out. How many dollars were on hand Wednesday morning? Ans. $3,693.

MULTIPLICATION.

43. To **multiply** a number is to *add* it to itself a certain number of times.

44. **Multiplication** is the process of multiplying one number by another.

The number thus added to itself, or the number to be multiplied, is called the **multiplicand.**

The number which shows how many times the *multiplicand* is to be taken, or the number by which we *multiply*, is called the **multiplier.**

The result obtained by multiplying is called the **product.**

45. In the following table, the product of any two numbers (neither of which exceeds 12) may be found:

1 times 1 is 1	2 times 1 is 2	3 times 1 is 3
1 times 2 is 2	2 times 2 is 4	3 times 2 is 6
1 times 3 is 3	2 times 3 is 6	3 times 3 is 9
1 times 4 is 4	2 times 4 is 8	3 times 4 is 12
1 times 5 is 5	2 times 5 is 10	3 times 5 is 15
1 times 6 is 6	2 times 6 is 12	3 times 6 is 18
1 times 7 is 7	2 times 7 is 14	3 times 7 is 21
1 times 8 is 8	2 times 8 is 16	3 times 8 is 24
1 times 9 is 9	2 times 9 is 18	3 times 9 is 27
1 times 10 is 10	2 times 10 is 20	3 times 10 is 30
1 times 11 is 11	2 times 11 is 22	3 times 11 is 33
1 times 12 is 12	2 times 12 is 24	3 times 12 is 36
4 times 1 is 4	5 times 1 is 5	6 times 1 is 6
4 times 2 is 8	5 times 2 is 10	6 times 2 is 12
4 times 3 is 12	5 times 3 is 15	6 times 3 is 18
4 times 4 is 16	5 times 4 is 20	6 times 4 is 24
4 times 5 is 20	5 times 5 is 25	6 times 5 is 30
4 times 6 is 24	5 times 6 is 30	6 times 6 is 36
4 times 7 is 28	5 times 7 is 35	6 times 7 is 42
4 times 8 is 32	5 times 8 is 40	6 times 8 is 48
4 times 9 is 36	5 times 9 is 45	6 times 9 is 54
4 times 10 is 40	5 times 10 is 50	6 times 10 is 60
4 times 11 is 44	5 times 11 is 55	6 times 11 is 66
4 times 12 is 48	5 times 12 is 60	6 times 12 is 72
7 times 1 is 7	8 times 1 is 8	9 times 1 is 9
7 times 2 is 14	8 times 2 is 16	9 times 2 is 18
7 times 3 is 21	8 times 3 is 24	9 times 3 is 27
7 times 4 is 28	8 times 4 is 32	9 times 4 is 36
7 times 5 is 35	8 times 5 is 40	9 times 5 is 45
7 times 6 is 42	8 times 6 is 48	9 times 6 is 54
7 times 7 is 49	8 times 7 is 56	9 times 7 is 63
7 times 8 is 56	8 times 8 is 64	9 times 8 is 72
7 times 9 is 63	8 times 9 is 72	9 times 9 is 81
7 times 10 is 70	8 times 10 is 80	9 times 10 is 90
7 times 11 is 77	8 times 11 is 88	9 times 11 is 99
7 times 12 is 84	8 times 12 is 96	9 times 12 is 108
10 times 1 is 10	11 times 1 is 11	12 times 1 is 12
10 times 2 is 20	11 times 2 is 22	12 times 2 is 24
10 times 3 is 30	11 times 3 is 33	12 times 3 is 36
10 times 4 is 40	11 times 4 is 44	12 times 4 is 48
10 times 5 is 50	11 times 5 is 55	12 times 5 is 60
10 times 6 is 60	11 times 6 is 66	12 times 6 is 72
10 times 7 is 70	11 times 7 is 77	12 times 7 is 84
10 times 8 is 80	11 times 8 is 88	12 times 8 is 96
10 times 9 is 90	11 times 9 is 99	12 times 9 is 108
10 times 10 is 100	11 times 10 is 110	12 times 10 is 120
10 times 11 is 110	11 times 11 is 121	12 times 11 is 132
10 times 12 is 120	11 times 12 is 132	12 times 12 is 144

This table should be carefully committed to memory.

Since 0 has no value, the product of 0 and any number is 0.

46. The sign of multiplication is ×. It is read **times** or **multiplied by.** Thus, 9 × 6 is read *9 times 6*, or *9 multiplied by 6.*

47. It matters not in what order the numbers to be multiplied together are placed. Thus, 6 × 9 is the same as 9 × 6.

48. To multiply a number by one figure only:

EXAMPLE.—Multiply 425 by 5.

SOLUTION.—

$$\begin{array}{rr} \textit{multiplicand} & 425 \\ \textit{multiplier} & 5 \\ \hline \textit{product} & 2125 \text{ Ans.} \end{array}$$

EXPLANATION.—For convenience, the multiplier is generally written under the right-hand figure of the multiplicand. On looking in the multiplication table, we see that 5 × 5 is 25. Multiplying the first figure at the right of the multiplicand, or 5, by the multiplier 5, it is seen that 5 × 5 units is 25 units, or 2 tens and 5 units. Write the 5 units in units place in the product and reserve the 2 tens to add to the product of tens. Looking in the multiplication table again, we see that 5 × 2 is 10. Multiplying the second figure of the multiplicand by the multiplier 5, we see that 5 times 2 tens is 10 tens, which, plus the 2 tens reserved, is 12 tens, or 1 hundred and 2 tens. Write the 2 tens in tens place and reserve the 1 hundred to add to the product of hundreds. Again, we see by the multiplication table that 5 × 4 is 20. Multiplying the third, or last, figure of the multiplicand by the multiplier 5, we see that 5 times 4 hundreds is 20 hundreds, which, plus the 1 hundred reserved, is 21 hundreds, or 2 thousands and 1 hundred, which we write in thousands and hundreds places, respectively.

Hence, the product is 2,125.

This result is the same as adding 425 five times. Thus,

$$\begin{array}{rr} & 425 \\ & 425 \\ & 425 \\ & 425 \\ & 425 \\ \hline \textit{sum} & 2125 \text{ Ans.} \end{array}$$

EXAMPLES FOR PRACTICE.

49. Find the product of:

		Ans.
(*a*)	61,483 × 6.	368,898.
(*b*)	12,375 × 5.	61,875.
(*c*)	10,426 × 7.	72,982.
(*d*)	10,835 × 3.	32,505.
(*e*)	98,376 × 4.	393,504.
(*f*)	10,873 × 8.	86,984.
(*g*)	71,543 × 9.	643,887.
(*h*)	218,734 × 2.	437,468.

1. A stationary engine makes 5,520 revolutions per hour. Running 9 hours a day, 5 days in the week, and 5 hours on Saturday, how many revolutions would it make in 4 weeks? Ans. 1,104,000 rev.

2. An engineer earns $650 a year and his average expenses are $548. How much could he save in 8 years at that rate? Ans. $816.

3. The connection between an engine and boiler is made up of 5 lengths of pipe, three of which are 12 feet long, one 2 feet 6 inches long, and one 8 feet 6 inches long. If the pipe weighs 9 pounds per foot, what is the total weight of the pipe used? Ans. 423 lb.

---

50. To multiply a number by two or more figures:

EXAMPLE.—Multiply 475 by 234.

```
SOLUTION.—   multiplicand      4 7 5
               multiplier      2 3 4
                             -------
                             1 9 0 0
                           1 4 2 5
                           9 5 0
                         -----------
                  product 1 1 1 1 5 0  Ans.
```

EXPLANATION.—For convenience, the multiplier is generally written under the multiplicand, placing units under units, tens under tens, etc.

We cannot multiply by 234 at one operation; we must, therefore, multiply by the *parts* and then *add* the **partial products.**

The parts by which we are to multiply are 4 units, 3 tens, and 2 hundreds. 4 times 475 = 1,900, the *first partial* *product;* 3 times 475 = 1,425, the *second partial product, the right-hand figure of which is written directly under the figure multiplied by*, or 3; 2 times 475 = 950, the *third partial product*, the right-hand figure of which is written directly under the figure multiplied by, or 2.

The sum of these three partial products is 111,150, which is the *entire product.*

51. Rule.—I. *Write the multiplier under the multiplicand, so that units are under units, tens under tens, etc.*

II. *Begin at the right and multiply each figure of the multiplicand by each successive figure of the multiplier, placing the right-hand figure of each partial product directly under the figure used as a multiplier.*

III. *The sum of the partial products will equal the required product.*

52. Proof.—*Review the work carefully, or multiply the multiplier by the multiplicand; if the results agree, the work is correct.*

53. When there is a *cipher* in the *multiplier*, multiply by it the same as with the other figures; since the result will be zero, place a cipher under the cipher in the multiplier and write the next partial product to the left of it, as shown below:

$$\begin{array}{rl} (a) & \\ 0 & \\ \times\ 0 & \\ \hline 0 & \text{Ans.} \end{array} \qquad \begin{array}{rl} (b) & \\ 2 & \\ \times\ 0 & \\ \hline 0 & \text{Ans.} \end{array} \qquad \begin{array}{rl} (c) & \\ 15 & \\ \times\ \ 0 & \\ \hline 0 & \text{Ans.} \end{array} \qquad \begin{array}{rl} (d) & \\ 708 & \\ \times\ \ \ 0 & \\ \hline 0 & \text{Ans.} \end{array}$$

$$\begin{array}{rl} (e) & \\ 3114 & \\ 203 & \\ \hline 9342 & \\ 62280 & \\ \hline 632142 & \text{Ans.} \end{array} \qquad \begin{array}{rl} (f) & \\ 4008 & \\ 305 & \\ \hline 20040 & \\ 120240 & \\ \hline 1222440 & \text{Ans.} \end{array} \qquad \begin{array}{rl} (g) & \\ 31264 & \\ 1002 & \\ \hline 62528 & \\ 3126400 & \\ \hline 31326528 & \text{Ans.} \end{array}$$

EXAMPLES FOR PRACTICE.

54. Find the product of:

(*a*)	3,842 × 26.	Ans.	(*a*)	**99,892.**
(*b*)	3,716 × 45.		(*b*)	**167,220.**
(*c*)	1,817 × 124.		(*c*)	**225,308.**
(*d*)	675 × 38.		(*d*)	**25,650.**
(*e*)	1,875 × 33.		(*e*)	**61,875.**
(*f*)	4,836 × 47.		(*f*)	**227,292.**
(*g*)	5,682 × 543.		(*g*)	**3,085,326.**
(*h*)	3,257 × 246.		(*h*)	**801,222.**
(*i*)	2,875 × 302.		(*i*)	**868,250.**
(*j*)	17,819 × 1,004.		(*j*)	**17,890,276.**
(*k*)	38,674 × 205.		(*k*)	**7,928,170.**
(*l*)	18,304 × 100.		(*l*)	**1,830,400.**
(*m*)	7,834 × 10.		(*m*)	**78,340.**
(*n*)	87,543 × 1,000.		(*n*)	**87,543,000.**
(*o*)	48,763 × 100.		(*o*)	**4,876,300.**

1. If the area of a steam-engine piston is 113 square inches, what is the total pressure upon it when the steam pressure is 85 pounds per square inch? Ans. 9,605 lb.

2. A steam engine which indicated 164 horsepower was found to consume 4 pounds of coal per horsepower per hour. Being replaced by a new engine of the same horsepower, another test was made, which showed a consumption of 3 pounds per horsepower per hour. What was the saving of coal for a year of 309 days, if the engine's average run was 14 hours a day? Ans. 709,464 lb.

3. Two steamers are 7,846 miles apart and are sailing towards each other, one at the rate of 18 miles an hour and the other at the rate of 15 miles an hour. How far apart will they be at the end of 205 hours? Ans. 1,081 mi.

DIVISION.

55. Division is the process of finding how many times one number is contained in another of the same kind.

The number to be *divided* is called the **dividend.**

The number by which we *divide* is called the **divisor.**

The number which shows how many times the divisor is contained in the dividend is called the **quotient.**

56. The sign of division is ÷. It is read **divided by.** 54 ÷ 9 is read *54 divided by 9.* Another way to write 54 divided by 9 is $\frac{54}{9}$. Thus, $54 \div 9 = 6$, or $\frac{54}{9} = 6$. The expression $\frac{54}{9}$ may be read either 54 divided by 9 or 54 *over* 9, the word "over" implying that there is a line between the two numbers indicating that the upper number is to be divided by the lower one. In both of these cases 54 is the ***dividend*** and 9 is the *divisor*.

Division is the reverse of multiplication.

57. To divide when the divisor consists of but one figure, proceed as in the following example:

EXAMPLE.—What is the quotient of $875 \div 7$?

```
                 divisor dividend quotient
SOLUTION.—            7)875(125  Ans.
                        7
                        -
                        17
                        14
                        --
                         35
                         35
                         --
            remainder     0
```

EXPLANATION.— 7 is contained in 8 hundreds, 1 hundred times. Place the 1 as the first, or left-hand, figure of the quotient. Multiply the divisor, 7, by the 1 hundred of the quotient, and place the product, 7 hundreds, under the 8 hundreds in the dividend, and subtract. Beside the remainder, 1, bring down the 7 tens, making 17 tens; 17 divided by 7 = 2 tens. Write the 2 as the second figure of the quotient. Multiply the divisor, 7, by the 2, and subtract the product from 17. Beside the remainder, 3, bring down the 5 units of the dividend, making 35 units. 7 is contained in 35, 5 times, and 5 is placed in the quotient. 5 times 7 = 35, which, subtracted from 35 under which it is placed, leaves 0. Therefore, the quotient is 125. This method is called **long division.**

58. In **short division,** only the divisor, dividend, and quotient are written, the operations being performed mentally.

$$\begin{array}{rl} & \textit{dividend} \\ \textit{divisor}\ \ 7) & \underline{8^{1}7^{3}5} \\ \textit{quotient} & 1\ 2\ 5\ \ \text{Ans.} \end{array}$$

The mental operation is as follows: 7 is contained in 8, 1 time and 1 remainder; 1 placed before 7 makes 17; 7 is contained in 17, 2 times and 3 over; the 3 placed before 5 makes 35; 7 is contained in 35, 5 times. These partial quotients, placed in order as they are found, make the entire quotient, 125.

59. If the divisor consists of *two or more* figures, proceed as in the following example:

EXAMPLE.—Divide 2,702,826 by 63.

SOLUTION.--

$$\begin{array}{rcccccccl} \textit{divisor} & \multicolumn{7}{c}{\textit{dividend}} & \textit{quotient} \\ 63) & 2 & 7 & 0 & 2 & 8 & 2 & 6 & (4\,2\,9\,0\,2\ \ \text{Ans.} \\ & 2 & 5 & 2 & & & & & \\ \cline{2-4} & & 1 & 8 & 2 & & & & \\ & & 1 & 2 & 6 & & & & \\ \cline{3-5} & & & 5 & 6 & 8 & & & \\ & & & 5 & 6 & 7 & & & \\ \cline{4-6} & & & & 1 & 2 & 6 & & \\ & & & & 1 & 2 & 6 & & \\ \cline{5-7} \end{array}$$

EXPLANATION.—As 63 is not contained in the first two figures, 27, we must use the first three figures, 270. Now, by trial we must find how many times 63 is contained in 270. 6 is contained in the first two figures of 270, 4 times. Place the 4 as the first, or left-hand, figure in the quotient. Multiply the divisor, 63, by 4, and subtract the product, 252, from 270. The remainder is 18, beside which we write the next figure of the dividend, 2, making 182. Now, 6 is contained in the first two figures of 182, 3 times, but on multiplying 63 by 3, we see that the product, 189, is too great, so we try 2 as the second figure of the quotient. Multiplying the divisor, 63, by 2 and subtracting the product, 126, from 182, the remainder is 56, beside which we bring down the next figure of the dividend, making 568. 6 is contained in 56 about 9 times. Multiply the divisor, 63, by 9 and subtract the product, 567, from 568. The remainder is 1, and bringing down the next figure of the dividend, 2, gives 12. As 12 is smaller than 63, we write 0 in the quotient and bring down the next figure, 6, making 126. 63 is contained in 126, 2 times, without a remainder. Therefore, 42,902 is the quotient.

60. Rule.—I. *Write the divisor at the left of the dividend, with a line between them.*

II. *Find how many times the divisor is contained in the lowest number of the left-hand figures of the dividend that will contain it, and write the result at the right for the first figure of the quotient.*

III. *Multiply the divisor by this quotient; write the product under the partial dividend used, and subtract, and to the remainder annex the next figure of the dividend. Divide as before, and thus continue until all the figures of the dividend have been used.*

IV. *If any partial dividend will not contain the divisor, write a cipher in the quotient, annex the next figure of the dividend, and proceed as before.*

V. *If there be at last a remainder, write it after the quotient, with the divisor underneath.*

61. Proof.—*Multiply the quotient by the divisor and add the remainder, if there be any, to the product. The result will be the dividend.* Thus,

divisor *dividend* *quotient*

$63\ )\ 4235\ (\ 67\frac{14}{63}$ Ans.

$$\begin{array}{llr}
 & & 378 \\
\cline{3-3}
 & & 455 \\
 & & 441 \\
\cline{3-3}
 & \textit{remainder} & 14 \\
\textbf{PROOF,} & \textit{quotient} & 67 \\
 & \textit{divisor} & 63 \\
\cline{3-3}
 & & 201 \\
 & & 402 \\
\cline{3-3}
 & & 4221 \\
 & \textit{remainder} & 14 \\
\cline{3-3}
 & \textit{dividend} & 4235
\end{array}$$

EXAMPLES FOR PRACTICE.

62. Divide the following:

(*a*)	126,498 by 58.	Ans.	(*a*)	2,181.
(*b*)	3,207,594 by 767.		(*b*)	4,182.
(*c*)	11,408,202 by 234.		(*c*)	48,753.
(*d*)	2,100,315 by 581.		(*d*)	3,615.
(*e*)	969,936 by 4,008.		(*e*)	242.
(*f*)	7,481,888 by 1,021.		(*f*)	7,328.
(*g*)	1,525,915 by 5,003.		(*g*)	305.
(*h*)	1,646,301 by 381.		(*h*)	4,321.

1. In a mile there are 5,280 feet. How many rails would it take to lay a double row 1 mile long, each rail being 30 feet long?
Ans. 352 rails.

2. How many rivets will be required for the longitudinal seams of a cylindrical boiler 20 feet long, the joint being double-riveted, and the rivets being spaced 4 inches apart? Ans. 120 rivets.

NOTE.—First find the length of the boiler in inches.

3. It requires 7,020,000 bricks to build a large foundry. How many teams will it require to draw the bricks in 60 days, if each team draws 6 loads per day and 1,500 bricks at a load? Ans. 13 teams.

NOTE.—Find how many loads 7,020,000 bricks make; then, how many days it will take one team to draw the bricks.

CANCELATION.

63. **Cancelation** is the process of shortening operations in division by casting out equal factors from both dividend and divisor.

64. The **factors** of a number are those numbers which, when multiplied together, will *equal that number*. Thus, 5 and 3 are the factors of 15, since 5 × 3 = 15. Likewise, 8 and 7 are the factors of 56, since 8 × 7 = 56.

65. A **prime number** is one which cannot be divided by any number except itself and 1. Thus, 2, 3, 11, 29, etc. are prime numbers.

66. A **prime factor** is any factor that is a prime number.

Any number that is not a prime is called a **composite** number, and may be produced by multiplying together its prime factors. Thus, 60 is a composite number, and is equal to the product of its prime factors, 2 × 2 × 3 × 5.

Numbers are said to be **prime to each other** when no two of them can be divided by any number except 1; the numbers themselves *may* be either prime or composite. Thus, the numbers 3, 5, and 11 are prime to one another; so also are 22, 25, and 21—all three of which are composite numbers.

67. *Canceling equal factors from both dividend and divisor does not change the quotient.*

Canceling a factor in both dividend and divisor is the same as *dividing them both by the same number*, which, by the principle of division, *does not change the quotient.*

Write the numbers forming the dividend above the line, and those forming the divisor below it.

68. EXAMPLE.—Divide 4 × 45 × 60 by 9 × 24.

SOLUTION.—Placing the dividend over the divisor, and canceling,

$$\frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{\underset{1}{\cancel{6}}}{\cancel{24}}} = \frac{50}{1} = 50. \quad \text{Ans.}$$

EXPLANATION.—The 4 in the dividend and the 24 in the divisor are both divisible by 4, since 4 divided by 4 equals 1, and 24 divided by 4 equals 6. Cross off the 4 and write the 1 over it; also cross off the 24 and write the 6 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times 45 \times 60}{9 \times \underset{6}{\cancel{24}}} =$$

60 in the dividend and 6 in the divisor are divisible by 6, since 60 divided by 6 equals 10, and 6 divided by 6 equals 1. Cross off the 60 and write 10 over it; also cross off the 6 and write 1 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times 45 \times \overset{10}{\cancel{60}}}{9 \times \underset{\underset{1}{\cancel{6}}}{\cancel{24}}} =$$

Again, 45 in the dividend and 9 in the divisor are divisible by 9, since 45 divided by 9 equals 5, and 9 divided by 9 equals 1. Cross off the 45 and write the 5 over it; also cross off the 9 and write the 1 under it. Thus,

$$\frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{\underset{1}{\cancel{6}}}{\cancel{24}}} =$$

Since there are no two remaining numbers (one in the dividend and one in the divisor) divisible by any number except 1, without a remainder, it is impossible to cancel further.

Multiply all the uncanceled numbers in the dividend together and divide their product by the product of all the uncanceled numbers in the divisor. The result will be the quotient. The product of all the uncanceled numbers in the dividend equals $5 \times 1 \times 10 = 50$; the product of all the uncanceled numbers in the divisor equals $1 \times 1 = 1$.

$$\text{Hence, } \frac{\overset{1}{\cancel{4}} \times \overset{5}{\cancel{45}} \times \overset{10}{\cancel{60}}}{\underset{1}{\cancel{9}} \times \underset{\underset{1}{\cancel{6}}}{\cancel{24}}} = \frac{1 \times 5 \times 10}{1 \times 1} = 50. \quad \text{Ans.}$$

It is usual to omit the 1's when canceling, instead of writing them as above.

69. Rule.—I. *Cancel the common factors from both the dividend and the divisor.*

II. *Then divide the product of the remaining factors of the dividend by the product of the remaining factors of the divisor, and the result will be the quotient.*

EXAMPLES FOR PRACTICE.

70. Divide:

			Ans.
(*a*)	14 × 18 × 16 × 40 by 7 × 8 × 6 × 5 × 3.	(*a*)	32.
(*b*)	3 × 65 × 50 × 100 × 60 by 30 × 60 × 13 × 10.	(*b*)	250.
(*c*)	8 × 4 × 3 × 9 × 11 by 11 × 9 × 4 × 3 × 8.	(*c*)	1.
(*d*)	164 × 321 × 6 × 7 × 4 by 82 × 321 × 7.	(*d*)	48.
(*e*)	50 × 100 × 200 × 72 by 1,000 × 144 × 100.	(*e*)	5.
(*f*)	48 × 63 × 55 × 49 by 7 × 21 × 11 × 48.	(*f*)	105.
(*g*)	110 × 150 × 84 × 32 by 11 × 15 × 100 × 64.	(*g*)	42.
(*h*)	115 × 120 × 400 × 1,000 by 23 × 1,000 × 60 × 800.	(*h*)	5.

ARITHMETIC.

(PART 2.)

FRACTIONS.

REMARK.—If a stick of wood is divided into, say, 12 equal parts, one of these parts is called a twelfth. If we take away 5 of these equal parts, we shall have left 7 parts or 7-twelfths. Since it would be very inconvenient to spell out the names of the number of parts into which an object has been (or is supposed to have been) divided, mathematicians invented, long ago, a kind of a shorthand method of expressing 7-twelfths, 25-forty-fifths, etc., viz.: they wrote the number of the equal parts taken or considered above a horizontal line, and called this number the *numerator;* then they wrote below the horizontal line the number which denoted the number of equal parts into which the object was supposed to be divided, and called it the *denominator.* Hence, instead of writing 7-twelfths, 25-forty-fifths, etc., they wrote $\frac{7}{12}$, $\frac{25}{45}$, etc.; but they read them the same as if they had been written the other way.

1. A **fraction** is *one or more equal parts of a unit. One-half, two-thirds, seven-fifths* are fractions.

2. *Two* numbers are required to express a fraction; one is called the **numerator** and the other the **denominator.**

3. The numerator is placed above the denominator with a line between them, as $\frac{2}{3}$. Here 3 is the denominator, and shows into how many *equal parts* the *unit*, or *one*, is divided. The numerator 2 shows how many of these *equal parts* are

For notice of copyright, see page immediately following the title page.

18. To reduce an improper fraction to a whole or a mixed number:

Example.—Reduce $\frac{21}{4}$ to a mixed number.

Solution.— 4 is contained in 21, 5 times and 1 remaining; as this is also divided by 4, its value is $\frac{1}{4}$. Therefore, $5 + \frac{1}{4}$, or $5\frac{1}{4}$, is the number. Ans.

19. Rule.—*Divide the numerator by the denominator, the quotient will be the whole number; the remainder, if there be any, will be the numerator of the fractional part, of which the denominator is the same as the denominator of the improper fraction.*

EXAMPLES FOR PRACTICE.

20. Reduce to whole or mixed numbers:

(*a*) $\frac{145}{6}$.		(*a*) $24\frac{1}{6}$.
(*b*) $\frac{185}{3}$.		(*b*) $61\frac{2}{3}$.
(*c*) $\frac{701}{6}$.	Ans.	(*c*) $116\frac{5}{6}$.
(*a*) $\frac{149}{3}$.		(*d*) $49\frac{2}{3}$.
(*e*) $\frac{76}{19}$.		(*e*) 4.
(*f*) $\frac{125}{25}$.		(*f*) 5.

21. A **common denominator** of two or more fractions is a number which will contain all the denominators of the fractions without a remainder. The **least common denominator** is the least number that will contain all the denominators of the fractions without a remainder.

22. To find the least common denominator:

Example.—Find the least common denominator of $\frac{1}{4}$, $\frac{1}{3}$, $\frac{1}{9}$, and $\frac{1}{16}$.

Solution.—We first place the denominators in a row, separated by commas.

$$\begin{array}{r}
2\,)\,\underline{4,\ \ 3,\ \ 9,\ 16} \\
2\,)\,\underline{2,\ \ 3,\ \ 9,\ \ \ 8} \\
3\,)\,\underline{1,\ \ 3,\ \ 9,\ \ \ 4} \\
3\,)\,\underline{1,\ \ 1,\ \ 3,\ \ \ 4} \\
4\,)\,\underline{1,\ \ 1,\ \ 1,\ \ \ 4} \\
1,\ \ 1,\ \ 1,\ \ \ 1
\end{array}$$

$2 \times 2 \times 3 \times 3 \times 4 = 144$, the least common denominator. Ans.

Explanation.—Divide each of them by some prime number which will divide at least two of them without a remainder (if possible), bringing down those denominators to the

REDUCTION OF FRACTIONS.

11. Reduction of fractions is the process of changing their forms without changing their *value*.

12. *A fraction is reduced to higher terms by multiplying both terms of the fraction by the same number.* Thus, $\frac{3}{4}$ is reduced to $\frac{6}{8}$ by multiplying both terms by 2.

$$\frac{3 \times 2}{4 \times 2} = \frac{6}{8}.$$

The *value* is not changed, for $\frac{3}{4} = \frac{6}{8}$.

13. *A fraction is reduced to lower terms by dividing both terms by the same number.* Thus, $\frac{8}{10}$ is reduced to $\frac{4}{5}$ by dividing both terms by 2.

$$\frac{8 \div 2}{10 \div 2} = \frac{4}{5}.$$

14. *A fraction is reduced to lowest terms when its numerator and denominator cannot be divided by the same number*, as $\frac{3}{4}$, $\frac{2}{3}$, $\frac{11}{17}$.

15. To reduce a whole number or a mixed number to an improper fraction:

EXAMPLE 1.—How many fourths in 5?

SOLUTION.—Since there are 4 fourths in 1 ($\frac{4}{4} = 1$), in 5 there will be 5×4 fourths, or 20 fourths; i. e., $5 \times \frac{4}{4} = \frac{20}{4}$. Ans.

EXAMPLE 2.—Reduce $8\frac{3}{4}$ to an improper fraction.

SOLUTION.— $8 \times \frac{4}{4} = \frac{32}{4}$. $\frac{32}{4} + \frac{3}{4} = \frac{35}{4}$. Ans.

16. Rule.—*Multiply the whole number by the denominator of the fraction, add the numerator to the product, and place the denominator under the result.*

EXAMPLES FOR PRACTICE.

17. Reduce to improper fractions:

(*a*)	$4\frac{1}{8}$.	Ans. (*a*)	$\frac{33}{8}$.
(*b*)	$5\frac{1}{9}$.	(*b*)	$\frac{46}{9}$.
(*c*)	$10\frac{3}{10}$.	(*c*)	$\frac{103}{10}$.
(*d*)	$37\frac{3}{4}$.	(*d*)	$\frac{151}{4}$.
(*e*)	$50\frac{4}{5}$.	(*e*)	$\frac{254}{5}$.

ADDITION OF FRACTIONS.

27. *Fractions cannot be added unless they have a common denominator.* We cannot add $\frac{3}{4}$ to $\frac{7}{8}$ as they now stand, since the denominators represent parts of different sizes. Fourths cannot be added to eighths.

The fractions should be reduced to the *least* common denominator, or the *least* number which will contain all the denominators.

28. Example.—Find the sum of $\frac{1}{2}$, $\frac{3}{4}$, and $\frac{5}{8}$.

Solution.—The *least common denominator*, or the *least number* which will contain all the denominators, is 8.

$$\tfrac{1}{2} = \tfrac{4}{8},\ \ \tfrac{3}{4} = \tfrac{6}{8},\ \text{ and }\ \tfrac{5}{8} = \tfrac{5}{8}.$$

Explanation.—As the denominator tells or indicates the *names* of the parts, the numerators only are added to obtain the total number of parts indicated by the denominator.

$$\tfrac{4}{8} + \tfrac{6}{8} + \tfrac{5}{8} = \frac{4 + 6 + 5}{8} = \frac{15}{8} = 1\tfrac{7}{8}. \quad \text{Ans.}$$

29. Example 1.—What is the sum of $12\frac{3}{4}$, $14\frac{5}{8}$, and $7\frac{5}{16}$?

Solution.—The least common denominator in this case is 16.

$$\begin{array}{l} 12\tfrac{3}{4} = 12\tfrac{12}{16} \\ 14\tfrac{5}{8} = 14\tfrac{10}{16} \\ \underline{7\tfrac{5}{16} = \ \ 7\tfrac{5}{16}} \\ \textit{sum}\ \ 33 + \tfrac{27}{16} = 33 + 1\tfrac{11}{16} = 34\tfrac{11}{16}. \quad \text{Ans.} \end{array}$$

The sum of the fractions $= \frac{27}{16}$ or $1\frac{11}{16}$, which added to the sum of the whole numbers $= 34\frac{11}{16}$.

Example 2.—What is the sum of 17, $13\frac{3}{16}$, $\frac{9}{32}$, and $3\frac{1}{4}$?

Solution.—The least common denominator is 32. $13\frac{3}{16} = 13\frac{6}{32}$, $3\frac{1}{4} = 3\frac{8}{32}$.

$$\begin{array}{rr} & 17 \\ & 13\tfrac{6}{32} \\ & \tfrac{9}{32} \\ & \underline{3\tfrac{8}{32}} \\ \textit{sum} & 33\tfrac{23}{32} \end{array} \quad \text{Ans.}$$

30. **Rule.—I.** *Reduce the given fractions to fractions having the least common denominator and write the sum of the numerators over the common denominator.*

II. ***When there are mixed numbers and whole numbers, add the fractions first, and if their sum is an improper fraction, reduce it to a mixed number and add the whole number with the other whole numbers.***

EXAMPLES FOR PRACTICE.

31. Find the sum of:

(*a*)	$\frac{4}{6}$, $\frac{7}{24}$, $\frac{5}{8}$.	Ans.	(*a*)	$1\frac{7}{12}$.
(*b*)	$\frac{3}{5}$, $\frac{6}{15}$, $\frac{24}{45}$.		(*b*)	$1\frac{8}{15}$.
(*c*)	$\frac{1}{2}$, $\frac{3}{8}$, $\frac{5}{16}$.		(*c*)	$1\frac{3}{16}$.
(*d*)	$\frac{5}{6}$, $\frac{11}{72}$, $\frac{13}{18}$.		(*d*)	$1\frac{51}{72}$.
(*e*)	$\frac{10}{11}$, $\frac{6}{33}$, $\frac{23}{66}$.		(*e*)	$1\frac{29}{66}$.
(*f*)	$\frac{23}{45}$, $\frac{11}{15}$, $\frac{14}{45}$.		(*f*)	$1\frac{5}{9}$.
(*g*)	$\frac{4}{11}$, $\frac{7}{33}$, $\frac{14}{22}$.		(*g*)	$1\frac{7}{33}$.
(*h*)	$\frac{3}{7}$, $\frac{14}{49}$, $\frac{2}{7}$.		(*h*)	1.

1. The weights of a number of castings were $412\frac{3}{4}$ pounds, $270\frac{1}{2}$ pounds, 1,020 pounds, $75\frac{1}{4}$ pounds, and $68\frac{1}{2}$ pounds. What was their total weight? Ans. 1,847 lb.

2. Four bolts are required, $2\frac{3}{4}$, $1\frac{7}{8}$, $2\frac{5}{16}$, and $1\frac{13}{32}$ inches long. How long a piece of iron will be required to cut them from, allowing $\frac{3}{4}$ of an inch altogether for cutting off and finishing the ends? Ans. $9\frac{3}{32}$ in.

SUBTRACTION OF FRACTIONS.

32. *Fractions cannot be subtracted without first reducing them to a common denominator.*

EXAMPLE.—Subtract $\frac{3}{8}$ from $\frac{13}{16}$.

SOLUTION.—The common denominator is 16.

$$\frac{3}{8} = \frac{6}{16}. \quad \frac{13}{16} - \frac{6}{16} = \frac{13 - 6}{16} = \frac{7}{16}. \quad \text{Ans.}$$

33. EXAMPLE.—From 7 take $\frac{5}{8}$.

SOLUTION.— $1 = \frac{8}{8}$; therefore, $7 = 6 + \frac{8}{8} = 6\frac{8}{8}$; $6\frac{8}{8} - \frac{5}{8} = 6\frac{3}{8}$. Ans.

34. EXAMPLE.—What is the difference between $17\frac{9}{16}$ and $9\frac{15}{32}$?

SOLUTION.—The common denominator of the fractions is 32. $17\frac{9}{16} = 17\frac{18}{32}$.

$$\begin{array}{rr} \textit{minuend} & 17\frac{18}{32} \\ \textit{subtrahend} & 9\frac{15}{32} \\ \hline \textit{difference} & 8\frac{3}{32} \end{array} \quad \text{Ans.}$$

35. [illegible]

[illegible] the fractions is 16. [illegible]

[illegible] Ans.

[illegible] the subtrahend is [illegible] minuend, it cannot be sub- [illegible] from the [illegible] the minuend [illegible] = [illegible]. Since 1 [illegible] = [illegible] — [illegible] = [illegible]

36. [illegible]

[illegible] Ans.

[illegible] the minuend [illegible] borrow 1, [illegible] was borrowed [illegible]

37. Rule. I. [illegible] *fractions having* [illegible] *from the* [illegible] *denominator.*

II. [illegible] *the fractions* [illegible] *remainders side by* [illegible]

III. [illegible] *greater than the fraction* [illegible] *whole number in the* [illegible] *minuend, from which subtract the fraction* [illegible]

IV. *When the minuend* [illegible] *borrow 1; reduce it to a fraction* [illegible] *the same as the denominator of the fraction in the* [illegible] *place it over that fraction for subtraction.*

EXAMPLES FOR PRACTICE.

38. Subtract:

(*a*)	$\frac{10}{24}$ from $\frac{11}{12}$.	Ans. (*a*)	$\frac{1}{2}$.
(*b*)	$\frac{7}{14}$ from $\frac{17}{28}$.	(*b*)	$\frac{3}{28}$.
(*c*)	$\frac{4}{30}$ from $\frac{5}{10}$.	(*c*)	$\frac{11}{30}$.
(*d*)	$\frac{15}{35}$ from $\frac{45}{70}$.	(*d*)	$\frac{3}{14}$.
(*e*)	$\frac{15}{16}$ from $\frac{57}{48}$.	(*e*)	$\frac{1}{4}$.
(*f*)	$13\frac{1}{4}$ from $30\frac{3}{4}$.	(*f*)	$17\frac{1}{2}$.
(*g*)	$12\frac{1}{8}$ from 27.	(*g*)	$14\frac{7}{8}$.
(*h*)	$5\frac{1}{4}$ from 30.	(*h*)	$24\frac{3}{4}$.

1. An engineer found that he had on hand $48\frac{1}{2}$ gallons of cylinder oil. During the following week he used $\frac{3}{4}$ of a gallon each day for three days, $\frac{7}{8}$ of a gallon on the fourth day, $\frac{15}{16}$ of a gallon on the fifth day, and $\frac{1}{2}$ of a gallon on the sixth day. How much oil remained at the end of the week? Ans. $43\frac{15}{16}$ gal.

2. The main line shaft of a manufacturing plant is run by an engine and waterwheel. A test of the plant showed that the engine was capable of developing $251\frac{4}{5}$ H. P. (horsepower), and the waterwheel, under full gate, $67\frac{1}{2}$ H. P. It was also found that the machinery consumed $210\frac{11}{16}$ H. P., and the friction of the shafting and belting was $32\frac{1}{7}$ H. P. How much power remained unused? Ans. $76\frac{263}{560}$ H. P.

MULTIPLICATION OF FRACTIONS.

39. *In multiplication of fractions it is not necessary to reduce the fractions to fractions having a common denominator.*

Multiplying the numerator or dividing the denominator multiplies the fraction.

EXAMPLE.—Multiply $\frac{3}{4}$ by 4.

SOLUTION.— $\frac{3}{4} \times 4 = \frac{3 \times 4}{4} = \frac{12}{4} = 3$. Ans.

Or, $\frac{3}{4} \times 4 = \frac{3}{4 \div 4} = \frac{3}{1} = 3$. Ans.

40. The word "of" in multiplication of fractions means the same as ×, or times. Thus,

$\frac{3}{4}$ of $4 = \frac{3}{4} \times 4 = 3$.

$\frac{1}{8}$ of $\frac{5}{16} = \frac{1}{8} \times \frac{5}{16} = \frac{5}{128}$.

EXAMPLE.—Multiply $\frac{3}{8}$ by 2.

SOLUTION.— $2 \times \frac{3}{8} = \frac{3 \times 2}{8} = \frac{6}{8} = \frac{3}{4}$. Ans.

Or, $2 \times \frac{3}{8} = \frac{3}{8 \div 2} = \frac{3}{4}$. Ans.

41. Example.—What is the product of $\frac{4}{16}$ and $\frac{7}{8}$?

Solution.— $\frac{4}{16}\times\frac{7}{8}=\frac{4\times 7}{16\times 8}=\frac{28}{128}=\frac{7}{32}$. Ans.

Or, by cancelation, $\frac{\cancel{4}\times 7}{\underset{4}{\cancel{16}}\times 8}=\frac{7}{4\times 8}=\frac{7}{32}$. Ans.

42. Example.—What is $\frac{4}{8}$ of $\frac{3}{4}$ of $\frac{16}{32}$?

Solution.— $\frac{\cancel{4}\times 3\times\cancel{16}}{8\times\cancel{4}\times\underset{2}{\cancel{32}}}=\frac{3}{8\times 2}=\frac{3}{16}$. Ans.

43. Example.—What is the product of $9\frac{3}{4}$ and $5\frac{5}{8}$?

Solution.— $9\frac{3}{4}=\frac{39}{4}$; $5\frac{5}{8}=\frac{45}{8}$.

$$\frac{39}{4}\times\frac{45}{8}=\frac{39\times 45}{4\times 8}=\frac{1755}{32}=54\frac{27}{32}. \text{ Ans.}$$

44. Example.—Multiply $15\frac{7}{8}$ by 3.

Solution.—

$$\begin{array}{r} 15\frac{7}{8} \\ 3 \\ \hline 47\frac{5}{8} \end{array} \quad \text{or} \quad \begin{array}{l} 15\frac{7}{8} \\ 3 \\ \hline 45+\frac{21}{8}=45+2\frac{5}{8}=47\frac{5}{8}. \text{ Ans.} \end{array}$$

45. Rule.—I. *Divide the product of the numerators by the product of the denominators. All factors common to the numerators and denominators should first be cast out by cancelation.*

II. *To multiply one mixed number by another, reduce them both to improper fractions.*

III. *To multiply a mixed number by a whole number, first multiply the fractional part by the multiplier, and if the product is an improper fraction, reduce it to a mixed number and add the whole number part to the product of the multiplier and the whole number.*

EXAMPLES FOR PRACTICE.

46. Find the product of:

			Ans.	
(*a*)	$7\times\frac{3}{16}$.		(*a*)	$1\frac{5}{16}$.
(*b*)	$14\times\frac{5}{16}$.		(*b*)	$4\frac{3}{8}$.
(*c*)	$\frac{21}{32}\times\frac{5}{14}$.		(*c*)	$\frac{15}{64}$.
(*d*)	$\frac{16}{27}\times 4$.		(*d*)	$2\frac{10}{27}$.
(*e*)	$\frac{19}{18}\times 7$.		(*e*)	$7\frac{7}{18}$.
(*f*)	$17\frac{18}{21}\times 7$.		(*f*)	125.
(*g*)	$\frac{105}{224}\times 32$.		(*g*)	15.
(*h*)	$\frac{15}{28}\times 14$.		(*h*)	$7\frac{1}{2}$.

1. A single belt can transmit $107\frac{3}{4}$ horsepower, but as it is desired to use more power, a double belt of the same width is substituted for it. Supposing the double belt to be capable of transmitting $\frac{10}{7}$ as much power as the single belt, how many horsepower can be used after the change? Ans. $153\frac{13}{14}$ H. P.

2. What is the weight of $2\frac{5}{8}$ miles of copper wire weighing $5\frac{3}{4}$ pounds per 100 feet? There are 5,280 feet in a mile. Ans. $796\frac{19}{20}$ lb.

3. The grate of a steam boiler contains $20\frac{1}{2}$ square feet. If the boiler burns $8\frac{3}{10}$ pounds of coal an hour per square foot of grate area and can evaporate $7\frac{1}{2}$ pounds of water an hour per pound of coal burned, how many pounds of water are evaporated by the boiler in 1 hour? Ans. $1,276\frac{1}{8}$ lb.

DIVISION OF FRACTIONS.

47. *In division of fractions it is not necessary to reduce the fractions to fractions having a common denominator.*

48. *Dividing the numerator or multiplying the denominator divides the fraction.*

EXAMPLE 1.—Divide $\frac{6}{8}$ by 3.

SOLUTION.—When *dividing the numerator*, we have

$$\frac{6}{8} \div 3 = \frac{6 \div 3}{8} = \frac{2}{8} = \frac{1}{4}. \text{ Ans.}$$

When *multiplying the denominator*, we have

$$\frac{6}{8} \div 3 = \frac{6}{8 \times 3} = \frac{6}{24} = \frac{1}{4}. \text{ Ans.}$$

EXAMPLE 2.—Divide $\frac{3}{16}$ by 2.

SOLUTION.— $\frac{3}{16} \div 2 = \frac{3}{16 \times 2} = \frac{3}{32}$. Ans.

EXAMPLE 3.—Divide $\frac{14}{32}$ by 7.

SOLUTION.— $\frac{14}{32} \div 7 = \frac{14 \div 7}{32} = \frac{2}{32} = \frac{1}{16}$. Ans.

49. To **invert** a fraction is to *turn it upside down;* that is, make the numerator and denominator *change places.*

Invert $\frac{3}{4}$ and it becomes $\frac{4}{3}$.

50. EXAMPLE.—Divide $\frac{9}{16}$ by $\frac{3}{16}$.

SOLUTION.—1. The fraction $\frac{3}{16}$ is contained in $\frac{9}{16}$, 3 times, for the denominators are the same, and one numerator is contained in the other 3 times. 2. If we now invert the divisor $\frac{3}{16}$ and multiply, the solution is

$$\frac{9}{16} \times \frac{16}{3} = \frac{\overset{3}{\cancel{9}} \times \cancel{16}}{\cancel{16} \times \cancel{3}} = 3. \text{ Ans.}$$

This brings the same quotient as in the first case.

51. Example.—Divide $\frac{3}{8}$ by $\frac{1}{4}$.

Solution.—We cannot divide $\frac{3}{8}$ by $\frac{1}{4}$, as in the first case above, for the denominators are not the same; therefore, we must solve as in the second case.

$$\frac{3}{8}\div\frac{1}{4}=\frac{3}{8}\times\frac{4}{1}=\frac{3\times\cancel{4}}{\underset{2}{\cancel{8}}\times 1}=\frac{3}{2}=1\tfrac{1}{2}.\quad\text{Ans.}$$

52. Example 1.—Divide 5 by $\frac{10}{16}$.

Solution.— $\frac{10}{16}$ inverted becomes $\frac{16}{10}$.

$$5\times\frac{16}{10}=\frac{\cancel{5}\times\overset{8}{\cancel{16}}}{\underset{2}{\cancel{10}}}=8.\quad\text{Ans.}$$

Example 2.—How many times is $3\frac{3}{4}$ contained in $7\frac{7}{16}$?

Solution.— $3\frac{3}{4}=\frac{15}{4}$; $7\frac{7}{16}=\frac{119}{16}$.

$\frac{15}{4}$ inverted equals $\frac{4}{15}$.

$$\frac{119}{16}\times\frac{4}{15}=\frac{119\times\cancel{4}}{\underset{4}{\cancel{16}}\times 15}=\frac{119}{60}=1\tfrac{59}{60}.\quad\text{Ans.}$$

53. **Rule.**—*Invert the divisor and proceed as in multiplication.*

54. We have learned that a line placed between two numbers indicates that the number above the line is to be divided by the number below it. Thus, $\frac{18}{3}$ shows that 18 is to be divided by 3. This is also true if a fraction or a fractional expression be placed above or below a line.

$\dfrac{9}{\frac{3}{8}}$ means that 9 is to be divided by $\frac{3}{8}$; $\dfrac{3\times 7}{\frac{8+4}{16}}$ means that 3 × 7 is to be divided by the value of $\frac{8+4}{16}$.

$\dfrac{\frac{1}{4}}{\frac{3}{8}}$ is the same as $\frac{1}{4}\div\frac{3}{8}$.

It will be noticed that there is a heavy line between the 9 and the $\frac{3}{8}$. This is necessary, since otherwise there would be nothing to show as to whether 9 was to be divided by $\frac{3}{8}$ or $\frac{9}{3}$ was to be divided by 8. Whenever a heavy line is used, as shown here, it indicates that *all above the line is to be divided by all below it.*

55. Whenever an expression like one of the three following ones is obtained, it may always be simplified by transposing the denominator from *above* to *below* the line, or from *below* to *above*, as the case may be, taking care, however, to indicate that the denominator when so transferred is a multiplier.

1. $\frac{\frac{3}{4}}{9} = \frac{3}{9 \times 4} = \frac{3}{36} = \frac{1}{12}$; for, regarding the fraction above the heavy line as the numerator of a fraction whose denominator is 9, $\frac{\frac{3}{4} \times 4}{9 \times 4} = \frac{3}{9 \times 4}$, as before.

2. $\frac{9}{\frac{3}{4}} = \frac{9 \times 4}{3} = 12$. The proof is the same as in the first case.

3. $\frac{\frac{5}{9}}{\frac{3}{4}} = \frac{5 \times 4}{3 \times 9} = \frac{20}{27}$; for, regarding $\frac{5}{9}$ as the numerator of a fraction whose denominator is $\frac{3}{4}$, $\frac{\frac{5}{9} \times 9}{\frac{3}{4} \times 9} = \frac{5}{\frac{3 \times 9}{4}}$; and $\frac{\frac{5}{\frac{3 \times 9}{4}} \times 4}{\frac{3 \times 9}{4} \times 4} = \frac{5 \times 4}{3 \times 9} = \frac{20}{27}$, as above.

This principle may be used to great advantage in cases like $\frac{\frac{1}{4} \times 310 \times \frac{27}{12} \times 72}{40 \times 4\frac{1}{2} \times 5\frac{1}{6}}$. Reducing the mixed numbers to fractions, the expression becomes $\frac{\frac{1}{4} \times 310 \times \frac{27}{12} \times 72}{40 \times \frac{9}{2} \times \frac{31}{6}}$. Now transferring the denominators of the fractions and canceling,

$$\frac{1 \times 310 \times 27 \times 72 \times 2 \times 6}{40 \times 9 \times 31 \times 4 \times 12} = \frac{1 \times \overset{\cancel{10}}{\cancel{310}} \times \overset{3}{\cancel{27}} \times \overset{\overset{3}{\cancel{6}}}{\cancel{72}} \times \cancel{2} \times \overset{3}{\cancel{6}}}{\underset{\underset{\cancel{2}}{\cancel{4}}}{\cancel{40}} \times \cancel{9} \times \cancel{31} \times \underset{2}{\cancel{4}} \times \cancel{12}}$$

$= \frac{27}{2} = 13\frac{1}{2}$.

Greater exactness in results can usually be obtained by using this principle than can be obtained by reducing the fractions to decimals. The principle, however, should not be employed *if a sign of addition or subtraction occurs either above or below the dividing line.*

EXAMPLES FOR PRACTICE.

56. Divide:

(*a*)	15 by $6\frac{3}{7}$.	Ans. (*a*)	$2\frac{1}{3}$.
(*b*)	30 by $\frac{3}{4}$.	(*b*)	40.
(*c*)	172 by $\frac{4}{5}$.	(*c*)	215.
(*d*)	$\frac{14}{18}$ by $1\frac{7}{16}$.	(*d*)	$\frac{112}{207}$.
(*e*)	$\frac{103}{6}$ by $14\frac{2}{3}$.	(*e*)	$1\frac{15}{88}$.
(*f*)	$\frac{142}{27}$ by $17\frac{1}{9}$.	(*f*)	$\frac{71}{231}$.
(*g*)	$\frac{14}{18}$ by $\frac{145}{72}$.	(*g*)	$\frac{56}{145}$.
(*h*)	$\frac{128}{18}$ by $72\frac{1}{3}$.	(*h*)	$\frac{64}{651}$.

1. A $\frac{3}{8}$-inch boiler plate containing 24 square feet of surface weighs $362\frac{4}{10}$ pounds. What is its weight per square foot? Ans. $15\frac{1}{10}$ lb.

2. A certain boiler has $927\frac{1}{2}$ square feet of heating surface, which is equal to 35 times the area of the grate. What is the area of the grate in square feet? Ans. $26\frac{1}{2}$ sq. ft.

3. If the distance around the rim of a locomotive driving wheel is $13\frac{1}{12}$ feet, how many revolutions will the wheel make in traveling 682 feet? Ans. $52\frac{20}{157}$ rev.

ARITHMETIC.

(PART 3.)

DECIMALS.

1. Decimals are *tenth* fractions; that is, the parts of a unit are expressed on the scale of ten, as *tenths*, *hundredths*, *thousandths*, etc.

2. The *denominator*, which is always 10, 100, 1,000, etc., is not expressed, as it would be in fractions, by writing it under the *numerator* with a line between them, as $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$, but is expressed by placing a *period* (.), which is called a **decimal point,** to the *left* of the *figures of the numerator*, to indicate that the figures on the right form the numerator of a fraction whose denominator is *ten*, *one hundred*, *one thousand*, etc.

3. The *reading* of a decimal number depends upon the number of decimal places in it, i. e., the number of figures to the right of the decimal point.

One decimal place expresses *tenths.*
Two decimal places express *hundredths.*
Three decimal places express *thousandths.*
Four decimal places express *ten-thousandths.*
Five decimal places express *hundred-thousandths.*
Six decimal places express *millionths.*

For notice of copyright, see page immediately following the title page.

and in decimals the *left-hand* side must be in line, which brings the decimal points directly under one another.

	whole numbers		*decimals*		*mixed numbers*	
	342		.342		342.032	
	4234		.4234		4234.5	
	26		.26		26.6782	
	3		.03		3.06	
sum	4605	Ans. *sum*	1.0554	Ans. *sum*	4606.2702	Ans.

8. EXAMPLE.—What is the sum of 242, .36, 118.725, 1.005, 6, and 100.1?

SOLUTION.—		
	242.	
	.36	
	118.725	
	1.005	
	6.	
	100.1	
sum	468.190	Ans.

9. Rule.—*Place the numbers to be added so that the decimal points will be directly under one another. Add as in whole numbers, and place the decimal point in the sum directly under the decimal points above.*

EXAMPLES FOR PRACTICE.

10. Find the sum of:

(*a*)	.2143, .105, 2.3042, and 1.1417.	Ans.	(*a*)	3.7652.
(*b*)	783.5, 21.473, .2101, and .7816.		(*b*)	805.9647.
(*c*)	21.781, 138.72, 41.8738, .72, and 1.413.		(*c*)	204.5078.
(*d*)	.3724, 104.15, 21.417, and 100.042.		(*d*)	225.9814.
(*e*)	200.172, 14.105, 12.1465, .705, and 7.2.		(*e*)	234.3285.
(*f*)	1,427.16, .244, .32, .032, and 10.0041.		(*f*)	1,437.7601.
(*g*)	2,473.1, 41.65, .7243, 104.067, and 21.073.		(*g*)	2,640.6143.
(*h*)	4,107.2, .00375, 21.716, 410.072, and .0345.		(*h*)	4,539.02625.

1. The estimated weights of the parts of a return-tubular boiler were as follows: shell, 3,626 pounds; tubes, 3,564.5 pounds; manhole cover, ring, and yoke, 270.34 pounds; stays, etc., 1,089.4 pounds; steam nozzles, 236.07 pounds; handhole covers and yokes, 120.25 pounds; feedpipe, 34.75 pounds; boiler supports, 350.6 pounds. What was the total estimated weight of the boiler? Ans. **9,291.91 lb.**

2. A bill for engine-room supplies had the following items: 1 waste can, $8.30; 20 feet of 4-inch belting, $11.20; 1 pipe wrench, $1.65; 12 pounds of waste, $0.84; 5 gallons of cylinder oil, $8.75; 20 gallons of machine oil, $24. How much did the bill amount to? Ans. **$54.74.**

SUBTRACTION OF DECIMALS.

11. For the same reason as in addition of decimals, the left-hand figures of decimal numbers are placed in line and the decimal points under each other.

EXAMPLE.—Subtract .132 from .3063.

SOLUTION.—
minuend .3 0 6 3
subtrahend .1 3 2
difference .1 7 4 3 Ans.

12. EXAMPLE.—What is the difference between 7.895 and .725?

SOLUTION.—
minuend 7.8 9 5
subtrahend .7 2 5
difference 7.1 7 0, or 7.17 Ans.

13. EXAMPLE.—Subtract .625 from 11.

SOLUTION.—
minuend 1 1.0 0 0
subtrahend .6 2 5
difference 1 0.3 7 5 Ans.

14. **Rule.**—*Place the subtrahend under the minuend, so that the decimal points will be directly under each other. Subtract as in whole numbers, and place the decimal point in the remainder directly under the decimal points above.*

When the figures in the decimal part of the subtrahend extend beyond those in the minuend, place ciphers in the minuend above them and subtract as before.

EXAMPLES FOR PRACTICE.

15. From:

			Ans.
(*a*)	407.385 take 235.0004.	(*a*)	172.3846.
(*b*)	22.718 take 1.7042.	(*b*)	21.0138.
(*c*)	1,368.17 take 13.6817.	(*c*)	1,354.4883.
(*d*)	70.00017 take 7.000017.	(*d*)	63.000153.
(*e*)	630.630 take .6304.	(*e*)	629.9996.
(*f*)	421.73 take 217.162.	(*f*)	204.568.
(*g*)	1.000014 take .00001.	(*g*)	1.000004.
(*h*)	.783652 take .542314.	(*h*)	.241338.

1. If the temperature of steam at 5 pounds pressure is 227.964 degrees and at 100 pounds pressure is 337.874 degrees, how many degrees hotter is the steam at the higher pressure?

Ans. 109.91 degrees.

2. The outside diameter of 2¼-inch wrought-iron pipe is 2.87 inches and the inside diameter is 2.46 inches. How thick is the pipe?

Ans. .41 ÷ 2 = .205 in.

3. In a cistern that will hold 326.5 barrels of water there are 178.625 barrels? How much does it lack of being full?

Ans. 147.875 bbl.

4. A wrought-iron rod is 2.53 inches in diameter. What must be the thickness of metal turned off, so that the rod will be 2.495 inches in diameter?

Ans. .035 ÷ 2 = .0175 in.

MULTIPLICATION OF DECIMALS.

16. In multiplication of decimals, we do not place the decimal points directly under each other as in addition and subtraction. We pay no attention for the time being to the decimal points. Place the multiplier under the multiplicand, so that the *right-hand* figure of the one is under the *right-hand* figure of the other, and proceed exactly as in multiplication of whole numbers. After multiplying, *count the number of decimal places in both multiplicand and multiplier, and point off the same number in the product.*

EXAMPLE.—Multiply .825 by 13.

SOLUTION.—

$$\begin{array}{rr} \textit{multiplicand} & .825 \\ \textit{multiplier} & 13 \\ \hline & 2475 \\ & 825 \\ \hline \textit{product} & 10.725 \end{array} \text{ Ans.}$$

In this example there are 3 decimal places in the multiplicand and none in the multiplier; therefore, 3 decimal places are pointed off in the product.

17. EXAMPLE.—What is the product of 426 and the decimal .005?

SOLUTION.—

$$\begin{array}{rr} \textit{multiplicand} & 426 \\ \textit{multiplier} & .005 \\ \hline \textit{product} & 2.130 \end{array} \text{, or 2.13 Ans.}$$

In this example there are 3 decimal places in the multiplier and none in the multiplicand; therefore, 3 decimal places are pointed off in the product.

18. It is not necessary to multiply by the ciphers on the *left* of a decimal; they merely *determine the number of decimal places.* Ciphers to the *right* of a decimal should be omitted, as they only make more figures to deal with and do not change the value.

19. EXAMPLE.—Multiply 1.205 by 1.15.

SOLUTION.—

$$\begin{array}{rr} \textit{multiplicand} & 1.205 \\ \textit{multiplier} & 1.15 \\ \cline{2-2} & 6025 \\ & 1205 \\ & 1205 \\ \cline{2-2} \textit{product} & 1.38575 \end{array}\ \text{Ans.}$$

In this example there are 3 decimal places in the multiplicand and 2 in the multiplier; therefore, 3 + 2, or 5, decimal places must be pointed off in the product.

20. EXAMPLE.—Multiply .232 by .001.

SOLUTION.—

$$\begin{array}{rr} \textit{multiplicand} & .232 \\ \textit{multiplier} & .001 \\ \cline{2-2} \textit{product} & .000232 \end{array}\ \text{Ans.}$$

In this example we multiply the multiplicand by the digit in the multiplier, which gives 232 for the product; but since there are 3 decimal places each in the multiplier and multiplicand, we must prefix 3 ciphers to the 232 to make 3 + 3, or 6, decimal places in the product.

21. Rule.—*Place the multiplier under the multiplicand, disregarding the position of the decimal points. Multiply as in whole numbers, and in the product point off as many decimal places as there are decimal places in both multiplier and multiplicand, prefixing ciphers if necessary.*

EXAMPLES FOR PRACTICE.

22. Find the product of:

(*a*)	.000492 × 4.1418.	Ans. (*a*)	.0020377656.
(*b*)	4,003.2 × 1.2.	(*b*)	4,803.84.
(*c*)	78.6531 × 1.03.	(*c*)	81.012693.
(*d*)	.3685 × .042.	(*d*)	.015477.
(*e*)	178,352 × .01.	(*e*)	1,783.52.
(*f*)	.00045 × .0045.	(*f*)	.000002025.
(*g*)	.714 × .00002.	(*g*)	.00001428.
(*h*)	.00004 × .008.	(*h*)	.00000032.

1. The stroke of an engine was found by measurement to be 2.987 feet. How many feet will the crosshead pass over in 600 revolutions? Ans. 3,584.4 ft.

2. If a steam pump delivers 2.39 gallons of water per stroke and runs at 51 strokes a minute, how many gallons of water would it pump in 58½ minutes? Ans. 7,130.565 gal.

3. Wishing to obtain the weight of a connecting-rod from a drawing, it was calculated that the rod contained 294.8 cubic inches of wrought iron, 63.5 cubic inches of brass, and 10.4 cubic inches of Babbitt. Assuming the weight of wrought iron to be .278 pound per cubic inch, of brass .303 pound, and of Babbitt .264 pound, what was the weight of the rod? Ans. 103.94 lb.

DIVISION OF DECIMALS.

23. In division of decimals we pay no attention to the decimal point until after the division is performed. The *number of decimal places in the dividend must equal (or be made to equal by annexing ciphers) the number of decimal places in the divisor. Divide exactly as in whole numbers. Subtract the number of decimal places in the divisor from the number of decimal places in the dividend, and point off as many decimal places in the quotient as are indicated by the remainder.*

EXAMPLE.—Divide .625 by 25.

```
                divisor dividend quotient
SOLUTION.—          2 5 ) .6 2 5 ( .0 2 5  Ans.
                           5 0
                           ———
                           1 2 5
                           1 2 5
                           —————
                remainder      0
```

In this example there are no decimal places in the divisor and 3 decimal places in the dividend; therefore, there are 3 minus 0, or 3, decimal places in the quotient. One cipher has to be prefixed to the 25 to make the 3 decimal places.

24. EXAMPLE.—Divide 6.035 by .05.

```
                    divisor dividend quotient
SOLUTION.—            .0 5 ) 6.0 3 5 ( 1 2 0.7   Ans.
                             5
                             -
                             1 0
                             1 0
                             ---
                                 3 5
                                 3 5
                                 ---
                  remainder        0
```

In this example we divide by 5, as if the cipher were not before it. There is 1 more decimal place in the dividend than in the divisor; therefore, 1 decimal place is pointed off in the quotient.

25. EXAMPLE.—Divide .125 by .005.

```
                       divisor dividend quotient
SOLUTION.—              .0 0 5 ) .1 2 5 ( 2 5   Ans.
                                  1 0
                                  ---
                                    2 5
                                    2 5 .
                                    ---
                     remainder        0
```

In this example there are the same number of decimal places in the dividend as in the divisor; therefore, the quotient has no decimal places and is a whole number.

26. EXAMPLE.—Divide 326 by .25.

```
                    divisor dividend quotient
SOLUTION.—           .2 5 ) 3 2 6.0 0 ( 1 3 0 4   Ans.
                            2 5
                            ---
                              7 6
                              7 5
                              ---
                                1 0 0
                                1 0 0
                                -----
                 remainder          0
```

In this problem two ciphers were annexed to the dividend, to make the number of decimal places equal to the number in the divisor. The quotient is a whole number.

27. EXAMPLE.—Divide .0025 by 1.25.

divisor *dividend* *quotient*
SOLUTION.— 1.25).00250(.002 Ans.
250
remainder 0

EXPLANATION.—In this example we are to divide .0025 by 1.25. Consider the dividend as a whole number, i. e., as 25 (disregarding the two ciphers at its left, for the present); also, consider the divisor as a whole number, i. e., as 125. It is clearly evident that the dividend, 25, will not contain the divisor, 125; we must, therefore, annex one cipher to the 25, thus making the dividend 250. 125 is contained twice in 250, so we place the figure 2 in the quotient. In pointing off the decimal places in the quotient, it must be remembered that there were only 4 decimal places in the dividend; but one cipher was annexed, thereby making 4 + 1, or 5, decimal places. Since there are 5 decimal places in the dividend and 2 decimal places in the divisor, we must point off 5 − 2, or 3, decimal places in the quotient. In order to point off 3 decimal places, two ciphers must be prefixed to the figure 2, thereby making .002 the quotient. It is not necessary to consider the ciphers at the left of a decimal when dividing, except when determining the position of the decimal point in the quotient.

28. Rule.—**I.** *Place the divisor to the left of the dividend and proceed as in division of whole numbers; in the quotient, point off as many decimal places as the number of decimal places in the dividend exceeds the number of decimal places in the divisor, prefixing ciphers to the quotient, if necessary.*

II. *If in dividing one whole number by another there be a remainder, the remainder can be placed over the divisor as a fractional part of the quotient; but it is generally better to annex ciphers to the remainder and continue dividing until there are three or four decimal places in the quotient, and*

then if there still be a remainder, terminate the quotient by the plus sign (+), which shows that it can be carried farther.

29. Example.—What is the quotient of 199 divided by 15?

divisor dividend quotient

Solution.— 15) 199 (13 + $\frac{4}{15}$ Ans.

15

49

45

remainder 4

Or, 15) 199.000 (13.266+ Ans.

15

49

45

40

30

100

90

100

90

remainder 10

$13\frac{4}{15} = 13.266+$

$\frac{4}{15} = .266+$

It very frequently happens, as in the above example, that the division will never terminate. In such cases, decide to how many decimal places the division is to be carried and carry the work one place farther. If the last figure of the quotient thus obtained is 5 or a greater number, increase the preceding figure by 1, and write after it the minus sign (−), thus indicating that the quotient is not quite as large as indicated; if the figure thus obtained is less than 5, write the plus sign (+) after the quotient, thus indicating that the number is slightly greater than as indicated. In the last example, had it been desired to obtain the answer correct to four decimal places, the work would have been carried to five places, obtaining 13.26666, and the answer would have been given as 13.2667−. This remark applies to any other calculation involving decimals, when it is desired to omit some of the figures in the decimal. Thus, if it is desired to retain three decimal places in the number .2471253, it would be expressed as .247+; if it was desired to retain five decimal places, it would be expressed as .24713−. Both the + and − signs are frequently omitted; they are seldom used outside of arithmetic, except in exact calculations, when it is desired to call particular attention to the fact that the result obtained is not quite exact.

EXAMPLES FOR PRACTICE.

30. Divide:

		Ans.
(*a*)	101.6688 by 2.36.	(*a*) 43.08.
(*b*)	187.12264 by 123.107.	(*b*) 1.52.
(*c*)	.08 by .008.	(*c*) 10.
(*d*)	.0003 by 3.75.	(*d*) .00008.
(*e*)	.0144 by .024.	(*e*) .6.
(*f*)	.00375 by 1.25.	(*f*) .003.
(*g*)	.004 by 400.	(*g*) .00001.
(*h*)	.4 by .008.	(*h*) 50.

1. In a steam-engine test of an hour's duration, the horsepower developed was found to be as follows at 10-minute intervals: 25.73, 25.64, 26.13, 25.08, 24.20, 26.7, 26.34. What was the average horsepower? Ans. 25.6886−, average.

NOTE.—Add the different horsepowers together and divide by the number of tests, or 7.

2. There are 31.5 gallons in a barrel. How many barrels are there in 2,787.75 gallons? Ans. 88.5 bbl.

3. A carload of 18.75 tons of coal cost $60.75. How much was it worth per ton? Ans. $3.24 per ton.

4. A keg of $\frac{9}{16}''$ × $1\frac{5}{8}''$ boiler rivets weighs 100 pounds and contains 595 rivets. What is the weight of one of the rivets? Ans. .168+ lb.

TO REDUCE A FRACTION TO A DECIMAL.

31. EXAMPLE 1.— $\frac{3}{4}$ equals what decimal?

SOLUTION.— $4\,)\,\underline{3.0\,0}$

.7 5, or $\frac{3}{4}$ = .75 Ans.

Example 2.—What decimal is equivalent to $\frac{7}{8}$?

Solution.—

```
8 ) 7.0 0 0 ( .8 7 5
    6 4
    ---
      6 0
      5 6      or 7/8 = .875.  Ans.
      ---
        4 0
        4 0
        ---
```

32. Rule.—*Annex ciphers to the numerator and divide by the denominator. Point off as many decimal places in the quotient as there are ciphers annexed.*

EXAMPLES FOR PRACTICE.

33. Reduce the following common fractions to decimals:

(*a*)	$\frac{15}{32}$.	Ans.	(*a*)	.46875.
(*b*)	$\frac{7}{8}$.		(*b*)	.875.
(*c*)	$\frac{21}{32}$.		(*c*)	.65625.
(*d*)	$\frac{51}{64}$.		(*d*)	.796875.
(*e*)	$\frac{4}{25}$.		(*e*)	.16.
(*f*)	$\frac{5}{8}$.		(*f*)	.625.
(*g*)	$\frac{10}{200}$.		(*g*)	.05.
(*h*)	$\frac{4}{1000}$.		(*h*)	.004.

---

34. To reduce inches to decimal parts of a foot:

Example.—What decimal part of a foot is 9 inches?

Solution.—Since there are 12 inches in 1 foot, 1 inch is $\frac{1}{12}$ of a foot and 9 inches is $9 \times \frac{1}{12}$, or $\frac{9}{12}$ of a foot. This reduced to a decimal by the above rule shows what decimal part of a foot 9 inches is.

```
1 2 ) 9.0 0 ( .7 5 of a foot.  Ans.
      8 4
      ---
        6 0
        6 0
        ---
```

35. Rule.—**I.** *To reduce inches to a decimal part of a foot, divide the number of inches by 12.*

II. *Should the resulting decimal be an unending one, and it is desired to terminate the division at some point, say the fourth decimal place, carry the division one place farther, and if the fifth figure is 5 or greater, increase the fourth figure by 1, omitting the signs + and –.*

EXAMPLES FOR PRACTICE.

36. Reduce to the decimal part of a foot:

	Ans.
(*a*) 3 in.	(*a*) .25 ft.
(*b*) 4½ in.	(*b*) .375 ft.
(*c*) 5 in.	(*c*) .4167 ft.
(*d*) 6⅝ in.	(*d*) .5521 ft.
(*e*) 11 in.	(*e*) .9167 ft.

1. The lengths of belting required to connect three countershafts with the main line shaft were found with the tape measure to be 27 feet 4 inches, 23 feet 8 inches, and 38 feet 6 inches. How many feet of belting were necessary? Ans. 89.5 ft.

2. The stroke of an engine is 14 inches. What is the length of the crank in feet measured from center of shaft to center of crankpin, knowing that length of crank is one-half the stroke? Ans. .5833+ ft.

3. A steam pipe fitted with an expansion joint was found to expand 1.668 inches when steam was admitted into it. How much was its expansion in decimal parts of a foot? Ans. .139 ft.

TO REDUCE A DECIMAL TO A FRACTION.

37. Example 1.—Reduce .125 to a fraction.

Solution.— $.125 = \frac{125}{1000} = \frac{5}{40} = \frac{1}{8}$. Ans.

Example 2.—Reduce .875 to a fraction.

Solution.— $.875 = \frac{875}{1000} = \frac{35}{40} = \frac{7}{8}$. Ans.

38. Rule.—*Under the figures of the decimal, place 1 with as many ciphers at its right as there are decimal places in the decimal, and reduce the resulting fraction to its lowest terms by dividing both numerator and denominator by the same number.*

EXAMPLES FOR PRACTICE.

39. Reduce the following to common fractions:

		Ans.
(*a*)	.375.	(*a*) $\frac{3}{8}$.
(*b*)	.625.	(*b*) $\frac{5}{8}$.
(*c*)	.3125.	(*c*) $\frac{5}{16}$.
(*d*)	.04.	(*d*) $\frac{1}{25}$.
(*e*)	.06.	(*e*) $\frac{3}{50}$.
(*f*)	.75.	(*f*) $\frac{3}{4}$.
(*g*)	.15625.	(*g*) $\frac{5}{32}$.
(*h*)	.875.	(*h*) $\frac{7}{8}$.

---

40. To express a decimal approximately as a fraction having a given denominator:

Example 1.—Express .5827 in 64ths.

Solution.— $.5827 \times \frac{64}{64} = \frac{37.2928}{64}$, say $\frac{37}{64}$.

Hence, $.5827 = \frac{37}{64}$, nearly. Ans.

Example 2.—Express .3917 in 12ths.

Solution.— $.3917 \times \frac{12}{12} = \frac{4.7004}{12}$, say $\frac{5}{12}$.

Hence, $.3917 = \frac{5}{12}$, nearly. Ans.

41. Rule.—*Reduce 1 to a fraction having the given denominator. Multiply the given decimal by the fraction so obtained, and the result will be the fraction required.*

EXAMPLES FOR PRACTICE.

42. Express:

		Ans.
(*a*)	.625 in 8ths.	(*a*) $\frac{5}{8}$.
(*b*)	.3125 in 16ths.	(*b*) $\frac{5}{16}$.
(*c*)	.15625 in 32ds.	(*c*) $\frac{5}{32}$.
(*d*)	.77 in 64ths.	(*d*) $\frac{49}{64}$.
(*e*)	.81 in 48ths.	(*e*) $\frac{39}{48}$.
(*f*)	.923 in 96ths.	(*f*) $\frac{89}{96}$.

---

43. The sign for dollars is $. It is read *dollars.* $25 is read *25 dollars.*

Since there are 100 cents in a dollar, 1 cent is 1 one-hundredth of a dollar; the first two figures of a decimal part of a dollar represent *cents*. Since a mill is $\frac{1}{10}$ of a cent, or $\frac{1}{1000}$ of a dollar, the third figure represents *mills*.

Thus, $25.16 is read *twenty-five dollars and sixteen cents;* $25.168 is read *twenty-five dollars sixteen cents and eight mills.*

SIGNS OF AGGREGATION.

44. The **vinculum** ——, **parenthesis** ( ), **brackets** [ ], and **brace** { } are called **symbols of aggregation,** and are used to include numbers which are to be considered together; thus, $13 \times \overline{8 - 3}$, or $13 \times (8 - 3)$, shows the 3 is to be taken from 8 before multiplying by 13.

$$13 \times \overline{8 - 3} = 13 \times 5 = 65. \quad \text{Ans.}$$
$$13 \times (8 - 3) = 13 \times 5 = 65. \quad \text{Ans.}$$

When the vinculum or parenthesis is not used, we have

$$13 \times 8 - 3 = 104 - 3 = 101. \quad \text{Ans.}$$

45. In any series of numbers connected by the signs $+$, $-$, $\times$, and $\div$, the operations indicated by the signs must be performed in order from left to right, *except* that no addition or subtraction may be performed if a sign of multiplication or division *follows* the number on the *right* of a sign of addition or subtraction until the indicated multiplication or division has been performed. In all cases the sign of multiplication takes the precedence, the reason being that when two or more numbers or expressions are connected by the sign of multiplication, the numbers thus connected are regarded as factors of the product indicated, and not as separate numbers.

EXAMPLE.—What is the value of $4 \times 24 - 8 + 17$?

SOLUTION.—Performing the operations in order from left to right, $4 \times 24 = 96$; $96 - 8 = 88$; $88 + 17 = 105$. Ans.

46. EXAMPLE.—What is the value of the following expression: $1{,}296 \div 12 + 160 - 22 \times 3\frac{1}{2}$?

SOLUTION.— $1{,}296 \div 12 = 108$; $108 + 160 = 268$; here we cannot subtract 22 from 268 because the sign of multiplication follows 22; hence, multiplying 22 by $3\frac{1}{2}$, we get 77, and $268 - 77 = 191$. Ans.

Had the above expression been written $1{,}296 \div 12 + 160 - 22 \times 3\frac{1}{2} \div 7 + 25$, it would have been necessary to have divided $22 \times 3\frac{1}{2}$ by 7 before subtracting, and the final result would have been $22 \times 3\frac{1}{2} = 77$; $77 \div 7 = 11$; $268 - 11 = 257$; $257 + 25 = 282$. Ans. In other words, it is necessary to perform *all* the multiplication or division included between the signs + and −, or − and +, before adding or subtracting. Also, had the expression been written $1{,}296 \div 12 + 160 - 24\frac{1}{2} \div 7 \times 3\frac{1}{2} + 25$, it would have been necessary to have multiplied $3\frac{1}{2}$ by 7 before dividing $24\frac{1}{2}$, since the sign of multiplication takes the precedence, and the final result would have been $3\frac{1}{2} \times 7 = 24\frac{1}{2}$; $24\frac{1}{2} \div 24\frac{1}{2} = 1$; $268 - 1 = 267$; $267 + 25 = 292$. Ans.

It likewise follows that if a succession of multiplication and division signs occurs, the indicated operations must not be performed in order, from left to right—the multiplication must be performed first. Thus, $24 \times 3 \div 4 \times 2 \div 9 \times 5 = \frac{1}{5}$. Ans. In order to obtain the same result that would be obtained by performing the indicated operations in order, from left to right, symbols of aggregation must be used. Thus, by using two vinculums, the last expression becomes $24 \times \overline{3 \div 4} \times \overline{2 \div 9} \times 5 = 20$, the same result that would be obtained by performing the indicated operations in order, from left to right.

EXAMPLES FOR PRACTICE.

47. Find the values of the following expressions:

(*a*) $(8 + 5 - 1) \div 4$.
(*b*) $5 \times 24 - 32$.
(*c*) $5 \times 24 \div 15$.
(*d*) $144 - 5 \times 24$.
(*e*) $(1{,}691 - 540 + 559) \div 3 \times 57$.
(*f*) $2{,}080 + 120 - 80 \times 4 - 1{,}670$.
(*g*) $(\overline{90 + 60} \div 25) \times 5 - 29$.
(*h*) $\overline{90 + 60} \div 25 \times 5$.

Ans.
(*a*) 3.
(*b*) 88.
(*c*) 8.
(*d*) 24.
(*e*) 10.
(*f*) 210.
(*g*) 1.
(*h*) 1.2.

ARITHMETIC.

(PART 4.)

PERCENTAGE.

DEFINITIONS AND PRINCIPLES.

1. In certain operations, particularly those pertaining to business, it is very convenient to regard the quantity on which we are to operate as being divided into 100 equal parts; thus, instead of using the ordinary fractions $\frac{1}{4}$, $\frac{3}{5}$, $\frac{2}{7}$, we use the equivalent fractions $\frac{25}{100}$, $\frac{60}{100}$, $\frac{28\frac{4}{7}}{100}$, or their equivalent decimals, .25, .60, .28$\frac{4}{7}$. This practice is a very convenient one in all computations involving United States money, because, since $1 equals 100 cents, it is easier to comprehend what part of the whole $\frac{35}{100}$ is than some other equivalent fraction, as $\frac{49}{140}$; it is also much easier to compute with fractions whose denominators are 100 than it is to compute with fractions whose denominators are composed of other figures.

2. Percentage is a term applied to those arithmetical operations in which the number or quantity to be operated upon is supposed to be divided into 100 equal parts.

3. The term **per cent.** means *by the hundred.* Thus, 8 per cent. of a number means 8 hundredths; i. e., $\frac{8}{100}$, or .08, of that number; 8 per cent. of 250 is $250 \times \frac{8}{100}$, or $250 \times .08 = 20$; 47 per cent. of 75 tons is $75 \times \frac{47}{100} = 75 \times .47 = 35.25$ tons. The statement that the population of a city has increased 22 per cent. in a given time, say from

For notice of copyright, see page immediately following the title page.

1880 to 1890, is equivalent to saying that the increase is 22 in every hundred, that is, for every 100 in 1880 there are 22 more, or 122, in 1890.

4. The **sign** of per cent. is %, and is read per cent. Thus, 6% is read *six per cent.*; 12½% is read *twelve and one-half per cent.*, etc.

5. When expressing the per cent. of a number to use in calculations, it is customary to express it decimally instead of fractionally. Thus, instead of expressing 6%, 25%, and 43% as $\frac{6}{100}$, $\frac{25}{100}$, and $\frac{43}{100}$, it is usual to express them as .06, .25, and .43.

6. The following table will show how any per cent. can be expressed either as a decimal or as a fraction:

1%	.01	$\frac{1}{100}$	¼%	.0025	$\frac{\frac{1}{4}}{100}$ or $\frac{1}{400}$
2%	.02	$\frac{2}{100}$ or $\frac{1}{50}$	½%	.005	$\frac{\frac{1}{2}}{100}$ or $\frac{1}{200}$
5%	.05	$\frac{5}{100}$ or $\frac{1}{20}$	1¼%	.0125	$\frac{1\frac{1}{4}}{100}$ or $\frac{1}{80}$
10%	.10	$\frac{10}{100}$ or $\frac{1}{10}$	6¼%	.06¼	$\frac{6\frac{1}{4}}{100}$ or $\frac{1}{16}$
25%	.25	$\frac{25}{100}$ or $\frac{1}{4}$	8⅓%	.08⅓	$\frac{8\frac{1}{3}}{100}$ or $\frac{1}{12}$
50%	.50	$\frac{50}{100}$ or $\frac{1}{2}$	12½%	.125	$\frac{12\frac{1}{2}}{100}$ or $\frac{1}{8}$
75%	.75	$\frac{75}{100}$ or $\frac{3}{4}$	16⅔%	.16⅔	$\frac{16\frac{2}{3}}{100}$ or $\frac{1}{6}$
100%	1.00	$\frac{100}{100}$ or 1	33⅓%	.33⅓	$\frac{33\frac{1}{3}}{100}$ or $\frac{1}{3}$
125%	1.25	$\frac{125}{100}$ or 1¼	37½%	.37½	$\frac{37\frac{1}{2}}{100}$ or $\frac{3}{8}$
150%	1.50	$\frac{150}{100}$ or 1½	62½%	.625	$\frac{62\frac{1}{2}}{100}$ or $\frac{5}{8}$
300%	3.00	$\frac{300}{100}$ or 3	87½%	.875	$\frac{87\frac{1}{2}}{100}$ or $\frac{7}{8}$

7. The names of the different terms used in percentage are: the *base*, the *rate* or *rate per cent.*, the *percentage*, the *amount*, and the *difference*.

8. The **base** is the number or quantity which is supposed to be divided into 100 equal parts.

9. The **rate per cent.** is that number of the 100 equal parts into which the base is supposed to be divided which is taken or considered. The **rate** is the number of hundredths of the base that is taken or considered. The distinction between the rate per cent. and the rate is this: The *rate per cent.* is always 100 times the *rate*. Thus, 7% of 125 and .07 of 125 amount in the end to the same thing: the former, 7, is the *rate per cent.*, the *number* of hundredths of 125 intended; the latter, .07, is the *rate*, the *part* of 125 that is to be found; 7% is used in *speech*, .07 is the form used in *computation*. So, also, 12½% = .125, ½% = .005, 1¾% = .0175. In the table just given, the numbers in the first column are rates per cent.; those in the second column are rates.

10. The **percentage** is the result obtained by multiplying the base by the rate. Thus, 7% of 125 = 125 × .07 = 8.75, the percentage.

11. The **amount** is the sum of the base and the percentage.

12. The **difference** is the remainder obtained when the percentage is subtracted from the base.

13. The terms amount and difference are ordinarily used when there is an increase or a decrease in the base. For example, suppose the population of a village is 1,500 and it increases 25 per cent. This means that for every 100 of the original 1,500 there is an increase of 25, or a total increase of 15 × 25 = 375. This increase added to the original population gives the amount, or the population after the increase. If the population had decreased 375, the final population would have been 1,500 − 375 = 1,125, and this would be the **difference.** The original population, 1,500, is the base on which the percentage is computed; the 25 is the rate per cent., and the increase or decrease, 375, is the percentage. If the base increases, the final value is the amount; and if it decreases, its final value is the difference.

CALCULATIONS INVOLVING PERCENTAGE.

14. From the foregoing it is evident that to find the percentage, the base must be multiplied by the rate. Hence, the following

Rule.—*To find the percentage, multiply the base by the rate.*

EXAMPLE.—Out of a lot of 300 boiler tubes 76% was used in a boiler. How many tubes were used ?

SOLUTION.—The rate is .76; the base is 300; hence, the number of tubes used, or the percentage, is by the above rule

$$300 \times .76 = 228 \text{ tubes. Ans.}$$

Expressing the rule as a

Formula, *percentage* = *base* × *rate.*

15. When the percentage and rate are given, the base may be found by dividing the percentage by the rate. For, suppose that 12 is 6%, or $\frac{6}{100}$, of some number; then 1%, or $\frac{1}{100}$, of the number, is 12 ÷ 6, or 2. Consequently, if 2 = 1%, or $\frac{1}{100}$, 100%, or $\frac{100}{100} = 2 \times 100 = 200$. But as the same result may be arrived at by dividing 12 by .06, since 12 ÷ .06 = 200, it follows that:

Rule.—*When the percentage and rate are given, to find the base, divide the percentage by the rate.*

Formula, *base* = *percentage* ÷ *rate.*

Example.— 76% of a lot of boiler tubes was used in the construction of a boiler. If the number of tubes used was 228, how many tubes were in the lot?

Solution.—Here 228 is the percentage and .76 is the rate; hence, applying the rule,

$$228 \div .76 = 300 \text{ tubes. Ans.}$$

16. When the base and percentage are given, to find the rate, the rate may be found by dividing the percentage by the base. For, suppose that it is desired to find what per cent. 12 is of 200. 1% of 200 is 200 × .01 = 2. Now, if 1% is 2, 12 is evidently as many per cent. as 2 is contained times in 12, or 12 ÷ 2 = 6%. But the same result may be obtained by dividing 12, the percentage, by 200, the base, since 12 ÷ 200 = .06 = 6%. Hence,

Rule.—*When the percentage and base are given, to find the rate, divide the percentage by the base, and the result will be the rate.*

Formula, *rate = percentage ÷ base.*

Example 1.—Out of a lot of 300 boiler tubes 228 were used. What per cent. of the total number was used?

Solution.—Here 300 is the base and 228 is the percentage; hence, applying rule,

$$\text{Rate} = 228 \div 300 = .76 = 76\%. \quad \text{Ans.}$$

Example 2.—What per cent. of 875 is 25?

Solution.—Here 875 is the base and 25 is the percentage; hence, applying rule,

$$\text{Rate} = 25 \div 875 = .02\tfrac{6}{7} = 2\tfrac{6}{7}\%. \quad \text{Ans.}$$

Proof.— $875 \times .02\tfrac{6}{7} = 25.$

EXAMPLES FOR PRACTICE.

17. What per cent of:

			Ans.
(*a*)	360 is 90?	(*a*)	25%.
(*b*)	900 is 360?	(*b*)	40%.
(*c*)	125 is 25?	(*c*)	20%.
(*d*)	150 is 750?	(*d*)	500%.
(*e*)	280 is 112?	(*e*)	40%.
(*f*)	400 is 200?	(*f*)	50%.
(*g*)	47 is 94?	(*g*)	200%.
(*h*)	500 is 250?	(*h*)	50%.

18. The amount may be found, when the base and rate are given, by multiplying the base by 1 plus the rate expressed decimally. For, suppose that it is desired to find the amount when 200 is the base and .06 is the rate. The percentage is 200 × .06 = 12, and, according to definition, Art. **11**, the amount is 200 + 12 = 212. But the same result may be obtained by multiplying 200 by 1 + .06, or 1.06, since 200 × 1.06 = 212. Hence,

Rule.—*When the base and rate are given, to find the amount, multiply the base by 1 plus the rate.*

Formula, *amount* = *base* × (1 + *rate*).

EXAMPLE.—If a man earned $725 in a year, and the next year 10% more, how much did he earn the second year?

SOLUTION.—Here 725 is the base and .10 is the rate, and the amount is required. Hence, applying the rule,

725 × 1.10 = $797.50. Ans.

19. When the base and rate are given, the difference may be found by multiplying the base by 1 minus the rate expressed decimally. For, suppose that it is desired to find the difference when the base is 200 and the rate is 6%. The percentage is 200 × .06 = 12; and, according to definition, Art. **12**, the difference = 200 − 12 = 188. But the same result may be obtained by multiplying 200 by 1 − .06, or .94, since 200 × .94 = 188. Hence,

Rule.—*When the base and rate are given, to find the difference, multiply the base by 1 minus the rate.*

Formula, *difference* = *base* × (1 − *rate*).

EXAMPLE.—Out of a lot of 300 boiler tubes all but 24% were used in one boiler; how many tubes were used?

SOLUTION.—Here 300 is the base, .24 is the rate, and it is desired to find the difference. Hence, applying the rule,

300 × (1 − .24) = 228 tubes. Ans.

20. When the amount and rate are given, the base may be found by dividing the amount by 1 plus the rate. For, suppose that it is known that 212 equals some number increased by 6% of itself. Then, it is evident that 212 equals 106% of the number (base) that it is desired to find. Consequently, if $212 = 106\%$, $1\% = \frac{212}{106} = 2$, and $100\% = 2 \times 100 = 200 =$ the base. But the same result may be obtained by dividing 212 by $1 + .06$ or 1.06, since $212 \div 1.06 = 200$. Hence,

Rule.—*When the amount and rate are given, to find the base, divide the amount by 1 plus the rate.*

Formula, *base* = *amount* ÷ (1 + *rate*).

EXAMPLE.—The theoretical discharge of a certain pump when running at a piston speed of 100 feet per minute is 278,910 gallons per day of 10 hours. Owing to leakage and other defects, this value is 25% greater than the actual discharge. What is the actual discharge ?

SOLUTION.—Here 278,910 equals the actual discharge (base) increased by 25% of itself. Consequently, 278,910 is the amount and 25% is the rate. Applying rule,

Actual discharge = 278,910 ÷ 1.25 = 223,128 gal. Ans.

21. When the difference and rate are given, the base may be found by dividing the difference by 1 minus the rate. For, suppose that 188 equals some number less 6% of itself. Then, 188 evidently equals $100 - 6 = 94\%$ of some number. Consequently, if $188 = 94\%$, $1\% = 188 \div 94 = 2$, and $100\% = 2 \times 100 = 200$. But the same result may be obtained by dividing 188 by $1 - .06$, or .94, since $188 \div .94 = 200$. Hence,

Rule.—*When the difference and rate are given, to find the base, divide the difference by 1 minus the rate.*

Formula, *base* = *difference* ÷ (1 − *rate*).

EXAMPLE 1.—From a lot of boiler tubes 76% was used in the construction of a boiler. If there were 72 tubes unused, how many tubes were in the lot ?

SOLUTION.—Here 72 is the difference and .76 is the rate. Applying rule,

72 ÷ (1 − .76) = 300 tubes. Ans.

EXAMPLE 2.—The theoretical number of foot-pounds of work per minute required to operate a boiler feed-pump is 127,344. If 30% of the total number actually required be allowed for friction, leakage, etc., how many foot-pounds are actually required to work the pump?

SOLUTION.—Here the number actually required is the base; hence, 127,344 is the difference and .30 is the rate. Applying the rule,

$$127{,}344 \div (1 - .30) = 181{,}920 \text{ foot-pounds. Ans.}$$

22. EXAMPLE.—A certain chimney gives a draft of 2.76 inches of water. By increasing the height 20 feet, the draft was increased to 3 inches of water. What was the gain per cent.?

SOLUTION.—Here it is evident that 3 inches is the amount and that 2.76 inches is the base. Consequently, 3 − 2.76 = .24 inch is the percentage, and it is required to find the rate. Hence, applying the rule given in Art. **16**,

$$\text{Gain per cent.} = .24 \div 2.76 = .087 = 8.7\%. \text{ Ans.}$$

23. EXAMPLE.—A certain chimney gave a draft of 3 inches of water. After an economizer had been put in, the draft was reduced to 1.2 inches of water. What was the loss per cent.?

SOLUTION.—Here it is evident that 1.2 inches is the difference (since it equals 3 inches diminished by a certain per cent. loss of itself) and 3 inches is the base. Consequently, 3 − 1.2 = 1.8 inches is the percentage. Hence, applying the rule given in Art. **16**,

$$\text{Loss per cent.} = 1.8 \div 3 = .60 = 60\%. \text{ Ans.}$$

24. To find the gain or loss per cent.:

Rule.—*Find the difference between the initial and the final value; divide this difference by the initial value.*

EXAMPLE.—If a man buys a steam engine for $1,860 and some time afterwards purchases a condenser for 25% of the cost of the engine, does he gain or lose, and how much per cent., if he sells both engine and condenser for $2,100?

SOLUTION.—The cost of the condenser was $1,860 × .25 = $465; consequently, the initial value, or cost, was $1,860 + $465 = $2,325. Since he sold them for $2,100, he lost $2,325 − $2,100 = $225. Hence, applying rule,

$$225 \div 2{,}325 = .0968 = 9.68\% \text{ loss. Ans.}$$

EXAMPLES FOR PRACTICE.

25. Solve the following:

(*a*)	What is $12\frac{1}{2}\%$ of \$900?	Ans.	(*a*)	\$112.50.
(*b*)	What is $\frac{4}{5}\%$ of 627?		(*b*)	5.016.
(*c*)	What is $33\frac{1}{3}\%$ of 54?		(*c*)	18.
(*d*)	101 is $68\frac{3}{4}\%$ of what number?		(*d*)	$146\frac{10}{11}$.
(*e*)	784 is $83\frac{1}{3}\%$ of what number?		(*e*)	940.8.
(*f*)	What % of 960 is 160?		(*f*)	$16\frac{2}{3}\%$.
(*g*)	What % of \$3,606 is $\$450\frac{3}{4}$?		(*g*)	$12\frac{1}{2}\%$.
(*h*)	What % of 280 is 112?		(*h*)	40%.

1. A steam plant consumed an average of 3,640 pounds of coal per day. The engineer made certain alterations which resulted in a saving of 250 pounds per day. What was the per cent. of coal saved? Ans. 7%, nearly.

2. If the speed of an engine running at 126 revolutions per minute should be increased $6\frac{1}{2}\%$, how many revolutions per minute would it then make? Ans. 134.19 rev.

3. The list price of an engine was \$1,400; of a boiler, \$1,150; and of the necessary fittings for the two, \$340. If 25% discount was allowed on the engine, 22% on the boiler, and $12\frac{1}{2}\%$ on the fittings, what was the actual cost of the plant? Ans. \$2,244.50.

4. If I lend a man \$1,100, and this is $18\frac{1}{2}\%$ of the amount that I have on interest, how much money have I on interest? Ans. \$5,945.95.

5. A test showed that an engine developed 190.4 horsepower, 15% of which was consumed in friction. How much power was available for use? Ans. 161.84 H. P.

6. By adding a condenser to a steam engine, the power was increased 14% and the consumption of coal per horsepower per hour was decreased 20%. If the engine could originally develop 50 horsepower and required $3\frac{1}{2}$ pounds of coal per horsepower per hour, what would be the total weight of coal used in an hour with the condenser, assuming the engine to run full power? Ans. 159.6 lb.

DENOMINATE NUMBERS.

26. A **denominate number** is a concrete number, and may be either simple or compound; as, 8 quarts; 5 feet 10 inches, etc. Denominate numbers are also called **compound numbers.**

27. A **simple denominate number** consists of units of but one denomination; as, 16 cents; 10 hours; 5 dollars, etc.

28. A **compound denominate number** consists of units of two or more denominations of a similar kind; as, 3 yards 2 feet 1 inch.

29. In *whole numbers* and in *decimals*, the law of increase and decrease is on the scale of 10, but in *compound*, or *denominate*, *numbers* the scale varies.

MEASURES.

30. A **measure** is a *standard unit*, established by *law* or *custom*, by which quantity of any kind is measured. The standard unit of **dry measure** is the Winchester bushel; of **weight**, the pound; of **liquid measure**, the gallon, etc.

31. Measures are of six kinds:

1. Extension.
2. Weight.
3. Capacity.
4. Time.
5. Angles.
6. Money or value.

MEASURES OF EXTENSION.

32. Measures of extension are used in measuring lengths, distances, surfaces, and solids.

LINEAR MEASURE.

TABLE.

		Abbreviation.
12 inches (in.)	= 1 foot	ft.
3 feet	= 1 yard	yd.
5.5 yards	= 1 rod	rd.
40 rods	= 1 furlong	fur.
8 furlongs	= 1 mile	mi.

in.	ft.	yd.	rd.	fur.	mi.
36 =	3 =	1			
198 =	16½ =	5.5 =	1		
7,920 =	660 =	220 =	40 =	1	
63,360 =	5,280 =	1,760 =	320 =	8 =	1

SQUARE MEASURE.

TABLE.

144 square inches (sq. in.)	= 1 square foot	sq. ft.
9 square feet	= 1 square yard	sq. yd.
30¼ square yards	= 1 square rod	sq. rd.
160 square rods	= 1 acre	A.
640 acres	= 1 square mile	sq. mi.

sq. mi.	A.	sq. rd.	sq. yd.	sq. ft.	sq. in.
1 =	640 =	102,400 =	3,097,600 =	27,878,400 =	4,014,489,600

CUBIC MEASURE.

TABLE.

1,728 cubic inches (cu. in.)	= 1 cubic foot	cu. ft.
27 cubic feet	= 1 cubic yard	cu. yd.
128 cubic feet	= 1 cord	cd.
24¾ cubic feet	= 1 perch	P.

cu. yd. cu. ft. cu. in.
1 = 27 = 46,656

MEASURES OF WEIGHT.

AVOIRDUPOIS WEIGHT.

TABLE.

16 ounces (oz.)	= 1 pound	lb.
100 pounds	= 1 hundredweight	cwt.
20 cwt., or 2,000 lb.	= 1 ton	T.

T. cwt. lb. oz.
1 = 20 = 2,000 = 32,000

33. The ounce is divided into halves, quarters, etc. Avoirdupois weight is used for weighing coarse and heavy articles.

LONG TON TABLE.

16 ounces	= 1 pound	lb.
112 pounds	= 1 hundredweight	cwt.
20 cwt., or 2,240 lb.	= 1 ton	T.

34. In all the calculations hereafter, 2,000 pounds will be considered 1 ton, unless the long ton (2,240 pounds) is especially mentioned.

TROY WEIGHT.

TABLE.

24 grains (gr.)	= 1 pennyweight	pwt.
20 pennyweights	= 1 ounce	oz.
12 ounces	= 1 pound	lb.

lb. oz. pwt. gr.
1 = 12 = 240 = 5,760

Troy weight is used in weighing gold and silver ware, jewels, etc. It is used by jewelers.

MEASURES OF CAPACITY.

LIQUID MEASURE.

TABLE.

4 gills (gi.)	=	1 pint	pt.
2 pints	=	1 quart	qt.
4 quarts	=	1 gallon	gal.
31½ gallons	=	1 barrel	bbl.
2 barrels, or 63 gallons	=	1 hogshead	hhd.

hhd.		bbl.		gal.		qt.		pt.		gi.
1	=	2	=	63	=	252	=	504	=	2,016

DRY MEASURE.

TABLE.

2 pints (pt.)	=	1 quart	qt.
8 quarts	=	1 peck	pk.
4 pecks	=	1 bushel	bu.

bu.		pk.		qt.		pt.
1	=	4	=	32	=	64

MEASURE OF TIME.

TABLE.

60 seconds (sec.)	=	1 minute	min.
60 minutes	=	1 hour	hr.
24 hours	=	1 day	da.
7 days	=	1 week	wk.
365 days 12 months	=	1 common year	yr.
366 days	=	1 leap year.	
100 years	=	1 century.	

NOTE.—It is customary to consider one month as 30 days.

MEASURE OF ANGLES OR ARCS.

TABLE.

60 seconds (″)	=	1 minute	′.
60 minutes	=	1 degree	°.
90 degrees	=	1 right angle or quadrant	∟.
360 degrees	=	1 circle	cir.

1 cir. = 360° = 21,600′ = 1,296,000″

MEASURE OF MONEY.

UNITED STATES MONEY.

TABLE.

1[illegible] mills m. = 1 cent [illegible]
1[illegible] cents = 1 dime [illegible]
1[illegible] dimes = 1 dollar $.
1[illegible] dollars = 1 eagle E.

E. $ [illegible] [illegible] [illegible]
1 = 10 = 100 = 1 000 = 1[illegible]00

MISCELLANEOUS TABLE.

12 things are 1 dozen
12 dozen are 1 gross
12 gross are 1 great gross
2 things are 1 pair
20 things are 1 score
1 league is 3 miles
1 fathom is 6 feet
1 meter is 39.37 inches
1 hand is [illegible]
1 palm is [illegible]
1 span is [illegible]
24 sheets are 1 quire
20 quires [illegible]
1 [illegible]
1 U. S. standard gallon also called a wine gallon [illegible]
1 U. S. standard gallon of water weighs [illegible]
1 cubic foot contains 7.48 U. S. standard gallons [illegible]
1 British imperial gallon of water weighs [illegible]

It will be of great advantage to the student [illegible] memorize all the above tables.

REDUCTION OF DENOMINATE NUMBERS.

35. Reduction [illegible] changing their denomination [illegible] They may be changed from a higher [illegible] tion or from a lower to a higher— [illegible]

2 hours = [illegible]
32 [illegible] = [illegible]

36. Principle.—[illegible] lower denominations by [illegible] nations by *dividing*.

To reduce denominate numbers to lower denominations:

37. Example.—Reduce 5 yd. 2 ft. 7 in. to inches.

Solution.—

$$\begin{array}{rlcc}
\text{yd.} & & \text{ft.} & \text{in.} \\
5 & & 2 & 7 \\
3 & & & \\
\hline
15 & \text{ft.} & & \\
2 & \text{ft.} & & \\
\hline
17 & \text{ft.} & & \\
12 & & & \\
\hline
34 & & & \\
17 & & & \\
\hline
204 & \text{in.} & & \\
7 & \text{in.} & & \\
\hline
211 & \text{in.} & \text{Ans.} &
\end{array}$$

Explanation.—Since there are 3 feet in 1 yard, in 5 yards there are 5 × 3, or 15, feet, and 15 feet + 2 feet = 17 feet. There are 12 inches in a foot; therefore, 12 × 17 = 204 inches, and 204 inches + 7 inches = 211 inches in 5 yards 2 feet 7 inches.

38. Example.—Reduce 6 hours to seconds.

Solution.—

$$\begin{array}{rl}
6 & \text{hr.} \\
60 & \\
\hline
360 & \text{min.} \\
60 & \\
\hline
21600 & \text{sec.} \quad \text{Ans.}
\end{array}$$

Explanation.—As there are 60 minutes in 1 hour, in 6 hours there are 6 × 60, or 360, minutes; as there are no minutes to add, we multiply 360 minutes by 60, to get the number of seconds.

39. In order to avoid mistakes, if any denomination be omitted, represent it by a cipher. Thus, before reducing 3 rods 6 inches to inches, insert a cipher for yards and a cipher for feet, as

rd.	yd.	ft.	in.
3	0	0	6

40. Rule.—*Multiply the number representing the highest denomination by the number of units in the next lower required* *to make one of the higher denomination, and to the product add the number of given units of that lower denomination. Proceed in this manner until the number is reduced to the required denomination.*

EXAMPLES FOR PRACTICE.

41. Reduce:

		Ans.	
(*a*)	4 rd. 2 yd. 2 ft. to feet.	(*a*)	74 ft.
(*b*)	4 bu. 3 pk. 2 qt. to quarts.	(*b*)	154 qt.
(*c*)	13 rd. 5 yd. 2 ft. to feet.	(*c*)	231.5 ft.
(*d*)	5 mi. 100 rd. 10 ft. to feet.	(*d*)	28,060 ft.
(*e*)	52 hhd. 24 gal. 1 pt. to pints.	(*e*)	26,401 pt.
(*f*)	5 cir. 16° 20' to minutes.	(*f*)	108,980'.
(*g*)	14 bu. to quarts.	(*g*)	448 qt.

To reduce lower to higher denominations:

42. **Example.**—Reduce 211 inches to higher denominations:

Solution.—

```
12)211 in.
    3)17 ft. + 7 in.
       5 yd. + 2 ft. + 7 in.  Ans.
```

Explanation.—There are 12 inches in 1 foot; therefore, 211 divided by 12 = 17 feet and 7 inches over. There are 3 feet in 1 yard; therefore, 17 feet divided by 3 = 5 yards and 2 feet over. The last quotient and the two remainders constitute the answer, 5 yards 2 feet 7 inches.

43. **Example.**—Reduce 14,135 gills to higher denominations.

Solution.—

```
4)14135
2) 3533 pt. 3 gi.
4) 1766 qt. 1 pt.
    441 gal. 2 qt.

31.5)441.0(14 bbl.
     315
     1260
     1260
```

Explanation.—There are 4 gills in 1 pint, and in 14,135 gills there are as many pints as 4 is contained times in 14,135, or 3,533 pints and 3 gills remaining. There are 2 pints in 1 quart, and in 3,533 pints there are 1,766 quarts and 1 pint remaining. There are 4 quarts in 1 gallon, and in 1,766 quarts there are 441 gallons and 2 quarts remaining. There are 31½ gallons in 1 barrel, and in 441 gallons there are 14 barrels.

The last quotient and the three remainders constitute the answer, 14 barrels 2 quarts 1 pint 3 gills.

44. **Rule.**—*Divide the number representing the denomination given by the number of units of this denomination required to make one unit of the next higher denomination. The remainder will be of the same denomination, but the quotient will be of the next higher. Divide this quotient by the number of units of its denomination required to make one unit of the next higher. Continue until the highest denomination is reached or until there is not enough of a denomination left to make one of the next higher. The last quotient and the remainders constitute the required result.*

EXAMPLES FOR PRACTICE.

45. Reduce to units of higher denominations:

(*a*) 7,460 sq. in.; (*b*) 7,580 sq. yd.; (*c*) 148,760 cu. in.; (*d*) 17,651″; (*e*) 8,000 gi.; (*f*) 36,450 lb.

Ans.
- (*a*) 5 sq. yd. 6 sq. ft. 116 sq. in.
- (*b*) 1 A. 90 sq. rd. 17 sq. yd. 4 sq. ft. 72 sq. in.
- (*c*) 3 cu. yd. 5 cu. ft. 152 cu. in.
- (*d*) 4° 54′ 11″.
- (*e*) 3 hhd. 61 gal.
- (*f*) 18 T. 4 cwt. 50 lb.

ADDITION OF DENOMINATE NUMBERS.

46. As in the case of abstract numbers, denominate numbers may be added, subtracted, multiplied, and divided.

47. Example.—Find the sum of 3 cwt. 46 lb. 12 oz.; 8 cwt. 12 lb. 13 oz.; 12 cwt. 50 lb. 13 oz.; 27 lb. 4 oz.

Solution.—

T.	cwt.	lb.	oz.	
0	3	46	12	
0	8	12	13	
0	12	50	13	
0	0	27	4	
1	4	37	10	Ans.

Explanation.—Begin to add at the right-hand column; 4 + 13 + 13 + 12 = 42 ounces; as 16 ounces make 1 pound, 42 ounces ÷ 16 = 2 pounds and a remainder of 10 ounces, or 2 pounds and 10 ounces. Place 10 ounces under the ounce column and add 2 pounds to the next, or pound, column. Then, 2 + 27 + 50 + 12 + 46 = 137 pounds; as 100 pounds make a hundredweight, 137 ÷ 100 = 1 hundredweight and a remainder of 37 pounds. Place the 37 under the pound column and add 1 hundredweight to the next, or hundredweight, column. Next, 1 + 12 + 8 + 3 = 24 hundredweight. 20 hundredweight make a ton; therefore, 24 ÷ 20 = 1 ton and 4 hundredweight remaining. Hence, the sum is 1 ton 4 hundredweight 37 pounds 10 ounces.

48. **Example.**—What is the sum of 2 rd. 3 yd. 2 ft. 5 in.; 6 rd. 1 ft. 10 in.; 17 rd. 11 in.; 4 yd. 1 ft.?

Solution.—

	rd.	yd.	ft.	in.	
	2	3	2	5	
	6	0	1	10	
	17	0	0	11	
	0	4	1	0	
	26	3½	0	2	
or	26	3	1	8	Ans.

Explanation.—The sum of the numbers in the first column = 26 inches, or 2 feet and 2 inches remaining. The sum of the numbers in the next column plus 2 feet = 6 feet, or 2 yards and 0 feet remaining. The sum of the numbers in the next column plus 2 yards = 9 yards, or 9 ÷ 5½ = 1 rod and 3½ yards remaining. The sum of the next column plus 1 rod = 26 rods. To avoid fractions in the sum, the ½ yard is reduced to 1 foot and 6 inches, which added to 26 rods 3 yards 0 feet and 2 inches = 26 rods 3 yards 1 foot 8 inches.

49. **Example.**—What is the sum of 47 ft. and 3 rd. 2 yd. 2 ft. 10 in.?

Solution.—When 47 ft. is reduced, it equals 2 rd. 4 yd. 2 ft., which can be added to 3 rd. 2 yd. 2 ft. 10 in. Thus,

	rd.	yd.	ft.	in.	
	3	2	2	10	
	2	4	2	0	
	6	1½	1	10	
or	6	2	0	4	Ans.

50. Rule.—*Place the numbers so that like denominations are under one another. Begin at the right-hand column and add. Divide the sum by the number of units of this denomination required to make one unit of the next higher. Place the remainder under the column added and carry the quotient to the next column. Continue in this manner until the highest denomination given is reached.*

EXAMPLES FOR PRACTICE.

51. What is the sum of:

(*a*) 25 lb. 7 oz. 15 pwt. 23 gr.; 17 lb. 16 pwt.; 15 lb. 4 oz. 12 pwt.; 18 lb. 16 gr.; 10 lb. 2 oz. 11 pwt. 16 gr.?

(*b*) 9 mi. 13 rd. 4 yd. 2 ft.; 16 rd. 5 yd. 1 ft. 5 in.; 16 mi. 2 rd. 3 in.; 14 rd. 1 yd. 9 in.?

(*c*) 3 cwt. 46 lb. 12 oz.; 12 cwt. $9\frac{1}{2}$ lb.; $2\frac{1}{4}$ cwt. $21\frac{5}{8}$ lb.?

(*d*) 10 yr. 9 mo. 1 wk. 3 da.; 42 yr. 6 mo. 7 da.; 7 yr. 9 mo. 2 wk. 4 da.; 17 yr. 17 da.?

(*e*) 17 T. 11 cwt. 49 lb. 14 oz.; 16 T. 47 lb. 13 oz.; 20 T. 13 cwt. 14 lb. 6 oz.; 11 T. 4 cwt. 16 lb. 12 oz.?

(*f*) 14 sq. yd. 8 sq. ft. 19 sq. in.; 105 sq. yd. 16 sq. ft. 240 sq. in.; 42 sq. yd. 28 sq. ft. 165 sq. in.?

Ans.
(*a*) 86 lb. 3 oz. 16 pwt. 7 gr.
(*b*) 25 mi. 47 rd. 1 ft. 5 in.
(*c*) 18 cwt. 2 lb. 14 oz.
(*d*) 78 yr. 1 mo. 3 wk. 3 da.
(*e*) 65 T. 9 cwt. 28 lb. 13 oz.
(*f*) 167 sq. yd. 136 sq. in.

SUBTRACTION OF DENOMINATE NUMBERS.

52. Example.—From 21 rd. 2 yd. 2 ft. $6\frac{1}{2}$ in. take 9 rd. 4 yd. $10\frac{1}{4}$ in.

Solution.—

rd.	yd.	ft.	in.	
21	2	2	$6\frac{1}{2}$	
9	4	0	$10\frac{1}{4}$	
11	$3\frac{1}{2}$	1	$8\frac{1}{4}$	Ans.

Explanation.—Since $10\frac{1}{4}$ inches cannot be taken from $6\frac{1}{2}$ inches, we must borrow 1 foot, or 12 inches, from the 2 feet in the next column and add it to the $6\frac{1}{2}$. $6\frac{1}{2} + 12 = 18\frac{1}{2}$. $18\frac{1}{2}$ inches $-$ $10\frac{1}{4}$ inches $=$ $8\frac{1}{4}$ inches. Then, 0 foot from the 1 remaining foot = 1 foot. 4 yards cannot be taken from 2 yards; therefore, we borrow 1 rod, or 5½ yards, from 21 rods and add it to 2. 2 + 5½ = 7½; 7½ − 4 = 3½ yards. 9 rods from 20 rods = 11 rods. Hence, the remainder is 11 rods 3½ yards 1 foot 8¼ inches.

To avoid fractions as much as possible, we reduce the ½ yard to inches, obtaining 18 inches; this added to 8¼ inches gives 26¼ inches, which equals 2 feet 2¼ inches. Then, 2 feet + 1 foot = 3 feet = 1 yard, and 3 yards + 1 yard = 4 yards. Hence, the above answer becomes 11 rods 4 yards 0 feet 2¼ inches.

53. Example.—What is the difference between 3 rd. 2 yd. 2 ft. 10 in. and 47 ft.?

Solution.— 47 ft. = 2 rd. 4 yd. 2 ft.

	rd.	yd.	ft.	in.	
	3	2	2	10	
	2	4	2	0	
	0	3½	0	10	
or		3	2	4	Ans.

To find (approximately) the interval of time between two dates:

54. Example.—How many years, months, days, and hours between 4 o'clock P. M. of June 15, 1868, and 10 o'clock A. M., September 28, 1891?

Solution.—

yr.	mo.	da.	hr.	
1891	9	28	10	
1868	6	15	16	
23	3	12	18	Ans.

Explanation.—Counting 24 hours in 1 day, 4 o'clock P. M. is the 16th hour from the beginning of the day, or midnight. September is the 9th month and June is the 6th month. After placing the earlier date under the later date, subtract as in the previous problems. Count 30 days as 1 month.

55. Rule.—*Place the smaller quantity under the larger quantity, with like denominations under each other. Beginning at the right, subtract successively the number in the subtrahend in each denomination from the one above and place the differences underneath. If the number in the minuend of any* *denomination is less than the number under it in the subtrahend, one unit must be borrowed from the minuend of the next higher denomination, reduced, and added to it.*

EXAMPLES FOR PRACTICE.

56. From:

(*a*) 125 lb. 8 oz. 14 pwt. 18 gr. take 96 lb. 9 oz. 10 pwt. 4 gr.
(*b*) 126 hhd. 27 gal. take 104 hhd. 14 gal. 1 qt. 1 pt.
(*c*) 65 T. 14 cwt. 64 lb. 10 oz. take 16 T. 11 cwt. 14 oz.
(*d*) 148 sq. yd. 16 sq. ft. 142 sq. in. take 132 sq. yd. 136 sq. in.
(*e*) 100 bu. take 28 bu. 2 pk. 5 qt. 1 pt.
(*f*) 14 mi. 36 rd. 5 yd. 13 ft. 11 in. take 3 mi. 29 rd. 4 ft. 10 in.

Ans.
(*a*) 28 lb. 11 oz. 4 pwt. 14 gr.
(*b*) 22 hhd. 12 gal. 2 qt. 1 pt.
(*c*) 49 T. 3 cwt. 63 lb. 12 oz.
(*d*) 16 sq. yd. 16 sq. ft. 6 sq. in.
(*e*) 71 bu. 1 pk. 2 qt. 1 pt.
(*f*) 11 mi. 7 rd. 5 yd. 9 ft. 1 in.

MULTIPLICATION OF DENOMINATE NUMBERS.

57. Example.—Multiply 7 lb. 5 oz. 13 pwt. 15 gr. by 12.

Solution.—	lb.	oz.	pwt.	gr.	
	7	5	13	15	
				12	
	89	8	3	12	Ans.

Explanation.— 15 grains × 12 = 180 grains. 180 ÷ 24 = 7 pennyweights and 12 grains remaining. Place the 12 in the grain column and carry the 7 pennyweights to the next column. Now, 13 × 12 + 7 = 163 pennyweights; 163 ÷ 20 = 8 ounces and 3 pennyweights remaining. Then, 5 × 12 + 8 = 68 ounces; 68 ÷ 12 = 5 pounds and 8 ounces remaining. Then, 7 × 12 + 5 = 89 pounds. The entire product is 89 pounds 8 ounces 3 pennyweights 12 grains.

58. Rule.—*Multiply the number representing each denomination by the multiplier, and reduce each product to the next higher denomination, writing the remainders under each denomination, and carry the quotient to the next, as in addition of denominate numbers.*

59. Note.—In multiplication and division of denominate numbers, it is sometimes easier to reduce the number to the lowest denomination given before multiplying or dividing, especially if the multiplier or divisor is a decimal. Thus, in the above example, had the multiplier been 1.2, the easiest way to multiply would have been to reduce the number to grains; then, multiply by 1.2, and reduce the product to higher denominations. For example, 7 lb. 5 oz. 13 pwt. 15 gr. = 43,047 gr. 43,047 × 1.2 = 51,656.4 gr. = 8 lb. 11 oz. 12 pwt. 8.4 gr. Also, 43,047 × 12 = 516,564 gr. = 89 lb. 8 oz. 3 pwt. 12 gr., as before. The student may use either method.

EXAMPLES FOR PRACTICE.

60. Multiply:

(*a*) 15 cwt. 90 lb. by 5; (*b*) 12 yr. 11 mo. 3 da. by 14; (*c*) 11 mi. 145 rd. by 20; (*d*) 12 gal. 4 pt. by 9; (*e*) 8 cd. 76 cu. ft. by 15; (*f*) 4 hhd. 3 gal. 1 qt. 1 pt. by 12.

Ans.
(*a*) 79 cwt. 50 lb.
(*b*) 180 yr. 11 mo. 2 wk.
(*c*) 229 mi. 20 rd.
(*d*) 112 gal. 2 qt.
(*e*) 128 cd. 116 cu. ft.
(*f*) 48 hhd. 40 gal. 2 qt.

DIVISION OF DENOMINATE NUMBERS.

61. EXAMPLE 1.—Divide 48 lb. 11 oz. 6 pwt. by 8.

SOLUTION.—

	lb.	oz.	pwt.	gr.	
8)	48	11	6	0	
	6 lb.	1 oz.	8 pwt.	6 gr.	Ans.

EXPLANATION.—After placing the quantities as above, proceed as follows: 8 is contained in 48 6 times without a remainder. 8 is contained in 11 ounces once with 3 ounces remaining. 3 × 20 = 60; 60 + 6 = 66 pennyweights; 66 pennyweights ÷ 8 = 8 pennyweights and 2 pennyweights remaining; 2 × 24 grains = 48 grains; 48 grains ÷ 8 = 6 grains. Therefore, the entire quotient is 6 pounds 1 ounce 8 pennyweights 6 grains.

EXAMPLE 2.—A silversmith melted up 2 lb. 8 oz. 10 pwt. of silver, which he made into 6 spoons; what was the weight of each spoon?

SOLUTION.—

	lb.	oz.	pwt.		
6)	2	8	10		
		5 oz.	8 pwt.	8 gr.	Ans.

EXPLANATION.—Since we cannot divide 2 pounds by 6, we reduce it to ounces. 2 pounds = 24 ounces, and 24 ounces + 8 ounces = 32 ounces; 32 ounces ÷ 6 = 5 ounces and

2 ounces over. 2 ounces = 40 pennyweights. 40 pennyweights + 10 pennyweights = 50 pennyweights, and 50 pennyweights ÷ 6 = 8 pennyweights and 2 pennyweights over. 2 pennyweights = 48 grains, and 48 grains ÷ 6 = 8 grains. Hence, each spoon contains 5 ounces 8 pennyweights 8 grains.

62. EXAMPLE.—Divide 820 rd. 4 yd. 2 ft. by 112.

```
SOLUTION.—          rd.   yd.  ft. rd.  yd.  ft.  in.
              1 1 2)8 2 0  4    2 ( 7    1    2  5.143  Ans.
                    7 8 4
                    -----
                      3 6 rd. rem.
                      5.5
                    -----
                    1 8 0
                  1 8 0
                  -------
                  1 9 8.0 yd.
                      4   yd.
                  -----
            1 1 2)2 0 2 yd. ( 1 yd.
                  1 1 2
                  -----
                    9 0 yd. rem.
                      3
                  -----
                  2 7 0 ft.
                      2 ft.
                  -----
            1 1 2)2 7 2 ft. ( 2 ft.
                  2 2 4
                  -----
                    4 8 ft. rem.
                    1 2
                    ---
                    9 6
                  4 8
                  -----
            1 1 2)5 7 6.0 0 0 0 in. ( 5.1 4 2 8+ in., or 5.1 4 3 in.
                  5 6 0
                  -----
                    1 6 0
                    1 1 2
                    -----
                      4 8 0
                      4 4 8
                      -----
                        3 2 0
                        2 2 4
                        -----
                          9 6 0
                          8 9 6
                          -----
                            6 4
```

Explanation.—The first quotient is 7 rods with 36 rods remaining. 5.5 × 36 = 198 yards; 198 yards + 4 yards = 202 yards; 202 yards ÷ 112 = 1 yard and 90 yards remaining. 90 × 3 = 270 feet; 270 feet + 2 feet = 272 feet; 272 feet ÷ 112 = 2 feet and 48 feet remaining; 48 × 12 = 576 inches; 576 inches ÷ 112 = 5.143 inches, nearly.

The preceding example is solved by long division, because the numbers are too large to deal with mentally. Instead of expressing the last result as a decimal, it might have been expressed as a common fraction. Thus, $576 \div 112 = 5\frac{16}{112} = 5\frac{1}{7}$ inches. The chief advantage of using a common fraction is that if the quotient be multiplied by the divisor, the result will always be the same as the original dividend.

63. Rule.—*Find how many times the divisor is contained in the first or highest denomination of the dividend. Reduce the remainder (if any) to the next lower denomination and add to it the number in the given dividend expressing that denomination. Divide this new dividend by the divisor. The quotient will be the next denomination in the quotient required. Continue in this manner until the lowest denomination is reached. The successive quotients will constitute the entire quotient.*

EXAMPLES FOR PRACTICE.

64. Divide:

(*a*) 376 mi. 276 rd. by 22; (*b*) 1,137 bu. 3 pk. 4 qt. 1 pt. by 10; (*c*) 84 cwt. 48 lb. 49 oz. by 16; (*d*) 78 sq. yd. 18 sq. ft. 41 sq. in. by 18; (*e*) 148 mi. 64 rd. 24 yd. by 12; (*f*) 100 T. 16 cwt. 18 lb. 11 oz. by 15; (*g*) 36 lb. 18 oz. 18 pwt. 14 gr. by 8; (*h*) 112 mi. 48 rd. by 100.

Ans. (*a*) 17 mi. $41\frac{7}{11}$ rd.; (*b*) 113 bu. 3 pk. 1 qt. $\frac{1}{2}$ pt.; (*c*) 5 cwt. 28 lb. $3\frac{1}{16}$ oz.; (*d*) 4 sq. rd. 4 sq. ft. $2\frac{5}{18}$ sq. in.; (*e*) 12 mi. 112 rd. 2 yd.; (*f*) 6 T. 14 cwt. 41 lb. $3\frac{14}{15}$ oz.; (*g*) 4 lb. 8 oz. 7 pwt. $7\frac{3}{4}$ gr.; (*h*) 1 mi. $38\frac{22}{25}$ rd.

1. On Monday, 1 T. 3 cwt. of coal are burned under a boiler; on Tuesday, 1 T. 1 cwt. 54 lb.; on Wednesday, 1 T. 2 cwt. 16 lb.; on Thursday, 1 T. 2 cwt. 70 lb.; on Friday, 1 T. 3 cwt. 43 lb.; on Saturday, 15 cwt. 68 lb. How much coal was burned during the week?

Ans. 6 T. 8 cwt. 51 lb.

2. 504 gal. 2 qt. of water are drawn from a tank containing 30 hhd. 4 gal. 3 qt. of water. How much water remains?

Ans. 22 hhd. 4 gal. 1 qt.

3. A main line shaft is made up of 5 lengths, as follows: 16 ft. 3 in., 15 ft. 9 in., 15 ft. 2 in., 14 ft. 6 in., 12 ft. 10 in. If 10 hangers are used, one being at each end of the shaft, what is the distance between them, supposing them to be spaced equally? Ans. 8 ft. 3⅓ in.

4. If the distance around a flywheel is 47 ft. 3 in. and the belt extends ⅗ of the way around, what is the distance covered by the belt?

Ans. 28 ft. 4½ in.

5. A boiler shell is made up of three sheets, each 5 ft. 6$\frac{11}{16}$ in. long. If the lap at each of the two middle seams is 2½ in., what is the length of the shell? Ans. 16 ft. 3$\frac{1}{16}$ in.

6. In a return-tubular boiler the heating surface is divided as follows: outside of shell, 98 sq. ft. 9.8 sq. in.; heads, 5 sq. ft. 4½ sq. in.; tubes, 683 sq. ft. 10.75 sq. in. What is the total area of the heating surface in square feet and square inches? Ans. 786 sq. ft. 25.05 sq. in.

ARITHMETIC.

(PART 5.)

INVOLUTION.

1. If a product consists of equal factors, it is called a **power** of one of those equal factors, and one of the equal factors is called a **root** of the product. The power and the root are named according to the number of equal factors in the product. Thus, 3 × 3, or 9, is the *second power*, or **square,** of 3; 3 × 3 × 3, or 27, is the *third power*, or **cube,** of 3; 3 × 3 × 3 × 3, or 81, is the **fourth power** of 3. Also, 3 is the **second root,** or **square root,** of 9; 3 is the **third root,** or **cube root,** of 27; 3 is the **fourth root** of 81.

2. For the sake of brevity,

3 × 3 is written 3^2, and read **three square,**
or *three exponent two;*
3 × 3 × 3 is written 3^3, and read **three cube,**
or *three exponent three;*
3 × 3 × 3 × 3 is written 3^4, and read **three fourth,**
or *three exponent four;*

and so on.

A number written above and to the right of another number, to show how often the latter number is used as a factor, is called an **exponent.** Thus, in 3^{12}, the number ^{12} is the exponent, and shows that 3 is to be used as a factor twelve times; so that 3^{12} is a contraction for

$$3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3.$$

For notice of copyright, see page immediately following the title page.

In an expression like 3^5, the exponent ^{5} shows how often 3 is used as a factor. Hence, if the exponent of a number is unity, the number is used once as a factor; thus, $3^1 = 3$, $4^1 = 4$, $5^1 = 5$.

3. If the side of a square contains 5 inches, the area of the square contains 5×5, or 5^2, square inches. If the edge of a cube contains 5 inches, the volume of the cube contains $5 \times 5 \times 5$, or 5^3, cubic inches. It is for this reason that 5^2 and 5^3 are called the square and cube of 5, respectively.

4. To find any power of a number:

EXAMPLE 1.—What is the third power, or cube, of 35?

SOLUTION.— $35 \times 35 \times 35$,

$$\begin{array}{rr}
\text{or} & 35 \\
 & 35 \\
\cline{2-2}
 & 175 \\
 & 105 \\
\cline{2-2}
 & 1225 \\
 & 35 \\
\cline{2-2}
 & 6125 \\
 & 3675 \\
\cline{2-2}
\textit{cube} = & 42875 \quad \text{Ans.}
\end{array}$$

EXAMPLE 2.—What is the fourth power of 15?

SOLUTION.— $15 \times 15 \times 15 \times 15$,

$$\begin{array}{rr}
\text{or} & 15 \\
 & 15 \\
\cline{2-2}
 & 75 \\
 & 15 \\
\cline{2-2}
 & 225 \\
 & 15 \\
\cline{2-2}
 & 1125 \\
 & 225 \\
\cline{2-2}
 & 3375 \\
 & 15 \\
\cline{2-2}
 & 16875 \\
 & 3375 \\
\cline{2-2}
\textit{fourth power} = & 50625 \quad \text{Ans.}
\end{array}$$

EXAMPLE 3.— $1.2^3 =$ what?

SOLUTION.— $1.2 \times 1.2 \times 1.2$,

$$\begin{array}{lr} \text{or} & 1.2 \\ & \underline{1.2} \\ & 1.44 \\ & \underline{1.2} \\ & 288 \\ & \underline{144} \\ \textit{cube} = & 1.728 \end{array} \quad \text{Ans.}$$

EXAMPLE 4.—What is the third power, or cube, of $\frac{3}{8}$?

SOLUTION.— $(\frac{3}{8})^3 = \frac{3^3}{8^3} = \frac{3}{8} \times \frac{3}{8} \times \frac{3}{8} = \frac{3 \times 3 \times 3}{8 \times 8 \times 8} = \frac{27}{512}$. Ans.

5. Rule.—I. *To raise a whole number or a decimal to any power, use it as a factor as many times as there are units in the exponent.*

II. *To raise a fraction to any power, raise both the numerator and denominator to the power indicated by the exponent.*

EXAMPLES FOR PRACTICE.

6. Raise the following to the powers indicated:

(*a*)	85^2.	Ans.	(*a*)	7,225.
(*b*)	$(\frac{12}{13})^2$.		(*b*)	$\frac{144}{169}$.
(*c*)	6.5^2.		(*c*)	42.25.
(*d*)	14^4.		(*d*)	38,416.
(*e*)	$(\frac{3}{4})^3$.		(*e*)	$\frac{27}{64}$.
(*f*)	$(\frac{5}{6})^3$.		(*f*)	$\frac{125}{216}$.
(*g*)	$(\frac{7}{2})^3$.		(*g*)	$\frac{343}{8}$.
(*h*)	1.4^5.		(*h*)	5.37824.

EVOLUTION.

7. Evolution is the reverse of involution. It is the process of finding the root of a number which is considered as a power.

8. The **square root** of a number is that number which, when used twice as a factor, produces the number.

Thus, 2 is the square root of 4, since 2×2, or $2^2 = 4$.

9. The **cube root** of a number is that number which, when used three times as a factor, produces the number.

Thus, 3 is the cube root of 27, since $3 \times 3 \times 3$, or $3^3 = 27$.

10. The **radical sign** $\sqrt{\ }$, when placed before a number, indicates that some root of that number is to be found. The vinculum is almost always used in connection with the radical sign, as shown in Art. **11.**

11. The **index** of the root is a *small figure* placed *over* and to the *left* of the *radical sign*, to show what root is to be found.

Thus, $\sqrt[2]{100}$ denotes the *square root* of 100.
$\sqrt[3]{125}$ denotes the *cube root* of 125.
$\sqrt[4]{256}$ denotes the *fourth root* of 256, and so on.

12. When the square root is to be extracted, the index is generally omitted. Thus, $\sqrt{100}$ indicates the square root of 100. Also, $\sqrt{225}$ indicates the square root of 225.

SQUARE ROOT.

13. The *largest* number that can be written with *one* figure is 9, and $9^2 = 81$; the *largest* number that can be written with *two* figures is 99, and $99^2 = 9{,}801$; with *three* figures 999, and $999^2 = 998{,}001$; with *four* figures 9,999, and $9{,}999^2 = 99{,}980{,}001$, etc.

In *each* of the above it will be noticed that the square of the number contains just *twice* as many figures as the number.

In order to find the square root of a number, the first step is to find how many figures there will be in the root. This is done by pointing off the number into *periods* of *two* figures each, *beginning at the decimal point.* The number of periods will indicate the number of figures in the root.

Thus, the square root of 83,740,801 must contain 4 figures, since, pointing off the periods, we get 83'74'08'01, or 4 periods; consequently, there must be 4 figures in the root. In like manner, the square root of 50,625 must contain 3 figures, since there are (5'06'25) 3 periods. The extreme left-hand period may contain either 1 or 2 figures, according to the size of the number squared.

14. EXAMPLE.—Find the square root of 31,505,769.

SOLUTION.—

$$\begin{array}{lrlr}
(a) & 5 & & 31'50'57'69\,(\overset{\text{root}}{5\,6\,1\,3}\ \text{Ans.}\\
 & \underline{5} & (b) & \underline{25}\\
(d) & 100 & (c) & 650\\
 & \underline{6} & & \underline{636}\\
 & 106 & (e) & 1457\\
 & \underline{6} & & \underline{1121}\\
 & 1120 & & 33669\\
 & \underline{1} & & \underline{33669}\\
 & 1121 & & \\
 & \underline{1} & & \\
 & 11220 & & \\
 & \underline{3} & & \\
 & 11223 & &
\end{array}$$

EXPLANATION.—Pointing off into periods of two figures each, it is seen that there are 4 figures in the root. Now, find the largest single number whose square is less than or equal to 31, the first period. This is evidently 5, since $6^2 = 36$, which is greater than 31. Write it to the right, as in long division, and also to the left, as shown at (*a*). This is the first figure of the root. Now, multiply the 5 at (*a*) by the 5 in the root, and write the result under the first period, as shown at (*b*). Subtract and obtain 6 as a remainder.

Bring down the next period, 50, and annex it to the remainder 6, as shown at (*c*), which we call the **dividend**. Add the root already found to the 5 at (*a*), getting 10, and annex a cipher to this 10, thus making it 100, as shown at (*d*), which we call the **trial divisor**. Divide the dividend (*c*) by the trial divisor (*d*) and obtain 6, which is *probably* the next figure of the root. Write 6 in the root, as shown, and also add it to 100, the trial divisor, making it 106. This is called the **complete divisor**.

Multiply this by 6, the second figure in the root, and subtract the result from the dividend (*c*). The remainder is 14, to which annex the next period, making it 1457, as shown at (*e*), which we call the **new dividend.** Add the second figure of the root to the complete divisor, 106, and annex a cipher, thus getting 1120. Dividing 1457 by 1120, we get 1 as the next figure of the root. Adding this last figure of the root to 1120, multiplying the result by it, and subtracting from 1457, the remainder is 336.

Annexing the next and last period, 69, the result is 33669. Now, adding the last figure of the root to 1121 and annexing a cipher as before, the result is 11220. Dividing 33669 by 11220, the result is 3, the fourth figure in the root. Adding it to 11220 and multiplying the sum by it, the result is 33669. Subtracting, there is no remainder; hence, $\sqrt{31,505,769} = 5,613$.

15. The square of any number wholly decimal always contains twice as many figures as the number squared. For example, $.1^2 = .01$, $.13^2 = .0169$, $.751^2 = .564001$, etc.

16. It will also be noticed that the number squared is always less than the decimal. Hence, the square root of a number wholly decimal is greater than the number itself. If it be required to find the square root of a decimal, and the decimal has not an even number of figures in it, annex a cipher. The best way to determine the number of figures in the root of a decimal is to begin at the decimal point, and, going towards the *right*, point off the decimal into periods of two figures each. Then, if the last period contains but one figure, annex a cipher.

17. EXAMPLE.—What is the square root of .000576 ?

```
                                        root
SOLUTION.—       2       .0 0'0 5'7 6 ( .0 2 4  Ans.
                 2          4
                 —          —
                 4 0        1 7 6
                   4        1 7 6
                 ———        —————
                 4 4
```

Explanation.—Beginning at the decimal point and pointing off the number into periods of two figures each, it is seen that the first period is composed of ciphers; hence, the first figure of the root must be a cipher. The remaining portion of the solution should be perfectly clear from what has preceded.

18. If the number is not a perfect power, the root will consist of an interminable number of decimal places. The result may be carried to any required number of decimal places by annexing periods of two ciphers each to the number.

19. **Example.**—What is the square root of 3? Find the result to five decimal places.

		root
Solution.—	1	3.0 0'0 0'0 0'0 0'0 0 (1.7 3 2 0 5+ Ans.
	1	1
	2 0	2 0 0
	7	1 8 9
	2 7	1 1 0 0
	7	1 0 2 9
	3 4 0	7 1 0 0
	3	6 9 2 4
	3 4 3	1 7 6 0 0 0 0
	3	1 7 3 2 0 2 5
	3 4 6 0	2 7 9 7 5
	2	
	3 4 6 2	
	2	
	3 4 6 4 0 0	
	5	
	3 4 6 4 0 5	

Explanation.—Annexing 5 periods of two ciphers each to the right of the decimal point, the first figure of the root is 1. To get the second figure, we find that, in dividing 200 by 20, it is 10. This is evidently too large.

Trying 9, we add 9 to 20 and multiply 29 by 9; the result is 261, a result which is considerably larger than 200; hence, 9 is too large. In the same way, it is found that 8 is also too large. Trying 7, 7 times 27 are 189, a result smaller than 200; therefore, 7 is the second figure of the root. The next two figures, 3 and 2, are easily found. The fifth figure in the root is a cipher, since the trial divisor, 34640, is greater than the new dividend, 17600. In a case of this kind we annex another cipher to 34640, thereby making it 346400, and bring down the next period, making the 17600, 1760000. The next figure of the root is 5, and, as we now have five decimal places, we will stop.

The square root of 3 is, then, 1.73205+.

20. Example.—What is the square root of .3 to five decimal places?

```
                                                      root
SOLUTION.—      5             .3 0'0 0'0 0'0 0'0 0( .5 4 7 7 2+   Ans.
                5              2 5
               ---             ---
              1 0 0              5 0 0
                  4              4 1 6
              -----              -----
              1 0 4                8 4 0 0
                  4                7 6 0 9
              -----                -------
              1 0 8 0                7 9 1 0 0
                    7                7 6 6 2 9
              -------                ---------
              1 0 8 7                  2 4 7 1 0 0
                    7                  2 1 9 0 8 4
              -------                  -----------
              1 0 9 4 0                  2 8 0 1 6
                      7
              ---------
              1 0 9 4 7
                      7
              ---------
              1 0 9 5 4 0
                        2
              -----------
              1 0 9 5 4 2
```

Explanation.—In the above example we annex a cipher to .3, making the first period .30, since every period of a decimal, as was mentioned before, must have two figures in it. The remainder of the work should be perfectly clear.

21. If it is required to find the square root of a mixed number, begin at the decimal point and point off the periods both to the right and to the left. The manner of finding the root will then be exactly the same as in the previous cases.

22. Example.—What is the square root of 258.2449?

Solution.—

$$\begin{array}{rll}
 & & \textit{root} \\
1 & 2'58.24'49 & (16.07 \text{ Ans.} \\
1 & 1 & \\
\hline
20 & 158 & \\
6 & 156 & \\
\hline
26 & \quad 22449 & \\
6 & \quad 22449 & \\
\hline
3200 & & \\
7 & & \\
\hline
3207 & &
\end{array}$$

Explanation.—In the above example, since 320 is greater than 224, we place a cipher for the third figure of the root and annex a cipher to 320, making it 3200. Then, bringing down the next period, 49, 7 is found to be the fourth figure of the root. Since there is no remainder, the square root of 258.2449 is 16.07.

23. Proof. — *To prove square root, square the result obtained. If the number is an exact power, the square of the root will equal it; if it is not an exact power, the square of the root will very nearly equal it.*

24. Rule.—I. *Begin at units place and separate the number into periods of two figures each, proceeding from left to right with the decimal part, if there be any.*

II. *Find the greatest number whose square is contained in the first, or left-hand, period. Write this number as the first figure in the root; also, write it at the left of the given number.*

Multiply this number at the left by the first figure of the root, and subtract the result from the first period; then, annex the second period to the remainder.

III. *Add the first figure of the root to the number in the first column on the left and annex a cipher to the result; this is the trial divisor. Divide the dividend by the trial* which will answer the student's purposes fully as well as the exact method, and is far easier to learn and apply.

28. In applying this approximate method, we find the first three or four figures of the root by means of a table in a manner to be described presently and then find more figures, if necessary, as described in Arts. **35** and **36.**

29. In any number, the figures beginning with the first digit* at the left and ending with the last digit at the right are called the **significant figures** of the number. Thus, the number 405,800 has the four significant figures 4, 0, 5, 8; and the number .000090067 has the five significant figures 9, 0, 0, 6, and 7.

The part of a number consisting of its significant figures is called the **significant part** of the number. Thus, in the number 28,070, the significant part is 2807; in the number .00812, the significant part is 812; and in the number 170.3, the significant part is 1703.

In speaking of the significant figures or of the significant part of a number, we consider the figures, in their proper order, from the first digit at the left to the last digit at the right, but we pay no attention to the position of the decimal point. Hence, *all numbers that differ only in the position of the decimal point have the same significant part.* For example, .002103, 21.03, 21,030, and 210,300 have the same significant figures 2, 1, 0, and 3, and the same significant part 2103.

30. In the same manner as in the case of square root, it can be shown that the periods into which a number is divided whose cube root is to be extracted, must contain three figures, except that the first (left-hand) period of a whole or mixed number may contain one, two, or three figures. Hence, the first step in extracting the cube root is to point off the number into periods of three figures each, beginning at the decimal point and proceeding to the left and to the right. The remaining steps are best illustrated by an example.

---

* A cipher is not a digit.

SQUARES AND CUBES.

No.	Square.	Cube.	No.	Square.	Cube.	No.	Square.	Cube.
1	1	1	34	11'56	39'304	67	44'89	300'763
2	4	8	35	12'25	42'875	68	46'24	314'432
3	9	27	36	12'96	46'656	69	47'61	328'509
4	16	64	37	13'69	50'653	70	49'00	343'000
5	25	125	38	14'44	54'872	71	50'41	357'911
6	36	216	39	15'21	59'319	72	51'84	373'248
7	49	343	40	16'00	64'000	73	53'29	389'017
8	64	512	41	16'81	68'921	74	54'76	405'224
9	81	729	42	17'64	74'088	75	56'25	421'875
10	1'00	1'000	43	18'49	79'507	76	57'76	438'976
11	1'21	1'331	44	19'36	85'184	77	59'29	456'533
12	1'44	1'728	45	20'25	91'125	78	60'84	474'552
13	1'69	2'197	46	21'16	97'336	79	62'41	493'039
14	1'96	2'744	47	22'09	103'823	80	64'00	512'000
15	2'25	3'375	48	23'04	110'592	81	65'61	531'441
16	2'56	4'096	49	24'01	117'649	82	67'24	551'368
17	2'89	4'913	50	25'00	125'000	83	68'89	571'787
18	3'24	5'832	51	26'01	132'651	84	70'56	592'704
19	3'61	6'859	52	27'04	140'608	85	72'25	614'125
20	4'00	8'000	53	28'09	148'877	86	73'96	636'056
21	4'41	9'261	54	29'16	157'464	87	75'69	658'503
22	4'84	10'648	55	30'25	166'375	88	77'44	681'472
23	5'29	12'167	56	31'36	175'616	89	79'21	704'969
24	5'76	13'824	57	32'49	185'193	90	81'00	729'000
25	6'25	15'625	58	33'64	195'112	91	82'81	753'571
26	6'76	17'576	59	34'81	205'379	92	84'64	778'688
27	7'29	19'683	60	36'00	216'000	93	86'49	804'357
28	7'84	21'952	61	37'21	226'981	94	88'36	830'584
29	8'41	24'389	62	38'44	238'328	95	90'25	857'375
30	9'00	27'000	63	39'69	250'047	96	92'16	884'736
31	9'61	29'791	64	40'96	262'144	97	94'09	912'673
32	10'24	32'768	65	42'25	274'625	98	96'04	941'192
33	10'89	35'937	66	43'56	287'496	99	98'01	970'299

EXAMPLE.—Extract the cube root of 160,524.

SOLUTION.—Pointing off into periods of three figures each, we have 160'524, two periods of three figures each; hence, the whole-number part of the root will contain two figures. Referring to the table of Squares and Cubes, and looking in the columns headed "*Cube*" (these columns contain the cubes of the numbers in the columns headed "No."), we see that the given number falls between 157,464, the cube of 54, and 166,375, the cube of 55. Hence, the required cube root is 54+. Now find the difference between the two numbers in the table between which the given number falls, and call the result the **first difference,** in this case 166,375 − 157,464 = 8,911. Also find the difference between the given number and the smaller of the two numbers in the table between which it falls, and call the result the **second difference,** in this case 160,524 − 157,464 = 3,060. Now divide the second difference by the first difference and carry the result to three decimal places, increasing the second figure of the decimal by 1 if the third figure is 5 or greater, thus: 3,060 ÷ 8,911 = .343+, or .34. The decimal .34 is the next two figures of the root. Hence, the cube root of 160,524, or $\sqrt[3]{160{,}524}$, is 54.34 to four significant figures. Ans.

31. Had the given number been 160.524, .160524, .000160524, or 160,524,000, the figures of the root would have been exactly the same, the position of the decimal point only being changed, the reason being that there are the *same number of figures in the left-hand period of the significant part of the given number.* Hence, $\sqrt[3]{160.524} = 5.434+$, $\sqrt[3]{.160524} = .5434+$, $\sqrt[3]{.000160524} = .05434+$, and $\sqrt[3]{160{,}524{,}000} = 543.4+$.

Suppose, however, that the given number had been 16,052.4, 16.0524, or .0160524, etc.; then, pointing if off into periods, in the usual manner, we should have 16'052.400, 16.052'400, or .016'052'400, etc., and the left-hand period of the significant part of the number contains only two significant figures, instead of three, as in the former case; in other words, the left-hand period is 16 instead of 160. Since the cube root of 16 is entirely different from the cube root of 160, it is evident that great care must be taken to point off the eriods correctly.

XAMPLE.— $\sqrt[3]{16{,}052.4} = ?$

OLUTION.—Pointing off into periods of three figures each, we have [illegible]400, annexing two ciphers to complete the right-hand decimal iod. Referring to the table, we see that only two periods are given; therefore drop the third period and find the cube root of 16'052. e period dropped will in no case affect the root when only four figures e found; but had the period dropped been 500 or larger, 1 would have [illegible] added to the last figure of the second period. Referring to the lies between 15,625, the cube of 25, and 17,576, the cube of 26. The first difference is 17,576 − 15,625 = 1,951; the second difference is 16,052 − 15,625 = 427; and 427 ÷ 1,951 = .218+, or .22. Hence, $\sqrt[3]{16,052.4} = 25.22$ to four figures. Ans.

32. Example.—What is the cube root of .003?

Solution.—Since the number is entirely decimal, the root is entirely decimal. The given number has but one period and but one figure in that period to be considered, since the two ciphers on the left are not significant figures. The figures of the root will be exactly the same as those in the cube root of 3 or the cube root of 3,000; hence, for convenience, we neglect the decimal point for the present and find the cube root of 3,000. Referring to the table, 3'000 lies between 2'744, the cube of 14, and 3'375, the cube of 15; the first two figures of the root are, therefore, 14. The first difference is 3,375 − 2,744 = 631; the second difference is 3,000 − 2,744 = 256; and 256 ÷ 631 = .405+ or .41. Therefore, the first four figures of the root are 1441. To locate the decimal point, we employ the following principle: *The cube root of any number wholly decimal will have as many ciphers between the decimal and the first significant figure of the root as there are cipher periods of three ciphers each between decimal point and the first significant figure of the given number.* In the present example there are no cipher periods between the decimal point of the given number, .003, and the first significant figure; hence, $\sqrt[3]{.003} = .1441$. Ans.

33. We extracted the cube root of .003 by assuming it to be 3,000 instead of 3, in order to obtain two periods and get two figures of the root from the table. The fourth figure of the root just found is not quite correct—it ought to have been 2 instead of 1—in other words, the fourth figure cannot always be relied upon as being absolutely exact; it may be 1 or 2 less.

34. Rule.—*Point off the number into periods of three figures each, beginning at the decimal point, proceeding to the left and right. Consider the first two periods and find in the table in the columns headed "Cube" the two numbers between which the first two periods of the given number falls. The first two figures of the root will be found opposite the smaller of the two numbers in the nearest column to the left, headed "No." Find the difference between the two numbers in the column headed "Cube"; also find the difference between the smaller of these numbers and the first two periods of the given number. Divide the second difference by the first, carrying* *the quotient to three figures and increasing the second figure by 1 if the third figure is 5 or greater. The first two figures of this quotient will be the third and fourth figures of the root.*

35. As stated in Art. **33**, the fourth figure cannot always be depended upon; hence, three figures of the root are all that we can be absolutely sure of as being correct. If it should be necessary to find more than three or four figures of the root, it can be easily done as follows:

Find the first four figures as above described and locate the decimal point. Divide the given number by the root as found and carry the quotient to 6 or 7 figures. Divide this quotient by the root, and carry the quotient to 6 figures. Then add the last quotient and the two divisors together and divide the sum by 3; the result will be the root correct to 5 or 6 figures.

36. EXAMPLE.—Find the cube root of 160,524 to 7 figures.

SOLUTION.—In Art. **30**, the root to four figures was found to be 54.34. Proceeding as in Art. **35**, 160,524 ÷ 54.34 = 2,954.067; 2,954.067 ÷ 54.34 = 54.3626; 54.34 + 54.34 + 54.36266 = 163.04266; 163.0426 ÷ 3 = 54.34753. Therefore, $\sqrt[3]{160,524} = 54.34753$, correct to 7 figures. Ans.

37. The method illustrated in the example of Art. **37** is perfectly general, but if 5 figures only of the root are required, the fourth and fifth figures may be determined in an easier manner by adding a correction to the root as found by applying the rule of Art. **34.** The correction is determined as follows: When dividing the second difference by the first difference carry the quotient to 4 decimal places, increasing the fourth figure by 1 if the fifth figure is 5 or greater. Consider the quotient as a decimal and subtract it from 1. Multiply the remainder by the quotient and divide the product by the first two figures of the root, as found from the table, carrying the quotient to four decimal places. Then add this quotient to the quotient previously found, and the sum will be the third, fourth, and fifth figures of the root. Thus, referring to Art. **30**, the quotient is .3434; 1 − .3434 = .6566; .3434 × .6566 ÷ 54 = .0042; .3434 + .0042 = .3476. The root is, therefore, 54.348 to five figures.

RATIO.

38. Suppose that it is desired to compare two numbers, say 20 and 4. If we wish to know how many times larger 20 is than 4, we divide 20 by 4 and obtain 5 for the quotient; thus, $20 \div 4 = 5$. Hence, we say that 20 is 5 times as large as 4, i. e., 20 contains 5 times as many units as 4. Again, suppose we desire to know what part of 20 is 4. We then divide 4 by 20 and obtain $\frac{1}{5}$; thus, $4 \div 20 = \frac{1}{5}$, or .2. Hence, 4 is $\frac{1}{5}$, or .2, of 20. This operation of comparing two numbers is termed *finding the ratio* of the two numbers. Ratio, then, is a comparison. It is evident that the two numbers to be compared must be expressed in the same unit; in other words, the two numbers must both be abstract numbers or concrete numbers of the same kind. For example, it would be absurd to compare 20 horses with 4 birds, or 20 horses with 4. Hence, **ratio** may be defined as a comparison between two numbers of the same kind.

39. A ratio may be *expressed* in three ways; thus, if it is desired to compare 20 and 4 and express this comparison as a ratio, it may be done as follows: $20 \div 4$; $20 : 4$, or $\frac{20}{4}$. All three are read *the ratio of 20 to 4*. The ratio of 4 to 20 would be expressed thus: $4 \div 20$; $4 : 20$, or $\frac{4}{20}$. The first method of expressing a ratio, although correct, is seldom or never used; the second form is the one oftenest met with, while the third form, called the fractional form, possesses great advantages to students of algebra and of higher mathematical subjects. The second form is better adapted to arithmetical subjects and is one we shall ordinarily adopt.

40. The **terms** of a ratio are the two numbers to be compared; thus, in the above ratio, 20 and 4 are the terms. When both terms are considered together, they are called a **couplet;** when considered separately, the first term is called the **antecedent** and the second term the **consequent.** Thus, in the ratio 20 : 4, 20 and 4 form a couplet, and 20 is the antecedent and 4 the consequent.

41. A ratio may be **direct** or **inverse.** The *direct ratio* of 20 to 4 is 20 : 4, while the *inverse ratio* of 20 to 4 is 4 : 20. The direct ratio of 4 to 20 is 4 : 20, and the inverse ratio is 20 : 4. An inverse ratio is sometimes called a **reciprocal ratio.** The **reciprocal** of a number is 1 divided by the number. Thus, the reciprocal of 17 is $\frac{1}{17}$; of $\frac{3}{8}$ is $1 \div \frac{3}{8} = \frac{8}{3}$; i. e., the reciprocal of a fraction is the fraction inverted. Hence, the inverse ratio of 20 to 4 may be expressed as 4 : 20, or as $\frac{1}{20} : \frac{1}{4}$. Both have equal values; for, $4 \div 20 = \frac{1}{5}$, and $\frac{1}{20} \div \frac{1}{4} = \frac{1}{20} \times \frac{4}{1} = \frac{1}{5}$.

42. The term **vary** implies a ratio. When we say that two numbers vary as some other two numbers, we mean that the ratio between the first two numbers is the same as the ratio between the other two numbers.

43. The **value** of a ratio is the result obtained by performing the division indicated. Thus, the value of the ratio 20 : 4 is 5—it is the quotient obtained by dividing the antecedent by the consequent. The value of a ratio is always an abstract number, regardless of whether the terms are abstract or concrete numbers

44. When a ratio is expressed in words, as the ratio of 20 to 4, the first number named is always regarded as the antecedent and the second as the consequent, without regard to whether the ratio itself is direct or inverse. *When not otherwise specified, all ratios are understood to be direct.* To express an inverse ratio, the simplest way of doing it is to express it as if it were a direct ratio, with the first number named as the antecedent, and then transpose the antecedent to the place occupied by the consequent and the consequent to the place occupied by the antecedent; or if expressed in the fractional form, invert the fraction. Thus, to express the inverse ratio of 20 to 4, first write it 20 : 4, and then, transposing the terms, as 4 : 20; or as $\frac{20}{4}$, and then inverting, as $\frac{4}{20}$. Or, the reciprocals of the numbers may be taken, as explained above. To **invert** a ratio is to transpose its terms.

45. Instead of expressing the value of a ratio by a single number, as above, it is convenient to express it by means of another ratio in which the consequent is 1. Thus, suppose that it is desired to find the ratio of the weights of two pieces of iron, one weighing 45 pounds and the other weighing 30 pounds. The ratio of the heavier to the lighter is then 45 : 30, an inconvenient expression. Using the fractional form, we have $\frac{45}{30}$. Dividing both terms by 30,[*] the consequent, we obtain $\frac{1\frac{1}{2}}{1}$, or $1\frac{1}{2} : 1$. This is the same result as obtained above, for $1\frac{1}{2} \div 1 = 1\frac{1}{2}$, and $45 \div 30 = 1\frac{1}{2}$.

PROPORTION.

46. Proportion is an equality of ratios, the equality being indicated by the double colon (::) or by the sign of equality (=). Thus, to write in the form of a proportion the two equal ratios, 8 : 4 and 6 : 3, which both have the same value, 2, we may employ one of the three following forms:

$$8 : 4 :: 6 : 3 \qquad (1)$$

$$8 : 4 = 6 : 3 \qquad (2)$$

$$\frac{8}{4} = \frac{6}{3} \qquad (3)$$

47. The first form is the one most extensively used, by reason of its having been exclusively employed in all the older works on mathematics. The second and third forms are being adopted by all modern writers on mathematical subjects, and in time will probably entirely supersede the first form. In this arithmetic we shall adopt the second form, unless some statement can be made clearer by using the third form.

---

[*] This evidently does not alter the value of the ratio, since by the laws of fractions, both numerator and denominator may be divided by the same number without changing the value of the fraction.

48. A proportion may be *read* in two ways. The old way to read the above proportion was : *8 is to 4 as 6 is to 3;* the new way is: *the ratio of 8 to 4 equals the ratio of 6 to 3.* The student may read it either way, but we recommend the latter.

49. Each ratio of a proportion is termed a **couplet.** In the above proportion, 8 : 4 is a couplet, and so is 6 : 3.

50. The numbers forming the proportion are called **terms**; and they are numbered consecutively from left to right, thus:

$$\overset{\textit{first}}{8} : \overset{\textit{second}}{4} = \overset{\textit{third}}{6} : \overset{\textit{fourth}}{3}$$

Hence, in any proportion, the ratio of the first term to the second term equals the ratio of the third term to the fourth term.

51. The first and fourth terms of a proportion are called the **extremes,** and the second and third terms the **means.** Thus, in the foregoing proportion, 8 and 3 are the extremes and 4 and 6 are the means.

52. A **direct proportion** is one in which both couplets are direct ratios.

53. An **inverse proportion** is one which requires one of the couplets to be expressed as an inverse ratio. Thus, 8 is to 4 inversely as 3 is to 6 must be written 8 : 4 = 6 : 3; i. e., the second ratio (couplet) must be inverted.

54. Proportion forms one of the most useful sections of arithmetic. In our grandfathers' arithmetics, it was called "The rule of three."

55. **Rule.**—*In any proportion, the product of the extremes equals the product of the means.*

Thus, in the proportion,

$$17 : 51 = 14 : 42.$$

$17 \times 42 = 51 \times 14$, since both products equal 714.

56. **Rule.**—*The product of the extremes divided by either mean gives the other mean.*

Example.—What is the third term of the proportion $17 : 51 = \quad : 42$?

Solution.—Applying the rule, $17 \times 42 = 714$, and $714 \div 51 = 14$. Ans.

57. **Rule.**—*The product of the means divided by either extreme gives the other extreme.*

Example.—What is the first term of the proportion $\quad : 51 = 14 : 42$?

Solution.—Applying the rule, $51 \times 14 = 714$, and $714 \div 42 = 17$. Ans.

58. When stating a proportion in which one of the terms is unknown, represent the missing term by a letter, as x. Thus, the last example would be written

$$x : 51 = 14 : 42,$$

and for the value of x we have $x = \frac{51 \times 14}{42} = 17$.

59. The principle of all calculations in proportion is this: *Three of the terms are always given and the remaining one is to be found.*

60. Example.—If 4 men can earn \$25 in one week, how much can 12 men earn in the same time?

Solution.—The required term must bear the same relation to the given term of the same kind as one of the remaining terms bears to the other remaining term. We can then form a proportion by which the required term may be found.

The first question the student must ask himself in every calculation by proportion is: "What is it I want to find?" In this case it is dollars. We have two sets of men, one set earning \$25, and we want to know how many dollars the other set earns. It is evident that the *amount* 12 men earn bears the same relation to the *amount* that 4 men earn as 12 men bears to 4 men. Hence, we have the proportion, the amount 12 men earn is to \$25 as 12 men is to 4 men; or, since either extreme equals the product of the means divided by the other extreme, we have

$$\text{The amount 12 men earn} : \$25 = 12 \text{ men} : 4 \text{ men},$$

or the amount 12 men earn $= \frac{\$25 \times 12}{4} = \$75$. Ans.

Since it matters not which place x, or the required term, occupies, the problem could be stated in any of the following forms, the value of x being the same in each:

(*a*) **\$25 : the amount 12 men earn = 4 men : 12 men; or the amount 12 men earn = $\frac{\$25 \times 12}{4}$, or \$75, since either mean equals the product of the extremes divided by the other mean.**

(*b*) **4 men : 12 men = \$25 : the amount 12 men earn; or the amount that 12 earn = $\frac{\$25 \times 12}{4}$, or \$75, since either extreme equals the product of the means divided by the other extreme.**

(*c*) **12 men : 4 men = the amount 12 men earn : \$25; or the amount that 12 men earn = $\frac{\$25 \times 12}{4}$, or \$75, since either mean equals the product of the extremes divided by the other mean.**

61. If the proportion is an inverse one, first form it as though it were a direct proportion and then invert one of the couplets.

EXAMPLES FOR PRACTICE.

62. Find the value of x in each of the following:

(*a*)	\$16 : \$64 = x : \$4.		(*a*)	x = \$1.
(*b*)	x : 85 = 10 : 17.		(*b*)	x = 50.
(*c*)	24 : x = 15 : 40.		(*c*)	x = 64.
(*d*)	18 : 94 = 2 : x.	Ans.	(*d*)	x = 10$\frac{4}{9}$.
(*e*)	\$75 : \$100 = x : 100.		(*e*)	x = 75.
(*f*)	15 pwt. : x = 21 : 10.		(*f*)	x = 7$\frac{1}{7}$ pwt.
(*g*)	x : 75 yd. = \$15 : \$5.		(*g*)	x = 225 yd.

1. If 75 pounds of lead cost \$2.10, what would 125 pounds cost at the same rate? Ans. \$3.50.

2. If A does a piece of work in 4 days and B does it in 7 days, how long will it take A to do what B does in 63 days? Ans. 36 days.

3. The circumferences of any two circles are to each other as their diameters. If the circumference of a circle 7 inches in diameter is 22 inches, what will be the circumference of a circle 31 inches in diameter? Ans. 97$\frac{3}{7}$ in.

INVERSE PROPORTION.

63. In Art. **53,** an inverse proportion was defined as one which required one of the couplets to be expressed as an inverse ratio. Sometimes the word *inverse* occurs in the statement of the example; in such cases the proportion can be written directly, merely inverting one of the couplets.

But it frequently happens that only by carefully studying the conditions of the example can it be ascertained whether the proportion is direct or inverse. When in doubt, the student can always satisfy himself as to whether the proportion is direct or inverse by first ascertaining what is required, and stating the proportion as a direct proportion. Then, in order that the proportion may be true, *if the first term is smaller than the second term, the third term must be smaller than the fourth; or if the first term is larger than the second term, the third term must be larger than the fourth term.* Keeping this in mind, the student can always tell whether the required term will be larger or smaller than the other term of the couplet to which the required term belongs. Having determined this, the student then refers to the example and ascertains from its conditions whether the required term is to be larger or smaller than the other term of the same kind. If the two determinations agree, the proportion is direct; otherwise, it is inverse, and one of the couplets must be inverted.

64. EXAMPLE.—A's *rate* of doing work is to B's as 5 : 7; if A does a piece of work in 42 days, in what time will B do it?

SOLUTION.—The required term is the number of days it will take B to do the work. Hence, stating as a direct proportion,

$$5 : 7 = 42 : x.$$

Now, since 7 is greater than 5, x will be greater than 42. But, referring to the statement of the example, it is easy to see that B works faster than A; hence it will take B a *less* number of days to do the work than A. Therefore, the proportion is an inverse one, and should be stated

$$5 : 7 = x : 42,$$

from which $x = \frac{5 \times 42}{7} = 30$ days. Ans.

Had the example been stated thus: The time that A requires to do a piece of work is to the time that B requires, as 5 : 7; A can do it in 42 days, in what time can B do it? it is evident that it would take B a longer time to do the work than it would A; hence, x would be greater than 42, and the proportion would be direct, the value of x being $\frac{7 \times 42}{5} = 58.8$ days.

EXAMPLES FOR PRACTICE.

65. Solve the following:

1. If a pump which discharges 4 gal. of water per min. can fill a tank in 20 hr., how long will it take a pump discharging 12 gal. per min. to fill it? Ans. 6⅔ hr.

2. The circular seam of a boiler requires 50 rivets when the pitch is 2½ in.: how many would be required if the pitch were 3⅛ in.? Ans. 40.

3. The spring hangers on a certain locomotive are 2½ in. wide and ¾ in. thick; those on another engine are of same sectional area, but are 3 in. wide; how thick are they? Ans. ⅝ in.

4. A locomotive with driving wheels 16 ft. in circumference runs a certain distance in 5,000 revolutions; how many revolutions would it make in going the same distance if the wheels were 22 ft. in circumference (no allowance for slip being made in either case)? Ans. 3,636$\frac{4}{11}$ rev.

UNIT METHOD.

66. In the older books on arithmetic, a large number of problems were solved by proportion; but these problems can be solved much more easily by the **unit method,** which we now proceed to explain by means of examples.

EXAMPLE 1.—If a pump discharging 4 gallons of water per minute can fill a tank in 20 hours, how long will it take a pump discharging 12 gallons per minute to fill the tank?

SOLUTION.—A pump discharging 4 gallons per minute fills the tank in 20 hours. Therefore, a pump discharging 1 gallon per minute fills it in 4 × 20 hours. Hence, a pump discharging 12 gallons per minute fills it in $\frac{4 \times 20 \text{ hours}}{12} = \frac{20 \text{ hours}}{3} = 6\frac{2}{3}$ hr. Ans.

EXAMPLE 2.—If 4 men earn $65.80 in 7 days, how much can 14 men, paid at the same rate, earn in 12 days?

SOLUTION.— 4 men in 7 days earn $65.80.

Therefore, 1 man in 7 days earns $\frac{\$65.80}{4}$.

Therefore, 1 man in 1 day earns $\frac{\$65.80}{4 \times 7}$.

Therefore, 1 man in 12 days earns $\frac{\$65.80 \times 12}{4 \times 7}$.

Therefore, 14 men in 12 days earn $\frac{\$65.80 \times 12 \times 14}{4 \times 7}$.

Canceling,

14 men in 12 days earn $65.80 × 3 × 2 = $65.80 × 6 = $394.80. Ans.

67. The student will notice that in the solution of these examples, in the successive steps, the operations of multiplication and division were merely indicated, and no multiplication or division was performed until the very last, and then the answer was obtained easily by cancelation. In arithmetical calculations, the student should make it an invariable habit to indicate the multiplications and divisions that occur in the successive steps of a solution, and not to perform these operations until the very last. Then, he will probably be able to use the principle of cancelation.

Example.—If a block of granite 8 feet long, 5 feet wide, and 3 feet thick weighs 7,200 pounds, what is the weight of a block of granite 12 feet long, 8 feet wide, and 5 feet thick?

Solution.—If a block 8 feet long, 5 feet wide, and 3 feet thick weighs 7,200 pounds, a block 1 foot long, 5 feet wide, and 3 feet thick weighs $\frac{7{,}200}{8}$ pounds; a block 1 foot long, 1 foot wide, and 3 feet thick weighs $\frac{7{,}200}{8 \times 5}$; and a block 1 foot long, 1 foot wide, and 1 foot thick weighs $\frac{7{,}200}{8 \times 5 \times 3}$ pounds. Therefore, by the same reasoning, a block 12 feet long, 8 feet wide, and 5 feet thick weighs

$$\frac{7{,}200 \times 12 \times 8 \times 5}{8 \times 5 \times 3} \text{ pounds} = \frac{7{,}200 \times \overset{4}{\cancel{12}} \times \cancel{8} \times \cancel{5}}{\cancel{8} \times \cancel{5} \times \cancel{3}} = 28{,}800 \text{ lb.} \quad \text{Ans.}$$

EXAMPLES FOR PRACTICE.

1. If a pump discharges 90,000 gallons of water in 20 hours, in what time will it discharge 144,000 gallons? Ans. 32 hr.

2. When the barometer stands at 30 inches, the pressure of the atmosphere is 14.7 pounds per square inch. What is the atmospheric pressure per square inch when the barometer stands at 29.5 inches? Give answer correct to three figures. Ans. 14.5.

MENSURATION AND USE OF LETTERS IN FORMULAS.

FORMULAS.

1. The term **formula,** as used in mathematics and in technical books, may be defined as *a rule in which symbols are used instead of words;* in fact, a formula may be regarded as a shorthand method of expressing a rule. Any formula can be expressed in words, and when so expressed it becomes a rule.

Formulas are much more convenient than rules; they show at a glance all the operations that are to be performed; they do not require to be read three or four times, as is the case with most rules, to enable one to understand their meaning; they take up much less space, both in the printed book and in one's note book, than rules; in short, whenever a rule can be expressed as a formula, the formula is to be preferred.

As the term "quantity" is a very convenient one to use, we will define it. In mathematics, the word **quantity** is applied to anything that it is desired to subject to the ordinary operations of addition, subtraction, multiplication, etc., when we do not wish to be more specific and state exactly what the thing is. Thus, we can say "two or more numbers," or "two or more quantities"; the word quantity is more general in its meaning than the word number.

2. The signs used in formulas are the ordinary signs indicative of operations and the signs of aggregation. All

For notice of copyright, see page immediately following the title page.

these signs are explained in arithmetic, but some of them will here be explained in order to refresh the student's memory.

3. The signs indicative of operations are six in number, viz.: $+$, $-$, $\times$, $\div$, $|$, $\sqrt{}$.

Division is indicated by the sign $\div$, or by placing a straight line between the two quantities. Thus, $25 \mid 17$, $25 / 17$, and $\frac{25}{17}$ all indicate that 25 is to be divided by 17. When both quantities are placed on the same horizontal line, the straight line indicates that the quantity on the left is to be divided by that on the right. When one quantity is below the other, the straight line between indicates that the quantity above the line is to be divided by the one below it.

The sign ($\sqrt{}$) indicates that some root of the quantity on the right is to be taken; it is called the **radical sign.** To indicate what root is to be taken, a small figure, called the **index,** is placed within the sign, this being always omitted when the square root is to be indicated. Thus, $\sqrt{25}$ indicates that the square root of 25 is to be taken; $\sqrt[3]{25}$ indicates that the cube root of 25 is to be taken; etc.

4. The signs of aggregation are four in number; viz., ——, ( ), [ ], and { }, respectively called the **vinculum,** the **parenthesis,** the **brackets,** and the **brace;** they are used when it is desired to indicate that all the quantities included by them are to be subjected to the same operation. Thus, if we desire to indicate that the sum of 5 and 8 is to be multiplied by 7, and we do not wish to actually add 5 and 8 before indicating the multiplication, we may employ any one of the four signs of aggregation as here shown: $\overline{5+8} \times 7$, $(5+8) \times 7$, $[5+8] \times 7$, $\{5+8\} \times 7$. The vinculum is placed above those quantities which are to be treated as one quantity and subjected to the same operations.

5. While any one of the four signs may be used as shown above, custom has restricted their use somewhat. The vinculum is rarely used except in connection with the radical sign. Thus, instead of writing $\sqrt[3]{}\,(5+8)$, $\sqrt[3]{}\,[5+8]$, or $\sqrt[3]{}\,\{5+8\}$ for the cube root of 5 plus 8, all of which would be correct, the vinculum is nearly always used, $\sqrt[3]{5+8}$.

In cases where but one sign of aggregation is needed (except, of course, when a root is to be indicated), the parenthesis is always used. Hence, $(5+8)\times 7$ would be the usual way of expressing the product of 5 plus 8, and 7.

If two signs of aggregation are needed, the brackets and parenthesis are used, so as to avoid having a parenthesis within a parenthesis, the brackets being placed outside. For example, $[(20-5)\div 3]\times 9$ means that the difference between 20 and 5 is to be divided by 3, and this result multiplied by 9.

If three signs of aggregation are required, the brace, brackets, and parenthesis are used, the brace being placed outside, the brackets next, and the parenthesis inside. For example, $\{[(20-5)\div 3]\times 9-21\}\div 8$ means that the quotient obtained by dividing the difference between 20 and 5 by 3 is to be multiplied by 9, and that after 21 has been subtracted from the product thus obtained, the result is to be divided by 8.

Should it be necessary to use all four of the signs of aggregation, the brace would be put outside, the brackets next, the parenthesis next, and the vinculum inside. For example, $\{[(\overline{20-5}\div 3)\times 9-21]\div 8\}\times 12$.

6. As stated in *Arithmetic*, when several quantities are connected by the various signs indicating addition, subtraction, multiplication, and division, the operation indicated by the sign of multiplication must always be performed first. Thus, $2+3\times 4$ equals 14, 3 being multiplied by 4 before adding to 2. Similarly, $10\div 2\times 5$ equals 1, since 2×5 equals 10, and $10\div 10$ equals 1. Hence, in the above case, if the brace were omitted, the result would be $\frac{1}{4}$, whereas, by inserting the brace, the result is 36.

Following the sign of multiplication comes the sign of division in order of importance. For example, $5-9\div 3$ equals 2, 9 being divided by 3 before subtracting from 5. The signs of addition and subtraction are of equal value; that is, if several quantities are connected by plus and minus signs, the indicated operations may be performed in the order in which the quantities are placed.

[illegible] There is one other sign used, which is neither a sign of aggregation nor a sign indicating an operation to be performed; it is =, and is called the sign of equality; it means that all on one side of it is exactly equal to all on the other side. For example, [illegible]

[illegible] Having called particular attention to certain signs used in formulas, the formulas themselves will now be explained. First, consider the well-known rule for finding the horsepower of a steam engine, which may be stated as follows:

Divide the continued product of the mean effective pressure in pounds per square inch, the length of the stroke in feet, the area of the piston in square inches, and the number of strokes per minute, by 33,000; the result will be the horsepower.

This is a very simple rule, and very little, if anything, will be saved by expressing it as a formula, so far as clearness is concerned. The formula, however, will occupy a great deal less space, as we shall show.

An examination of the rule will show that four quantities (*viz.*, the mean effective pressure, the length of the stroke, the area of the piston, and the number of strokes) are multiplied together, and the result is divided by 33,000. Hence, the rule might be expressed as follows:

$$\textit{Horsepower} = \left(\begin{array}{c}\text{mean effective pressure}\\ \text{in pounds per square inch}\end{array}\right) \times \left(\begin{array}{c}\textbf{stroke}\\ \textbf{in feet}\end{array}\right) \times \left(\begin{array}{c}\text{area of piston}\\ \text{in square inches}\end{array}\right) \times \left(\begin{array}{c}\text{number of strokes}\\ \text{per minute}\end{array}\right) \div \mathbf{33{,}000}$$

This expression could be shortened by representing each quantity by a single letter; thus, representing horsepower by the letter "H," the mean effective pressure in pounds per square inch by "P," the length of stroke in feet by "L," the area of the piston in square inches by "A," the number of strokes per minute by "N," and substituting these letters for the quantities that they represent, the above expression would reduce to

$$H = \frac{P \times L \times A \times N}{33{,}000},$$

a much simpler and shorter expression. This last expression is called a *formula*.

9. The formula just given shows, as we stated in the beginning, that a formula is really a shorthand method of expressing a rule. It is customary, however, to omit the sign of multiplication between two or more quantities when they are to be multiplied together, or between a number and a letter representing a quantity, it being always understood that, when two letters are adjacent with no sign between them, the quantities represented by these letters are to be multiplied. Bearing this fact in mind, the formula just given can be further simplified to

$$H = \frac{PLAN}{33,000}.$$

10. The sign of multiplication, evidently, cannot be omitted between two or more numbers, as it would then be impossible to distinguish the numbers. A near approach to this, however, may be attained by placing a dot between the numbers which are to be multiplied together, and this is frequently done in works on mathematics when it is desired to economize space. In such cases it is usual to put the dot higher than the position occupied by the decimal point. Thus, $2 \cdot 3$ means the same as 2×3; $542 \cdot 749 \cdot 1,006$ indicates that the numbers 542, 749, and 1,006 are to be multiplied together.

It is also customary to omit the sign of multiplication in expressions similar to the following: $a \times \sqrt{b+c}$, $3 \times (b+c)$, $(b+c) \times a$, etc., writing them $a\sqrt{b+c}$, $3(b+c)$, $(b+c)a$, etc. The sign is not omitted when several quantities are included by a vinculum and it is desired to indicate that the quantities so included are to be multiplied by another quantity. For example, $3 \times \overline{b+c}$, $\overline{b+c} \times a$, $\sqrt{b+c} \times a$, etc. are always written as here printed.

11. Before proceeding further, we will explain one other device that is used by formula makers and which is apt to puzzle one who encounters it for the first time—it is the use of what mathematicians call *primes* and *subs.*, and what printers call *superior* and *inferior* characters. As a rule, formula makers designate quantities by the initial letters of the names of the quantities. For example, they represent volume by v, pressure by p, height by h, etc. This practice is to be commended, as the letter itself serves in many cases to identify the quantity which it represents. Some authors carry the practice a little further and represent all quantities of the same nature by the same letter throughout the book, always having the same letter represent the same thing. Now, this practice necessitates the use of the primes and subs. above mentioned when two quantities have the same name but represent different things. Thus, consider the word *pressure* as applied to steam at different stages between the boiler and the condenser. First, there is *absolute* pressure, which is equal to the gauge pressure in pounds per square inch plus the pressure indicated by the barometer reading (usually assumed in practice to be 14.7 pounds per square inch, when a barometer is not at hand). If this be represented by p, how shall we represent the gauge pressure? Since the absolute pressure is always greater than the gauge pressure, suppose we decide to represent it by a capital letter and the gauge pressure by a small (lower-case) letter. Doing so, P represents absolute pressure and p, gauge pressure. Further, there is usually a "drop" in pressure between the boiler and the engine, so that the initial pressure, or pressure at the beginning of the stroke, is less than the pressure at the boiler. How shall we represent the initial pressure? We may do this in one of three ways and still retain the letter p or P to represent the word pressure: First, by the use of the prime mark; thus, p' or P' (read *p prime* and *P major prime*) may be considered to represent the initial gauge pressure or the initial absolute pressure. Second, by the use of sub. figures; thus, p_1 or P_1 (read *p sub. one* and *P major sub. one*). Third, by the use of sub. letters; thus, p_i or P_i (read *p sub. i* and *P major sub. i*). In the same manner p'' (read *p second*), p_2, or p_r might be used to represent the gauge pressure at release, etc. The sub. letters have the advantage of still further identifying the quantity represented: in many instances, however, it is not convenient to use them, in which case primes and subs. are used instead. The prime notation may be continued as follows: p''', p^{iv}, p^{v}, etc.; it is inadvisable to use superior figures, for example, p^1, p^2, p^3, p^n, etc., as they are liable to be mistaken for exponents.

12. The main thing to be remembered by the student is that *when a formula is given in which the same letters occur several times, all like letters having the same primes or subs. represent the same quantities, while those which differ in any respect represent different quantities.* Thus, in the formula

$$t = \frac{w_1 s_1 t_1 + w_2 s_2 t_2 + w_3 s_3 t_3}{w_1 s_1 + w_2 s_2 + w_3 s_3},$$

w_1, w_2, and w_3 represent the weights of three different bodies; s_1, s_2, and s_3, their specific heats; and t_1, t_2, and t_3, their temperatures; while t represents the final temperature after the bodies have been mixed together. It should be noted that those letters having the *same* subs. refer to the same bodies. Thus, w_1, s_1, and t_1 all refer to one of the three bodies; w_2, s_2, t_2, to another body, etc.

It is very easy to apply the above formula when the values of the quantities represented by the different letters are known. All that is required is to substitute the numerical values of the letters and then perform the indicated operations. Thus, suppose that the values of w_1, s_1, and t_1 are, respectively, 2 pounds, .0951, and 80°; of w_2, s_2, and t_2, 7.8 pounds, 1, and 80°; and of w_3, s_3, and t_3, 3¼ pounds, .1138, and 780°; then, the final temperature t is, substituting these values for their respective letters in the formula,

$$t = \frac{2 \times .0951 \times 80 + 7.8 \times 1 \times 80 + 3\tfrac{1}{4} \times .1138 \times 780}{2 \times .0951 + 7.8 \times 1 + 3\tfrac{1}{4} \times .1138}$$
$$= \frac{15.216 + 624 + 288.483}{.1902 + 7.8 + .36985} = \frac{927.699}{8.36005} = 110.97°.$$

In substituting the numerical values, the signs of multiplication are, of course, written in their proper places; all the multiplications are performed before adding, according to the rule previously given.

13. The student should now be able to apply any formula involving only algebraic expressions that he may meet with, and which does not require the use of logarithms for its solution. We will, however, call his attention to one or two other facts that he may have forgotten.

Expressions similar to $\dfrac{160}{\dfrac{660}{25}}$ sometimes occur, the heavy line indicating that 160 is to be divided by the quotient obtained by dividing 660 by 25. If both lines were light, it would be impossible to tell whether 160 was to be divided by $\frac{660}{25}$, or whether $\frac{160}{660}$ was to be divided by 25. If this latter result were desired, the expression would be written $\dfrac{\dfrac{160}{660}}{25}$. In every case the heavy line indicates that all above it is to be divided by all below it.

In an expression like the following, $\dfrac{160}{7 + \dfrac{660}{25}}$, the heavy line is not necessary, since it is impossible to mistake the operation that is required to be performed. But, since $7 + \frac{660}{25} = \frac{175 + 660}{25}$, if we substitute $\frac{175 + 660}{25}$ for $7 + \frac{660}{25}$, the heavy line becomes necessary in order to make the resulting expression clear. Thus,

$$\frac{160}{7 + \dfrac{660}{25}} = \frac{160}{\dfrac{175 + 660}{25}} = \frac{160}{\dfrac{835}{25}}.$$

14. Fractional exponents are sometimes used instead of the radical sign. That is, instead of indicating the square, cube, fourth root, etc. of some quantity, as 37, by $\sqrt{37}$, $\sqrt[3]{37}$, $\sqrt[4]{37}$, etc., these roots are indicated by $37^{\frac{1}{2}}$, $37^{\frac{1}{3}}$, $37^{\frac{1}{4}}$, etc. Should the numerator of the fractional exponent be some quantity other than 1, this quantity, whatever it may be, indicates that the quantity affected by the exponent is to be raised to the power indicated by the numerator; the denominator is *always* the index of the root. Hence, instead of writing $\sqrt[3]{37^2}$ for the cube root of the square of 37, it may be written $37^{\frac{2}{3}}$, the denominator being the index of the root; in other words, $\sqrt[3]{37^2} = 37^{\frac{2}{3}}$. Likewise, $\sqrt[5]{(1 + a^2 b)^3}$ may also be written $(1 + a^2 b)^{\frac{3}{5}}$, a much simpler expression.

15. We will now give several examples showing how to apply some of the more difficult formulas that the student may encounter.

1. The area of any segment of a circle that is less than (or equal to) a semicircle is expressed by the formula

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r - h),$$

in which A = area of segment;
π = 3.1416;
r = radius;
E = angle obtained by drawing lines from the center to the extremities of arc of segment;
c = chord of segment;
h = height of segment.

Example.—What is the area of a segment whose chord is 10 inches long, angle subtended by chord is 83.46°, radius is 7.5 inches, and height of segment is 1.91 inches?

Solution.—Applying the formula just given,

$$A = \frac{\pi r^2 E}{360} - \frac{c}{2}(r - h) = \frac{3.1416 \times 7.5^2 \times 83.46}{360} - \frac{10}{2}(7.5 - 1.91)$$
$$= 40.968 - 27.95 = 13.018 \text{ sq. in., nearly. Ans.}$$

2. The area of any triangle may be found by means of the following formula, in which A = the area, and a, b, and c represent the lengths of the sides:

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2b}\right)^2}.$$

Example.—What is the area of a triangle whose sides are 21 feet, 46 feet, and 50 feet long?

Solution.—In order to apply the formula, suppose we let a represent the side that is 21 feet long; b, the side that is 50 feet long; and c, the side that is 46 feet long. Then, substituting in the formula

$$A = \frac{b}{2}\sqrt{a^2 - \left(\frac{a^2 + b^2 - c^2}{2\,b}\right)^2} = \frac{50}{2}\sqrt{21^2 - \left(\frac{21^2 + 50^2 - 46^2}{2 \times 50}\right)^2}$$

$$= \frac{50}{2}\sqrt{441 - \left(\frac{441 + 2{,}500 - 2{,}116}{100}\right)^2} = 25\sqrt{441 - \left(\frac{825}{100}\right)^2}$$

$$= 25\sqrt{441 - 8.25^2} = 25\sqrt{441 - 68.0625} = 25\sqrt{372.9375}$$

$$= 25 \times 19.312 = 482.8 \text{ sq. ft., nearly. Ans.}$$

The operations in the above examples have been extended much farther than was necessary; it was done in order to show the student every step of the process. The last formula is perfectly general, and the same answer would have been obtained had the 50-foot side been represented by a, the 46-foot side by b, and the 21-foot side by c.

3. The Rankine-Gordon formula for determining the least load in pounds that will cause a long column to break is

$$P = \frac{S\,A}{1 + q\,\frac{l^2}{G^2}},$$

in which $P =$ load (pressure) in pounds;

$S =$ ultimate strength (in pounds per square inch) of the material composing the column;

$A =$ area of cross-section of column in square inches;

$q =$ a factor (multiplier) whose value depends upon the shape of the ends of the column and on the material composing the column;

$l =$ length of column in inches;

$G =$ least radius of gyration of cross-section of column.

The values of S, q, and G^2 are given in printed tables in books in which this formula occurs.

EXAMPLE.—What is the least load that will break a hollow steel column whose outside diameter is 14 inches; inside diameter, 11 inches; length, 20 feet, and whose ends are flat?

SOLUTION.—For steel, $S = 150{,}000$, and $q = \frac{1}{25{,}000}$ for flat-ended steel columns; A, the area of the cross-section, $= .7854\,(d_1^2 - d_2^2) = .7854\,(14^2 - 11^2)$, d_1 and d_2 being the outside and inside diameters, respectively; $l = 20 \times 12 = 240$ inches; and $G^2 = \frac{d_1^2 + d_2^2}{16} = \frac{14^2 + 11^2}{16}$. Substituting these values in the formula

$$P = \frac{S\,A}{1 + q\,\frac{l^2}{G^2}} = \frac{150{,}000 \times .7854\,(14^2 - 11^2)}{1 + \frac{1}{25{,}000} \times \frac{240^2}{\frac{14^2 + 11^2}{16}}}$$

$$= \frac{150{,}000 \times 58.905}{1 + .1163} = \frac{8{,}835{,}750}{1.1163} = 7{,}915{,}211 \text{ lb.} \quad \text{Ans.}$$

4. Example.—When $A = 10$, $B = 8$, $C = 5$, and $D = 4$, what is the value of E in the following?

$$(a)\quad E = \sqrt[3]{\frac{B\,C\,D}{A\left(2 + \frac{D^2}{C^2}\right)}}; \qquad (b)\quad E = \frac{A - \frac{3}{4}D + \frac{4\,B^2}{A + C}}{A - \sqrt{\frac{2\,B^2}{A + 22}}}.$$

Solution.—(*a*) Substituting,

$$E = \sqrt[3]{\frac{8 \times 5 \times 4}{10\left(2 + \frac{4^2}{5^2}\right)}}.$$

To simplify the denominator, square the 4 and 5, add the resulting fraction to 2, and multiply by 10. Simplifying, we have

$$E = \sqrt[3]{\frac{160}{10\left(2 + \frac{16}{25}\right)}} = \sqrt[3]{\frac{160}{10 \times \frac{66}{25}}} = \sqrt[3]{\frac{160}{\frac{660}{25}}} = \sqrt[3]{\frac{200}{33}}.$$

Reducing the fraction to a decimal before extracting the cube root,

$$E = \sqrt[3]{6.0606} = 1.823. \quad \text{Ans.}$$

(*b*) Substituting,

$$E = \frac{10 - \frac{3}{4} \times 4 + \frac{4 \times 8^2}{10 + 5}}{10 - \sqrt{\frac{2 \times 8^2}{10 + 22}}} = \frac{10 - 3 + \frac{4 \times 64}{15}}{10 - \sqrt{\frac{2 \times 64}{32}}}$$

$$= \frac{7 + 17.066+}{10 - \sqrt{4}} = \frac{24.066+}{8} = 3.008+. \quad \text{Ans.}$$

16. In the preceding pages, the unknown quantity has always been represented by the single letter at the left of the sign of equality, while the letters at the right have represented known values from which the required values could be found. It is possible, however, to find the value of the quantity represented by any letter in a formula, if the values represented by all the others are known. For example, let it be required to find how many strokes per minute an engine having a piston area of 78.54 square inches must make in order to develop 60 horsepower, if the mean effective pressure is 40 pounds per square inch and the length of stroke is 1¼ ft. By substituting the given values in the formula $H = \frac{PLAN}{33{,}000}$, we have

$$60 = \frac{40 \times 1\tfrac{1}{4} \times 78.54 \times N}{33{,}000},$$

in which N, the number of strokes, is to be found.

But it is evident that the expression on the right of the sign of equality is equal to $\frac{40 \times 1\frac{1}{4} \times 78.54}{33{,}000} \times N$, a fraction whose numerator is composed of three factors. Reducing the numerator to a single number by performing the indicated multiplications, we obtain, after canceling,

$$60 = \frac{119}{1{,}000} \times N = .119\,N.$$

If 60 equals .119 N, then N equals 60 divided by .119; hence,

$$N = \frac{60}{.119} = 504.2 \text{ strokes per minute.}$$

The method of procedure is essentially the same when the unknown quantity occurs in the denominator of a formula. Thus, in the formula $f = \frac{m v^2}{r}$, suppose that $f = 375$, $m = 1.25$, and $v = 60$. Then, substituting,

$$375 = \frac{1.25 \times 60^2}{r} = \frac{4{,}500}{r}.$$

But, if 375 equals 4,500 divided by r, then $375 \times r = 4{,}500$; hence, r must equal 4,500 divided by 375, or $r = \frac{4{,}500}{375} = 12$.

EXAMPLES FOR PRACTICE.

Find the numerical values of x in the following formulas, when $A = 9$, $B = 8$, $d = 10$, $e = 3$, and $c = 2$:

1. $x = \frac{d + c^2}{d^2 - 40}$. Ans. $x = \frac{7}{30}$.

2. $x = \frac{\frac{2}{3}(A + e)}{c e}$. Ans. $x = 1\frac{1}{3}$.

3. $x = \sqrt{\frac{d^2}{2c}} + \sqrt{A B^2}$. Ans. $x = 29$.

4. $x = \frac{A e}{\sqrt{16 B c}} + \frac{5}{16}$. Ans. $x = 2$.

5. $x = (c + 2e)\left(\sqrt{B} - \frac{1}{c}\right) + \frac{e^2 - c^2}{e^2 + c^2}$. Ans. $x = 12\frac{5}{18}$.

6. $x = \sqrt{\frac{B c d}{A\left(2 + \frac{d^2}{c^2}\right)}}$. Ans. $x = .396+$.

MENSURATION.

17. **Mensuration** treats of the measurement of lines, angles, surfaces, and solids.

LINES AND ANGLES.

18. A **straight line** is one that does not change its direction throughout its whole length. To distinguish one straight line from another, two of its points are designated by letters. The line shown in Fig. 1 would be called the line *A B*.

FIG. 1

19. A **curved line** changes its direction at every point. Curved lines are designated by three or more letters, as the curved line *A B C*, Fig. 2.

FIG. 2.

20. **Parallel lines** (Fig. 3) are those which are equally distant from each other at all points.

FIG. 3.

21. A line is **perpendicular** to another (see Fig. 4) when it meets that line so as not to incline towards it on either side.

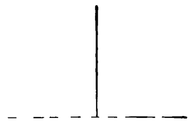


FIG. 4.

22. A **vertical line** is one that points towards the center of the earth, and is also known as a *plumb*-line.

23. A **horizontal line** (see Fig. 5) is one that makes a right angle with any vertical line.

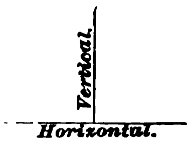


FIG. 5.

24. An **angle** is the opening between two lines which intersect or meet; the point of meeting is called the **vertex** of the angle. Angles are distinguished by naming the vertex and a point on each line. Thus, in Fig. 6, the angle formed by the lines $A B$ and $C B$ is called the angle $A B C$, or the angle $C B A$; the letter at the vertex is always placed in the middle. When an angle stands alone so that it cannot be mistaken for any other angle, only the vertex letter need be used. Thus, the angle referred to might be designated simply as the angle B.

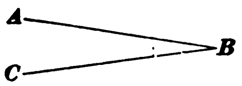


FIG. 6.

25. If one straight line meets another straight line at a point between its ends, as in Fig. 7, two angles, $A B C$ and $A B D$, are formed, which are called **adjacent angles.**

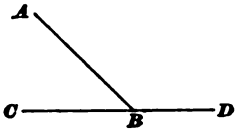


FIG. 7.

26. When these adjacent angles, $A B C$ and $A B D$, are equal, as in Fig. 8, they are called **right angles.**

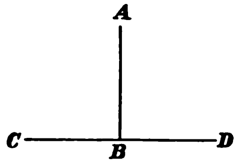


FIG. 8.

27. An **acute angle** is less than a right angle. $A B C$, Fig. 9, is an acute angle.

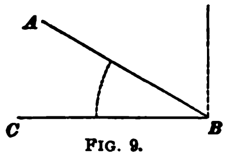


FIG. 9.

28. An **obtuse angle** is greater than a right angle. $A B D$ (Fig. 10) is an obtuse angle.

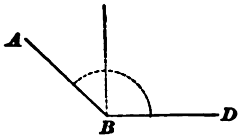


FIG. 10.

29. A **circle** (see Fig. 11) is a figure bounded by a curved line, called the **circumference**, every point of which is equally distant from a point within, called the **center.**

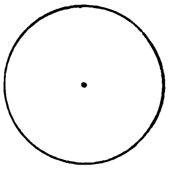


FIG. 11.

30. An **arc** of a circle is any part of its circumference; thus *a e b*, Fig. 12, is an arc of the circle.

Fig. 12.

31. The circumference of every circle is considered to be divided into 360 equal parts, or arcs, called **degrees**; every degree is subdivided into 60 equal parts, called **minutes**, and every minute is again divided into 60 equal parts, called **seconds**.

Since 1 degree is $\frac{1}{360}$ of any circumference, it follows that the length of a degree will be different in circles of different sizes, but the proportion of the length of an arc of 1 degree to the whole circumference will always be the same, viz., $\frac{1}{360}$ of the circumference.

Degrees, minutes, and seconds are denoted by the symbols °, ′, ″. Thus, the expression 37° 14′ 44″ is read 37 degrees 14 minutes 44 seconds.

32. The arcs of circles are used to measure angles. An angle having its vertex at the center of a circle is measured by the arc included between its sides; thus, in Fig. 13, the arc *F B* measures the angle *F O B*. If the arc *F B* contains 20°, or $\frac{20}{360}$ of the circumference, the angle *F O B* would be an angle of 20°; if it contained 20° 14′ 18″, it would be an angle of 20° 14′ 18″, etc.

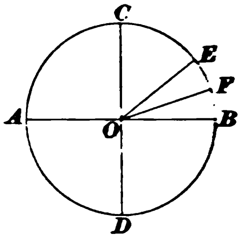


Fig. 13.

In the figure, if the line *C D* be drawn perpendicular to *A B*, the adjacent angles will be equal, and the circle will be divided into four equal angles, each of which will be a right angle. A right angle, therefore, is an angle of $\frac{360°}{4}$, or 90°; two right angles are measured by 180°, or half the circumference, and four right angles by the whole circumference, or 360°. One-half of a right angle, as *E O B*, is an angle of 45°. An *acute* angle may now be defined as an angle of *less* than 90°, and an *obtuse* angle as one of *more* than 90°. These values are important and should be remembered.

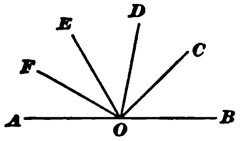


FIG. 14.

33. From the foregoing it will be evident that if a number of straight lines on the same side of a given straight line meet at the same point, the sum of all the angles formed is equal to two right angles, or 180°. Thus, in Fig. 14, angles $COB + DOC + EOD + FOE + AOF = 2$ right angles, or 180°.

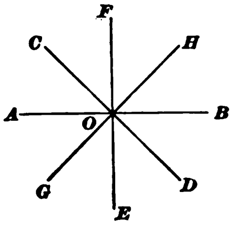


FIG. 15.

34. Also, if through a given point any number of straight lines be drawn, the sum of all the angles formed about the points of intersection equals four right angles, or 360°. Thus, in Fig. 15, angles $HOF + FOC + COA + AOG + GOE + EOD + DOB + BOH = 4$ right angles, or 360°.

EXAMPLE.—In a flywheel with 12 arms, how many degrees are there in the angle included between the center lines of any two arms, the arms being spaced equally ?

SOLUTION.—Since there are 12 arms, there are 12 angles, which together equal 360°. Hence, one angle equals $\frac{1}{12}$ of 360°, or $\frac{360°}{12} = 30°$. Ans.

EXAMPLES FOR PRACTICE.

1. How many seconds are in 32° 14′ 6″ ? Ans. 116,046 sec.

2. How many degrees, minutes, and seconds do 38,582 seconds amount to ? Ans. 10° 43′ 2″.

3. How many right angles are there in an angle of 170° ? Ans. $1\frac{8}{9}$ right angles.

4. In a pulley with five arms, what part of a right angle is included between the center lines of any two arms ? Ans. $\frac{4}{5}$ of a right angle.

5. If one straight line meets another so as to form an angle of 20° 10′, what does its adjacent angle equal ? Ans. 159° 50′.

6. If a number of straight lines meet a given straight line at a given point, all being on the same side of the given line, so as to form six equal angles, how many degrees are there in each angle ? Ans. 30°.

QUADRILATERALS.

35. A **plane figure** is any part of a plane or flat surface bounded by straight or curved lines.

36. A **quadrilateral** is a plane figure bounded by four straight lines.

37. A **parallelogram** is a quadrilateral whose opposite sides are parallel.

There are four kinds of parallelograms: the **square,** the **rectangle,** the **rhombus,** and the **rhomboid.**

38. A **rectangle** (Fig. 16) is a parallelogram whose angles are all right angles.

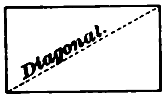


FIG. 16.

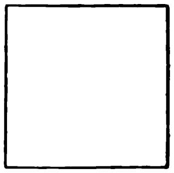


FIG. 17.

39. A **square** (Fig. 17) is a rectangle whose sides are all of the same length.

40. A **rhomboid** (Fig. 18) is a parallelogram whose opposite sides are equal and parallel, and whose angles are not right angles.

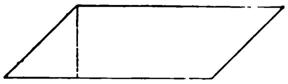


FIG. 18.

41. A **rhombus** (Fig. 19) is a parallelogram having equal sides, and whose angles are not right angles.

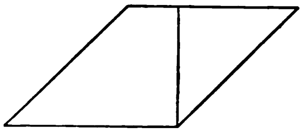


FIG. 19.

42. A **trapezoid** (Fig. 20) is a quadrilateral which has only two of its sides parallel.

FIG. 20.

43. The **altitude** of a parallelogram or a trapezoid is the perpendicular distance between the parallel lines, as shown by the vertical lines in Figs. 18, 19, and 20.

44. The **base** of *any* plane figure is the side on which it is supposed to stand.

45. The **area** of a surface is expressed by the number of unit squares it will contain.

46. A **unit square** is the square having a unit for its side. For example, if the unit is 1 inch, the unit square is the square each of whose sides measures 1 inch in length, and the area of a surface would be expressed by the number of square inches it would contain. If the unit were 1 foot, the unit square would measure 1 foot on each side, and the area of the given surface would be the number of square feet it would contain, etc.

The square that measures 1 inch on a side is called a **square inch,** and the one that measures 1 foot on a side is called a **square foot.** Square inch and square foot are abbreviated to sq. in. and sq. ft.

47. To find the area of any parallelogram:

Rule 1.—*Multiply the base by the altitude.*

NOTE.—Before multiplying, the base and altitude must be reduced to the same kind of units; that is, if the base should be given in feet and the altitude in inches, they could not be multiplied together until either the altitude had been reduced to feet or the base to inches. This principle holds throughout the subject of mensuration.

EXAMPLE 1.—The sides of a square piece of sheet iron are each 10¼ inches long. How many square inches does it contain?

SOLUTION.— 10¼ inches = 10.25 inches when reduced to a decimal. The base and altitude are each 10.25 inches. Multiplying them together, $10.25 \times 10.25 = 105.06+$ sq. in. Ans.

EXAMPLE 2.—What is the area in square rods of a piece of land in the shape of a rhomboid, one side of which is 8 rods long and whose length, measured on a line perpendicular to this side, is 200 feet?

SOLUTION.—The base is 8 rods and the altitude 200 feet. As the answer is to be in rods, the 200 feet should be reduced to rods. Reducing $200 \div 16\frac{1}{2} = 200 \div \frac{33}{2} = 12.12$ rods. Hence, area $= 8 \times 12.12 = 96.96$ sq. rd. Ans.

48. To find the area of a trapezoid:

Rule 2.—*Multiply one-half the sum of the parallel sides by the altitude.*

EXAMPLE.—A board 14 feet long is 20 inches wide at one end and 16 inches wide at the other. If the ends are parallel, how many square feet does the board contain?

SOLUTION.—One-half the sum of the parallel sides $= \frac{20 + 16}{2}$ $= 18$ inches $= 1\frac{1}{2}$ feet. The length of the board corresponds to the altitude of a trapezoid. Hence, $14 \times 1\frac{1}{2} = 21$ sq. ft. Ans.

49. Having given the area of a parallelogram and one dimension, to find the other dimension:

Rule 3.—*Divide the area by the given dimension.*

EXAMPLE.—What is the width of a parallelogram whose area is 212 square feet and whose length is $26\frac{1}{2}$ feet?

SOLUTION.— $212 \div 26\frac{1}{2} = 212 \div \frac{53}{2} = 8$ ft. Ans.

The following examples illustrate a few special cases:

EXAMPLE 1.—An engine room is 22 feet by 32 feet. The engine bed occupies a space of 3 feet by 12 feet; the flywheel pit, a space of 2 feet by 6 feet, and the outer bearing a space of 2 feet by 4 feet. How many square feet of flooring will be required for the room?

SOLUTION.—Area of engine bed $= 3 \times 12 = 36$ sq. ft.
Area of flywheel pit $= 2 \times 6 = 12$ sq. ft.
Area of outer bearing $= 2 \times 4 = 8$ sq. ft.
Total, 56 sq. ft.
Area of engine room $= 22 \times 32 = 704$ sq. ft.
$704 - 56 = 648$ sq. ft. of flooring required. Ans.

EXAMPLE 2.—How many square yards of plaster will it take to cover the sides and ceiling of a room 16×20 feet and 11 feet high, having four windows, each 7×4 feet, and three doors, each 9×4 feet over all, the baseboard coming 6 inches above the floor?

SOLUTION.—
Area of ceiling $= 16 \times 20 = 320$ sq. ft.
Area of end walls $= 2(16 \times 11) = 352$ sq. ft.
Area of side walls $= 2(20 \times 11) = 440$ sq. ft.
Total area $= 1{,}112$ sq. ft.

From the above must be deducted:
Windows $= 4(7 \times 4) = 112$ sq. ft.
Doors $= 3(9 \times 4) = 108$ sq. ft.

Baseboard less the width of three doors $= (72 - 12) \times \frac{6}{12} = 30$ sq. ft.

Total number of feet to be deducted $= 112 + 108 + 30 = 250$ sq. ft.

Hence, number of square feet to be plastered $= 1{,}112 - 250 = 862$ sq. ft., or $95\frac{7}{9}$ sq. yd. Ans.

EXAMPLE 3.—How many acres are contained in a rectangular tract of land 800 rods long and 520 rods wide?

SOLUTION.— $800 \times 520 = 416{,}000$ sq. rd. Since there are 160 square rods in 1 acre, the number of acres $= 416{,}000 \div 160 = 2{,}600$ acres. Ans.

EXAMPLES FOR PRACTICE.

1. What is the area in square feet of a rhombus whose base is 84 inches and whose altitude is 3 feet? Ans. 21 sq. ft.

2. A flat roof, 46 feet by 80 feet in size, is covered by tin roofing weighing one-half pound per square foot; what is the total weight of the roofing? Ans. 1,840 lb.

3. One side of a room measures 16 feet. If the floor contains 240 square feet, what is the length of the other side? Ans. 15 ft.

4. How many square feet in a board 12 feet long, 18 inches wide at one end, and 12 inches wide at the other end? Ans. 15 sq. ft.

5. How much would it cost to lay a sidewalk a mile long and 8 feet 6 inches wide, at the rate of 20 cents per square foot? How much at the rate of $1.80 per square yard? Ans. $8,976 in each case.

6. How many square yards of plastering will be required for the ceiling and walls of a room 10 ft. × 15 ft. and 9 feet high; the room contains one door 3½ ft. × 7 ft., three windows 3½ ft. × 6 ft., and a baseboard 8 inches high? Ans. 53.5 sq. yd.

THE TRIANGLE.

50. A **triangle** is a plane figure having three sides.

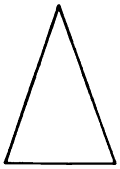


FIG. 21.

51. An **isosceles** triangle is one having two of its sides equal; see Fig. 21.

52. An **equilateral** triangle (Fig. 22) is one having all of its sides of the same length.

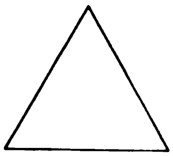


FIG. 22.

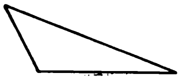


FIG. 23.

53. A **scalene** triangle (Fig. 23) is one having no two of its sides equal.

54. A **right-angled** triangle (Fig. 24) is any triangle having one right angle. The side opposite the right angle is called the **hypotenuse**. A right-angled triangle is now usually called a **right triangle**.

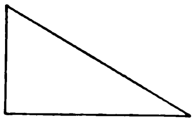


FIG. 24.

55. In any triangle the sum of the three angles equals two right angles, or 180°. Thus, in Fig. 25, the sum of the angles A, B, and C equals two right angles, or 180°. Hence, if any two angles of a triangle are given and it is required to find the third angle:

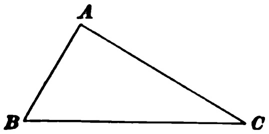


FIG. 25.

Rule 4.—*Add the two given angles and subtract their sum from 180°; the remainder will be the third angle.*

Example.—If two angles of a triangle are $48^\circ\ 16'$ and $47^\circ\ 50'$, what does the third angle equal?

Solution.—First reduce $48^\circ\ 16'$ and $47^\circ\ 50'$ to minutes, for convenience in adding and subtracting the angles. $48^\circ = 48 \times 60' = 2{,}880'$; $2{,}880' + 16' = 2{,}896'$; hence, $48^\circ\ 16' = 2{,}896'$. In like manner, $47^\circ\ 50' = 47 \times 60' + 50' = 2{,}820' + 50' = 2{,}870'$. Adding the two angles and subtracting the sum from 180° reduced to minutes, $2{,}896' + 2{,}870' = 5{,}766'$; $180^\circ = 180 \times 60' = 10{,}800'$; $10{,}800 - 5{,}766 = 5{,}034'$. Reducing this last number to degrees and minutes, $\frac{5{,}034}{60} = 83\frac{54}{60}^\circ = 83^\circ\ 54'$. Hence, the third angle in the triangle $= 83^\circ\ 54'$. Ans.

56. In any right triangle there can be but one right angle, and since the sum of all the angles is two right angles, it is evident that the sum of the two acute angles must equal one right angle, or 90°. Therefore, if in any right triangle one acute angle is known, to find the other acute angle:

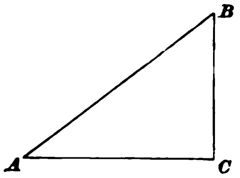


Fig. 26.

Rule 5.—*Subtract the known acute angle from 90°; the result will be the other acute angle.*

Example.—If one acute angle, as A, of the right triangle $A\,B\,C$, Fig. 26, equals 30°, what does the angle B equal?

Solution.— $90^\circ - 30^\circ = 60^\circ$. Ans.

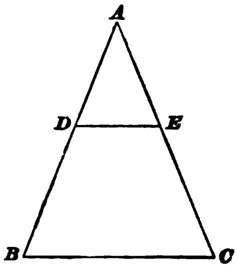


Fig. 27.

57. If a straight line be drawn through two sides of a triangle, parallel to the third side, a second triangle will be formed whose sides will be proportional to the corresponding sides of the first triangle. Thus, in the triangle $A\,B\,C$, Fig. 27, if the line $D\,E$ be drawn parallel to the side $B\,C$, the triangle $A\,D\,E$ will be formed and we shall have

(1) Side $A\,D$: side $D\,E$ = side $A\,B$: side $B\,C$; and,

(2) Side $A\,E$: side $D\,E$ = side $A\,C$: side $B\,C$; also,

(3) Side $A\,D$: side $A\,E$ = side $A\,B$: side $A\,C$.

EXAMPLE.—In Fig. 27, if $A B = 24$, $B C = 18$, and $D E = 8$, what does $A D$ equal?

SOLUTION.—Writing these values for the sides in (1),

$$A D : 8 = 24 : 18; \text{ whence, } A D = \frac{24 \times 8}{18} = 10\tfrac{2}{3}. \quad \text{Ans.}$$

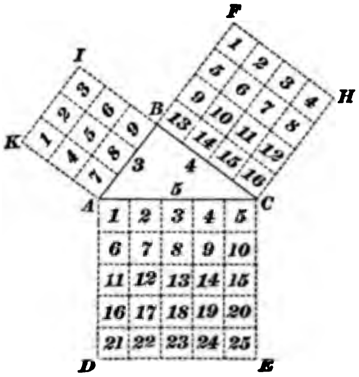


FIG. 28.

58. In any right triangle, the square described on the hypotenuse is equal to the sum of the squares described upon the other two sides. If $A B C$, Fig. 28, is a right triangle right-angled at B, then the square described upon the hypotenuse $A C$ is equal to the sum of the squares described upon the sides $A B$ and $B C$. Hence, having given the two sides forming the right angle in a right triangle, to find the hypotenuse:

Rule 6.—*Square each of the sides forming the right angle; add the squares together and take the square root of the sum.*

EXAMPLE.—If $A B = 3$ inches and $B C = 4$ inches, what is the length of the hypotenuse $A C$?

SOLUTION.—Squaring each of the given sides, $3^2 = 9$ and $4^2 = 16$. Taking the square root of the sum of 9 and 16, the hypotenuse $= \sqrt{9 + 16} = \sqrt{25} = 5$ in. Ans.

59. If the hypotenuse and one side are given, the other side can be found as follows:

Rule 7.—*Subtract the square of the given side from the square of the hypotenuse, and extract the square root of the remainder.*

EXAMPLE 1.—The side given is 3 inches, the hypotenuse is 5 inches; what is the length of the other side?

SOLUTION.— $3^2 = 9$; $5^2 = 25$. $25 - 9 = 16$, and $\sqrt{16} = 4$ in. Ans.

Example 2.—If from a church steeple which is 150 feet high a rope is to be attached to the top and to a stake in the ground, which is 85 feet from the center of the base (the ground being supposed to be level), what must be the length of the rope?

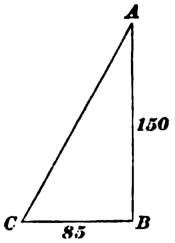


Fig. 29.

Solution.—In Fig. 29, *A B* represents the steeple, 150 feet high; *C* a stake 85 feet from the foot of the steeple, and *A C* the rope. Here we have a right triangle right-angled at *B*, and *A C* is the hypotenuse. The square of $A B = 150^2 = 22{,}500$; of $C B$, $85^2 = 7{,}225$. $22{,}500 + 7{,}225 = 29{,}725$; $\sqrt{29{,}725} = 172.4$ ft., nearly. Ans.

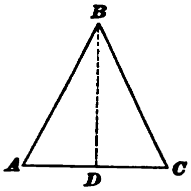


Fig. 30.

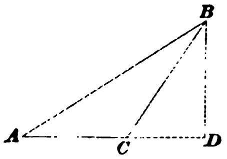


Fig. 31.

60. The **altitude** of any triangle is a line, as *B D*, drawn from the vertex *B* of the angle opposite the base *A C*, perpendicular to the base, as in Fig. 30, or to the base extended, as in Fig. 31.

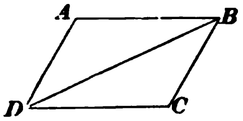


Fig. 32.

61. If in any parallelogram a straight line, called the **diagonal,** be drawn, connecting two opposite corners, it will divide the parallelogram into two equal triangles, as *A D B* and *D B C* in Fig. 32. The area of each triangle will equal one-half the area of the parallelogram, i. e., one-half the product of the base and the altitude. Hence, to find the area of any triangle:

Rule 8.—*Multiply the base by the altitude and divide the product by 2.*

Example.—What is the area in square feet of a triangle whose base is 18 feet and whose altitude is 7 feet 9 inches?

Solution.— 7 ft. 9 in. = $7\frac{3}{4}$ ft. = $\frac{31}{4}$ ft $18 \times \frac{31}{4} = 139\frac{1}{2}$, and one-half of $139\frac{1}{2} = 69\frac{3}{4}$ sq. ft. Ans.

To find the altitude or base of a triangle, having given the area and the base or altitude:

Rule 9.—*Multiply the area by 2 and divide by the given dimension.*

Example.—What must be the height of a triangular piece of sheet metal to contain 100 square inches, if the base is 10 inches long?

Solution.— $100 \times 2 = 200$; $200 \div 10 = 20$ in. Ans.

EXAMPLES FOR PRACTICE.

1. What is the area of a triangle whose base is 18 feet long and whose altitude is 10 feet 6 inches? Ans. 94.5 sq. ft.

2. Two angles of a scalene triangle together equal 100° 4′. What is the size of the third angle? Ans. 79° 56′.

3. One angle of a right triangle equals 20° 10′ 5″. What is the size of the other acute angle? Ans. 69° 49′ 55″.

4. A ladder 65 feet long reaches to the top of a wall when its foot is 25 feet from the wall. How high is the wall? Ans. 60 ft.

5. Draw a triangle, and through two of its sides draw a line parallel to the base. Letter the different lines, and then, without referring to the text, write out the proportions existing between the sides of the two triangles.

6. A triangular piece of sheet metal weighs 24 pounds. If the base of the triangle is 4 feet and its height 6 feet, how much does the metal weigh per square foot? Ans. 2 lb.

7. The area of a triangle is 16 square inches. If the altitude is 4 inches, what does the base measure? Ans. 8 in.

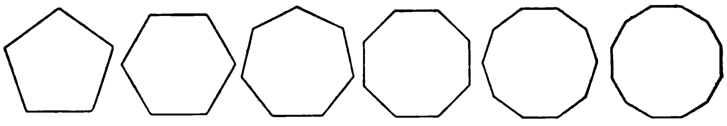
8. Two sides of a right triangle are 92 feet and 69 feet long. How long is the hypotenuse? Ans. 115 ft.

POLYGONS.

62. A **polygon** is a plane figure bounded by straight lines. The term is usually applied to a figure having more than four sides. The bounding lines are called the **sides**, and the sum of the lengths of all the sides is called the **perimeter** of the polygon.

63. A **regular polygon** is one in which all the sides and all the angles are equal.

64. A polygon of five sides is called a **pentagon**; one of six sides, a **hexagon**; one of seven sides, a **heptagon**, etc. Regular polygons having from five to twelve sides are



Pentagon. Hexagon. Heptagon. Octagon. Decagon. Dodecagon.

Fig. 33.

shown in Fig. 33. In any polygon, the sum of all the interior angles, as $A + B + C + D + E$, Fig. 34, equals 180° multiplied by a number which is two less than the number of sides in the polygon. Hence, to find the size of any one of the interior angles of a *regular* polygon:

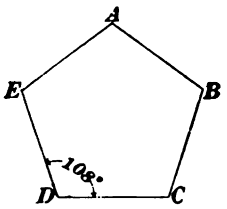


FIG. 34.

Rule 10.—*Multiply 180° by the number of sides less two and divide the result by the number of sides; the quotient will be the number of degrees in each interior angle.*

EXAMPLE 1.—If Fig. 34 is a regular pentagon, how many degrees are there in each interior angle?

SOLUTION.—In a pentagon there are five sides; hence, $5 - 2 = 3$ and $180 \times 3 = 540$; $540 \div 5 = 108°$ in each angle. Ans.

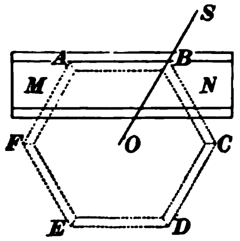


FIG. 35.

EXAMPLE 2.—It is desired to make a miter-box in which to cut a strip of molding to fit around a column having the shape of a regular hexagon. At what angle should the saw run across the miter-box?

SOLUTION.—In Fig. 35, let $A B$, $B C$, $C D$, etc. represent the pieces of molding as they will fit around the column. First find the size of one of the equal angles of the polygon by the above rule. Number of sides $= 6$; $6 - 2 = 4$; hence, $180 \times 4 = 720$, and $720 \div 6 = 120°$ in each angle. Now, let $M N$ represent the miter-box and $O S$ the direction in which the saw should run; then, $A B O$ is the angle made by the saw with the side of the miter-box; but as the polygon is a regular one, this angle is one-half the interior angle $A B C$, which we have found to be 120°. Hence, the saw should run at an angle of $\frac{120}{2} = 60°$ with the side of the miter-box. Ans.

65. The area of any regular polygon may be found by drawing lines from the center to each angle and computing the area of each triangle thus formed. Hence, to find the area of any regular polygon:

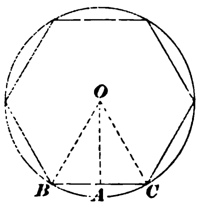


FIG. 36.

Rule 11.—*Multiply the length of a side by half the distance from the side to the center, and that product by the number of sides. The last product will be the area of the figure.*

Example.—In Fig. 36 the side $B\,C$ of the regular hexagon is 12 inches and the distance $A\,O$ is 10.4 inches; required the area of the polygon.

Solution.— $10.4 \div 2 = 5.2$; $12 \times 5.2 \times 6 = 374.4$ sq. in. Ans.

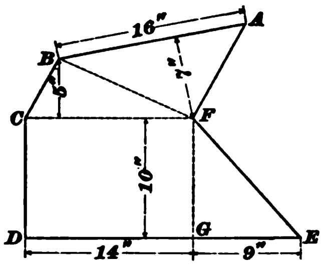


Fig. 37.

66. To obtain the area of any irregular polygon, draw diagonals dividing the polygon into triangles and quadrilaterals, and compute the areas of these separately; their sum will be the area of the figure.

Example.—It is required to find the area of the polygon $A\,B\,C\,D\,E\,F$, Fig. 37.

Solution.—Draw the diagonals $B\,F$ and $C\,F$ and the line $F\,G$ perpendicular to $D\,E$, dividing the figure into the triangles $A\,B\,F$, $B\,C\,F$, and $F\,G\,E$ and the rectangle $F\,C\,D\,G$. Let it be supposed that the altitudes of the figures and the lengths of the sides $A\,B$, $D\,G$, and $G\,E$ are as indicated in the polygon above. Then,

$$\text{Area } A\,B\,F = \frac{16 \times 7}{2} = 56 \text{ sq. in.}$$

$$\text{Area } B\,C\,F = \frac{14 \times 5}{2} = 35 \text{ sq. in.}$$

$$\text{Area } F\,C\,D\,G = 14 \times 10 = 140 \text{ sq. in.}$$

$$\text{Area } F\,G\,E = \frac{9 \times 10}{2} = 45 \text{ sq. in.}$$

Total area $= 56 + 35 + 140 + 45 = 276$ sq. in. Ans.

EXAMPLES FOR PRACTICE.

1. How many degrees are there in one of the angles of a regular octagon? Ans. 135°.

2. Find the area of the polygon $A\,B\,C\,D\,E\,F$ (see Fig. 37), supposing each of the given dimensions to be increased to $1\frac{1}{2}$ times the length given in the figure. Ans. 621 sq. in.

3. What is the area of a regular heptagon whose sides are 4 inches long, the distance from one side to the center being 4.15 inches? Ans. 58.1 sq. in.

4. At what angle should the saw run in a miter-box to cut strips to fit around the edge of a table top made in the shape of a regular pentagon? Ans. 54°.

THE CIRCLE.

67. A **circle** (Fig. 38) is a figure bounded by a curved line, called the **circumference**, every point of which is equally distant from a point within, called the **center**.

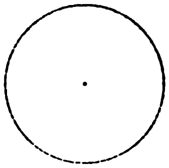


FIG. 38.

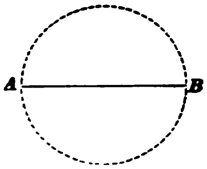


FIG. 39.

68. The **diameter** of a circle is a straight line passing through the center and terminated at both ends by the circumference; thus, *A B* (Fig. 39) is a diameter of the circle.

69. The **radius** of a circle, *A O* (Fig. 40), is a straight line drawn from the center *O* to the circumference. It is equal in length to one-half the diameter. The plural of radius is **radii**, and all radii of a circle are equal.

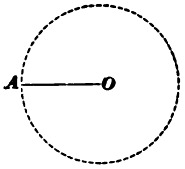


FIG. 40.

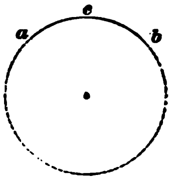


FIG. 41.

70. An **arc** of a circle (see *a e b*, Fig. 41) is any part of its circumference.

71. A **chord** is a straight line joining any two points in a circumference; or it is a straight line joining the extremities of an arc; thus, the straight line *A B*, Fig. 42, is a chord of the circle whose corresponding arc is *A E B*.

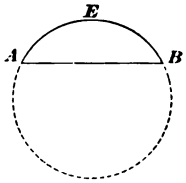


FIG. 42.

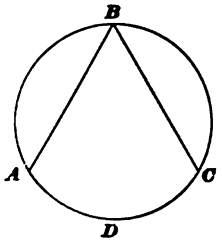


FIG. 43.

72. An **inscribed angle** is one whose vertex lies on the circumference of a circle and whose sides are chords. It is measured by one-half the intercepted arc. Thus, in Fig. 43, *A B C* is an inscribed angle, and it is measured by one-half the arc *A D C*.

Example.—If in Fig. 43, the arc *A D C* = $\frac{2}{5}$ of the circumference, what is the measurement of the inscribed angle *A B C*?

Solution.—Since the angle is an inscribed angle, it is measured by one-half the intercepted arc, or $\frac{2}{5} \times \frac{1}{2} = \frac{1}{5}$ of the circumference. The whole circumference = 360°; hence, $360° \times \frac{1}{5} = 72°$; therefore, angle *A B C* is an angle of 72°.

73. If a circle is divided into halves, each half is called a **semicircle,** and each half circumference is called a **semi-circumference.**

Any angle inscribed in a semicircle is a right angle, since it is measured by one-half a semi-circumference, or $180° \div 2 = 90°$. Thus, the angles *A D C* and *A B C*, Fig. 44, are right angles, since they are inscribed in a semicircle.

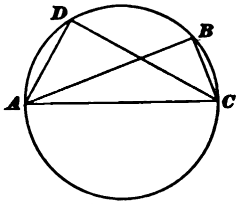


Fig. 44.

74. An **inscribed** polygon is one whose vertexes lie on the circumference of a circle and whose sides are chords, as *A B C D E*, Fig. 45.

The sides of an inscribed regular hexagon have the same length as the radius of the circle.

If, in any circle, a radius be drawn perpendicular to any chord, it bisects (cuts in halves) the chord. Thus, if the radius *O C*, Fig. 46, is perpendicular to the chord *A B*, *A D* = *D B*.

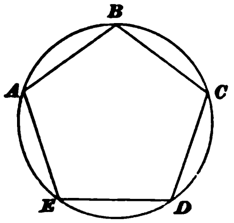


Fig. 45.

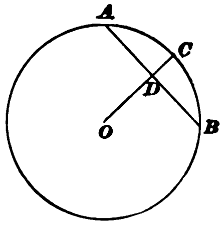


Fig. 46.

Example.—If a regular pentagon is inscribed in a circle and a radius is drawn perpendicular to one of the sides, what are the lengths of the two parts of the side, the perimeter of the pentagon being 27 inches?

Solution.—A pentagon has five sides, and since it is a regular pentagon, all the sides are of equal lengths; the perimeter of the pentagon, which equals the distance around it, or equals the sum of all the sides, is 27 inches. Therefore, the length of one side $= 27 \div 5 = 5\frac{2}{5}$ inches. Since the pentagon is an inscribed pentagon, its sides are chords, and as a radius perpendicular to a chord bisects it, we have $5\frac{2}{5} \div 2 = 2\frac{7}{10}$ inches, which equals the length of each of the parts of the side cut by a radius perpendicular to it. Ans.

75. If, from any point on the circumference of a circle, a perpendicular is let fall upon a diameter, it will divide the diameter into two parts, one of which will be in the same ratio to the perpendicular as the perpendicular is to the other part. That is, the perpendicular will be a *mean proportional* between the two parts of the diameter.

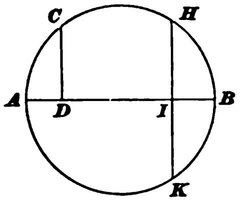


Fig. 47.

If $A\,B$, Fig. 47, is the given diameter and C any point on the circumference, then $A\,D : C\,D = C\,D : D\,B$, $C\,D$ being a mean proportional between $A\,D$ and $D\,B$.

Example.—If $H\,K = 30$ feet and $I\,B = 8$ feet, what is the diameter of the circle, $H\,K$ being perpendicular to $A\,B$?

Solution.— 30 feet ÷ 2 feet = 15 feet = $I\,H$. And $B\,I : I\,H = I\,H : I\,A$, or $8 : 15 = 15 : I\,A$.

Therefore, $I\,A = \frac{15^2}{8} = \frac{225}{8} = 28\frac{1}{8}$ feet and $I\,A + I\,B = 28\frac{1}{8} + 8 = 36\frac{1}{8}$ feet $= A\,B$, the diameter of the circle. Ans.

76. When the diameter of a circle and the lengths of the two parts into which it is divided are given, the length of the perpendicular may be found by multiplying the lengths of the two parts together and extracting the square root of the product.

Example.—In Fig. 47, the diameter of the circle $A\,B$ is $36\frac{1}{8}$ feet and the distance $B\,I$ is 8 feet; what is the length of the line $H\,K$?

Solution.—As the diameter of the circle is $36\frac{1}{8}$ feet and as $B\,I$ is 8 feet, $I\,A$ is equal to $36\frac{1}{8} - 8 = 28\frac{1}{8}$ feet. The two parts, therefore, are 8 and $28\frac{1}{8}$ feet, and their product $= 8 \times 28\frac{1}{8} = 8 \times \frac{225}{8} = 225$; the square root of their product $= \sqrt{225} = 15$ feet, and as $H\,K = I\,H + I\,K$, or $2\,I\,H$, $H\,K = 15 \times 2 = 30$ ft. Ans.

77. To find the circumference of a circle, the diameter being given:

Rule 12.—*Multiply the diameter by 3.1416.*

Example.—What is the circumference of a circle whose diameter is 15 inches?

Solution.— $15 \times 3.1416 = 47.124$ in. Ans.

78. To find the diameter of a circle, the circumference being given:

Rule 13.—*Divide the circumference by 3.1416.*

Example.—What is the diameter of a circle whose circumference is 65.973 inches?

Solution.— $65.973 \div 3.1416 = 21$ in. Ans.

79. To find the length of an arc of a circle:

Rule 14.—*Multiply the length of the circumference of the circle of which the arc is a part by the number of degrees in the arc and divide by 360.*

Example.—What is the length of an arc of 24°, the radius of the arc being 18 inches?

Solution. — $18 \times 2 = 36$ in. = the diameter of the circle. $36 \times 3.1416 = 113.1$ in., the circumference of the circle of which the arc is a part.

$113.1 \times \frac{24}{360} = 7.54$ in., or the length of the arc. Ans.

80. To find the area of a circle:

Rule 15.—*Square the diameter and multiply by .7854.*

Example.—What is the area of a circle whose diameter is 15 inches?

Solution.— $15^2 = 225$; and $225 \times .7854 = 176.72$ sq. in. Ans.

81. Given the area of a circle, to find its diameter:

Rule 16.—*Divide the area by .7854 and extract the square root of the quotient.*

Example 1.—The area of a circle = 17,671.5 square inches. What is its diameter in feet?

Solution.— $\sqrt{\frac{17,671.5}{.7854}} = 150$ inches.

$\frac{150}{12} = 12\frac{1}{2}$ feet, or the diameter. Ans.

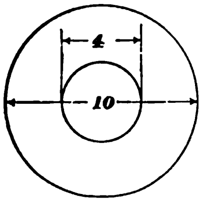


Fig. 48.

Example 2.—What is the area of a flat circular ring, Fig. 48, whose outside diameter is 10 inches and inside diameter is 4 inches?

Solution.—The area of the large circle = $10^2 \times .7854 = 78.54$ sq. in.; the area of the small circle = $4^2 \times .7854 = 12.57$ sq. in. The area of the ring is the difference between these areas, or $78.54 - 12.57 = 65.97$ sq. in. Ans.

82. To find the area of a sector (a **sector** of a circle is the area included between two radii and the circumference, as, for example, the area *B A C O*, Fig. 36):

Rule 17.—*Divide the number of degrees in the arc of the sector by 360. Multiply the result by the area of the circle of which the sector is a part.*

Example.—The number of degrees in the angle formed by drawing radii from the center of a circle to the extremities of the arc of the circle is 75°. The diameter of the circle is 12 inches; what is the area of the sector?

Solution.— $\frac{75}{360} = \frac{5}{24}$; and $12^2 \times .7854 = 113.1$ sq. in.

$113.1 \times \frac{5}{24} = 23.56$ sq. in., the area. Ans.

83. To find the area of a segment of a circle (a **segment** of a circle is the area included between a chord and its arc; for example, the area $A\,B\,C$, Fig. 49) when its chord and height are given. There is no exact method, except by applying principles of trigonometry. The following rule gives results that are exact enough for practical purposes.

Rule 18.—*Divide the diameter by the height of the segment; subtract .608 from the quotient and extract the square root of the remainder. This result multiplied by 4 times the square of the height of the segment and then divided by 3 will give the area, very nearly.*

The rule, expressed as a formula, is as follows, where D = the diameter of the circle and h = the height of the segment (see Fig. 49):

$$\text{Area of } A\,B\,C\,A = \frac{4\,h^2}{3}\sqrt{\frac{D}{h} - .608}.$$

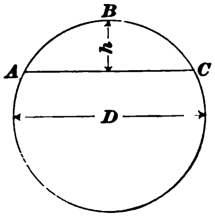


Fig. 49.

Example.—What is the area of the segment of a circle whose diameter is 54 inches, the height of the segment being 20 inches?

Solution.—Substituting in the formula,

$$\text{Area} = \frac{4 \times 20^2}{3}\sqrt{\frac{54}{20} - .608}. \quad \frac{4 \times 20^2}{3} = \frac{4 \times 400}{3}$$

$$= \frac{1{,}600}{3};\ \sqrt{\frac{54}{20} - .608} = \sqrt{2.092} = 1.447;\ \frac{1{,}600}{3} \times 1.447 = 771.7 \text{ sq. in.}$$

Ans.

Note.—Had the chord $A\,C$, Fig. 49, been given instead of the diameter, the diameter would have been found as explained in Art. **75.**

EXAMPLES FOR PRACTICE.

1. An angle inscribed in a circle intercepts one-third of the circumference. How many degrees are there in the angle? Ans. 60°.

2. Suppose that in Fig. 47, the diameter $AB = 15$ feet and the distance $BI = 3$ feet. What is the length of the line HK? Ans. 12 ft.

3. The diameter of a flywheel is 18 feet. What is the distance around it to the nearest 16th of an inch? Ans. 56 ft. $6\frac{9}{16}$ in.

4. A carriage wheel was observed to make $71\frac{5}{8}$ turns while going 300 yards. What was its diameter? Ans. 4 ft., nearly.

5. What is the length of an arc of 64°, the radius of the arc being 30 inches? Ans. 33.51 in.

6. Find the area of a circle 2 feet 3 inches in diameter. Ans. 3.976 sq. ft.

7. What must be the diameter of a circle to contain 100 square inches? Ans. 11.28 in.

8. Compute the area of a segment whose height is 11 inches and the radius of whose arc is 21 inches. Ans. 289.04 sq. in.

9. Find the area of a flat circular ring whose outside diameter is 12 inches and whose inside diameter is 6 inches. Ans. 84.82 sq. in.

THE PRISM AND CYLINDER.

84. A **solid,** or body, has three dimensions: length, breadth, and thickness. The sides which enclose it are called the **faces,** and their lines of intersection are called the **edges.**

85. A **prism** is a solid whose ends are equal and parallel polygons and whose sides are parallelograms. Prisms take their names from the form of their bases. Thus, a triangular prism is one having a triangle for its base; a hexagonal prism is one having a hexagon for its base, etc.

86. A **cylinder** is a body of uniform diameter whose ends are equal parallel circles.

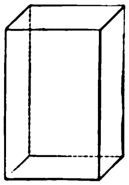


FIG. 50.

87. A **parallelopipedon** (Fig. 50) is a prism whose bases (ends) are parallelograms.

88. A **cube** (Fig. 51) is a prism whose faces and ends are squares. All the faces of a cube are equal.

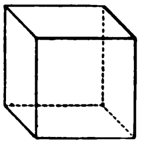


FIG. 51.

In the case of plane figures, we are concerned with perimeters and areas. In the case of solids, we are concerned with the areas of their outside surfaces and with their contents or volumes.

89. The **entire surface** of any solid is the area of the whole outside of the solid, including the ends.

The **convex surface** of a solid is the same as the entire surface, except that the areas of the ends are not included.

90. A **unit of volume** is a cube each of whose edges is equal in length to the unit. The **volume** is expressed by the number of times it will contain a *unit of volume*.

Thus, if the unit of length is 1 inch, the unit of volume will be the cube whose edges each measure 1 inch, this cube being 1 *cubic inch;* and the number of cubic inches the solid contains will be its volume. If the unit of length is 1 foot, the unit of volume will be 1 *cubic foot*, etc. Cubic inch, cubic foot, and cubic yard are abbreviated to cu. in., cu. ft., and cu. yd., respectively.

Instead of the word *volume*, the expression **cubical contents** is sometimes used.

91. To find the area of the convex surface of a prism or cylinder:

Rule 19.—*Multiply the perimeter of the base by the altitude.*

EXAMPLE 1.—A block of marble is 24 inches long and its ends are 9 inches square. What is the area of its convex surface?

SOLUTION. — $9 \times 4 = 36 =$ the perimeter of the base; $36 \times 24 = 864$ sq. in., the convex area. Ans.

To find the entire area of the outside surface, add the areas of the two ends to the convex area. Thus, the area of the two ends $= 9 \times 9 \times 2 = 162$ sq. in.; $864 + 162 = 1,026$ sq. in. Ans.

EXAMPLE 2.—How many square feet of sheet iron will be required for a pipe $1\frac{1}{2}$ feet in diameter and 10 feet long, neglecting the amount necessary for lapping?

SOLUTION.—The problem is to find the convex surface of a cylinder $1\frac{1}{2}$ feet in diameter and 10 feet long. The perimeter, or circumference, of the base $= 1\frac{1}{2} \times 3.1416 = 1.5 \times 3.1416 = 4.712$ ft. The convex surface $= 4.712 \times 10 = 47.12$ sq. ft. of metal. Ans.

92. To find the volume of a prism or a cylinder:

Rule 20.—*Multiply the area of the base by the altitude.*

EXAMPLE 1.—What is the weight of a length of wrought-iron shafting 16 feet long and 2 inches in diameter? Wrought iron weighs .28 pound per cubic inch.

SOLUTION.—The shaft is a cylinder 16 ft. long. The area of one end, or the base, $= 2^2 \times .7854 = 3.1416$ sq. in. Since the weight of the iron is given per cubic inch, the contents of the shaft must be found in cubic inches. The length, 16 ft., reduced to inches $= 16 \times 12 = 192$ in.; $3.1416 \times 192 = 603.19$ cu. in. = the volume. The weight $= 603.19 \times .28 = 168.89$ lb. Ans.

EXAMPLE 2.—Find the cubical contents of a hexagonal prism, Fig. 52, 12 inches long, each edge of the base being 1 inch long.

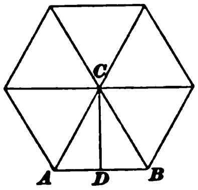


FIG. 52.

SOLUTION.—In order to obtain the area of one end, the distance CD from the center C to one side must be found.

In the right triangle CDA, side $AD = \frac{1}{2} AB$, or $\frac{1}{2}$ inch, and since the polygon is a hexagon, side CA = distance AB, or 1 inch (Art. **74**). Hence, CA being the hypotenuse, the length of side $CD = \sqrt{1^2 - (\frac{1}{2})^2} = \sqrt{1^2 - .5^2} = \sqrt{.75}$, or .866 inch. Area of triangle $ACB = \frac{1 \times .866}{2} = .433$ sq. in.; area of the whole polygon $= .433 \times 6 = 2.598$ sq. in. Hence, the contents of the prism $= 2.598 \times 12 = 31.176$ cu. in. Ans.

EXAMPLE 3.—It is required to find the number of cubic feet of steam space in the boiler shown in Fig. 53. The boiler is 16 feet long between heads, 54 inches in diameter, and the mean water-line MN is at a distance of 16 inches from the top of the boiler. The volume of the steam outlet casting may be neglected.

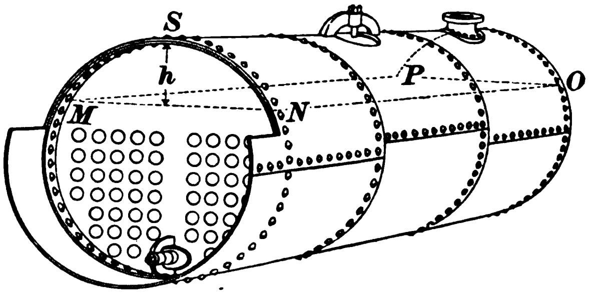


FIG. 53.

SOLUTION.—The volume of the steam space, which is that space within the boiler above the surface $MNOP$ of the water, is found by the rule for finding the volume of a prism or cylinder, the area MNS being the base and the length NO the altitude. First obtain the area of the segment MNS, whose height h is 16 inches, in square feet; then multiply the result by 16, the length of the boiler.

By the formula given in Art. **83**, the area of the segment =

$$\frac{4\,h^2}{3}\sqrt{\frac{D}{h}-.608}=\frac{4\times 16^2}{3}\sqrt{\frac{54}{16}-.608}.$$

$$\frac{4\times 16^2}{3}=341.33;\ \sqrt{\frac{54}{16}-.608}=\sqrt{2.767}=1.663.$$

Hence, the area $= 341.33 \times 1.663 = 567.63$ sq. in. This reduced to sq. ft. $= 567.63 \div 144 = 3.942$ sq. ft., and the volume therefore $= 3.942 \times 16 = 63.07$ cu. ft. Ans.

In the above solution, the space occupied by the stays is not considered, for sake of simplicity. They are not shown in the figure.

EXAMPLE 4.—In the above boiler there are 60 tubes, 3¼ inches outside diameter. How many gallons of water will it take to fill the boiler up to the mean water level, there being 231 cubic inches in a gallon?

SOLUTION.—Find the volume in cubic inches of that part of the boiler below the surface of the water *M N O P*, since the contents of a gallon is given in cubic inches, and from it subtract the volume of the tubes in cubic inches.

This may be done by first finding the *total* area of one end of the boiler in square inches, from it subtracting the area of the segment *M N S*, and the areas of the ends of the tubes in square inches, and then by multiplying the result by the length of the boiler *in inches*.

Total area of one end $= 54^2 \times .7854 = 2{,}290.23$ sq. in.

Area of segment *M N S*, as found in last example, $= 567.63$ sq. in.

Area of the end of one tube $= 3.25^2 \times .7854 = 8.2958$ sq. in.

Area of the ends of the 60 tubes $= 8.2958 \times 60 = 497.75$ sq. in.

Hence, the area to be subtracted $= 567.63 + 497.75 = 1{,}065.38$ sq. in.

Subtracting, $2{,}290.23 - 1{,}065.38 = 1{,}224.85$ sq. in. = net area.

The cubical contents $= 1{,}224.85 \times 16 \times 12 = 235{,}171.2$ cu. in. This divided by 231 will give the number of gallons; whence, $235{,}171.2 \div 231 = 1{,}018.06$ gal. of water. Ans.

EXAMPLES FOR PRACTICE.

1. Find the area in square inches of the convex surface of a bar of iron 4¼ inches in diameter and 8 feet 5 inches long. Ans. 1,348.53 sq. in.

2. Find the area of the entire surface of the above bar.
Ans. 1,376.9 sq. in.

3. What is the area of the entire surface of the hexagonal prism whose base is shown in Fig. 52? Ans. 77.196 sq. in.

4. A multitubular boiler has the following dimensions: diameter, 50 inches; length between heads, 15 feet; number of tubes, 56; outside diameter of tubes, 3 inches; distance of mean water-line from top of boiler, 16 inches. (*a*) Compute the steam space in cubic feet. (*b*) Find the number of gallons of water required to fill the boiler up to the mean water-line.

Ans. $\begin{cases} (a) & 56.4 \text{ cu. ft.} \\ (b) & 800 \text{ gal.} \end{cases}$

THE PYRAMID AND CONE.

93. A **pyramid** (Fig. 54) is a solid whose base is a polygon and whose sides are triangles uniting at a common point, called the **vertex.**

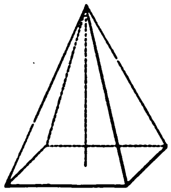


FIG. 54.

94. A **cone** (Fig. 55) is a solid whose base is a circle and whose convex surface tapers uniformly to a point called the **vertex.**

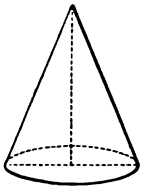


FIG. 55.

95. The **altitude** of a pyramid or cone is the perpendicular distance from the vertex to the base.

96. The **slant height** of a *pyramid* is a line drawn from the vertex perpendicular to one of the sides of the base. The slant height of a *cone* is any straight line drawn from the vertex to the circumference of the base.

97. To find the convex area of a pyramid or cone:

Rule 21.—*Multiply the perimeter of the base by one-half the slant height.*

EXAMPLE 1.—What is the convex area of a pentagonal pyramid if one side of the base measures 6 inches and the slant height is 14 inches?

SOLUTION.—The base of a pentagonal pyramid is a pentagon, and, consequently, has fives sides.

$6 \times 5 = 30$ inches, or the perimeter of the base. $30 \times \frac{14}{2} = 210$ sq. in., or the convex area. Ans.

EXAMPLE 2.—What is the entire area of a right cone whose slant height is 17 inches and whose base is 8 inches in diameter?

SOLUTION.—The perimeter of the base $= 8 \times 3.1416 = 25.1328$ in.

$$\begin{aligned} \text{Convex area} &= 25.1328 \times \tfrac{17}{2} = 213.63 \text{ sq. in.} \\ \text{Area of base} &= 8^2 \times .7854 = \underline{50.27} \text{ sq. in.} \\ \text{Entire area} &= 263.90 \text{ sq. in. Ans.} \end{aligned}$$

98. To find the volume of a pyramid or cone:

Rule 22.—*Multiply the area of the base by one-third of the altitude.*

EXAMPLE 1.—What is the volume of a triangular pyramid, each edge of whose base measures 6 inches and whose altitude is 8 inches?

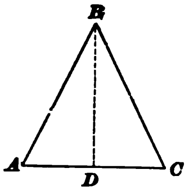


FIG. 56.

SOLUTION.—Draw the base as shown in Fig. 56; it will be an equilateral triangle, all of whose sides are 6 inches long.

Draw a perpendicular $B D$ from the vertex to the base; it will divide the base into two equal parts, since an equilateral triangle is also isosceles, and will be the altitude of the triangle. In order to obtain the area of the base, this altitude must be determined.

In the right triangle $B D A$, the hypotenuse $B A = 6$ inches and side $A D = 3$ inches, to find the other side,

$$B D = \sqrt{6^2 - 3^2} = 5.2 \text{ in., nearly.}$$

Area of the base, or $B A C$, $= \frac{6 \times 5.2}{2} = 15.6$ sq. in. Hence, the volume $= 15.6 \times \frac{8}{3} = 41.6$ cu. in. Ans.

EXAMPLE 2.—What is the volume of a cone whose altitude is 18 inches and whose base is 14 inches in diameter?

SOLUTION.—Area of the base $= 14^2 \times .7854 = 153.94$ sq. in. Hence, the volume $= 153.94 \times \frac{18}{3} = 923.64$ cu. in. Ans.

EXAMPLES FOR PRACTICE.

1. Find the convex surface of a square pyramid whose slant height is 28 inches and one edge of whose base is 7½ inches long. Ans. 420 sq. in.

2. What is the volume of a triangular pyramid, one edge of whose base measures 3 inches and whose altitude is 4 inches? Ans. 5.2 cu. in.

3. Find the volume of a cone whose altitude is 12 inches and the circumference of whose base is 31.416 inches. Ans. 314.16 cu. in.

NOTE.—Find the diameter of the base and then its area.

THE FRUSTUM OF A PYRAMID OR CONE.

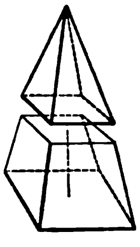


FIG. 57.

99. If a pyramid be cut by a plane, parallel to the base, so as to form two parts, as in Fig. 57, the lower part is called the **frustum** of the pyramid.

If a cone be cut in a similar manner, as in Fig. 58, the lower part is called the **frustum** of the cone.

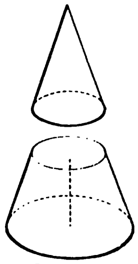


FIG. 58.

100. The upper end of the frustum of a pyramid or cone is called the **upper base,** and the lower end the **lower base.** The **altitude** of a frustum is the perpendicular distance between the bases.

101. To find the convex surface of a frustum of a pyramid or cone:

Rule 23.—*Multiply one-half the sum of the perimeters of the two bases by the slant height of the frustum.*

Example 1.—Given, the frustum of a triangular pyramid, in which one side of the lower base measures 10 inches, one side of the upper base measures 6 inches, and whose slant height is 9 inches; find the area of the convex surface.

Solution.— 10 in. × 3 = 30 in., the perimeter of the lower base.
6 in. × 3 = 18 in., the perimeter of the upper base.
$\frac{30 + 18}{2} = 24$ in., or one-half the sum of the perimeters of the two bases. $24 \times 9 = 216$ sq. in., the convex area. Ans.

Example 2.—If the diameters of the two bases of a frustum of a cone are 12 inches and 8 inches, respectively, and the slant height is 12 inches, what is the entire area of the frustum?

Solution.— $\frac{(12 \times 3.1416) + (8 \times 3.1416)}{2} \times 12 = 376.99$ sq. in., the area of the convex surface.

Area of the upper base = $8^2 \times .7854 = 50.27$ sq. in.
Area of the lower base = $12^2 \times .7854 = 113.1$ sq. in.
The entire area of the frustum = $376.99 + 50.27 + 113.1 = 540.36$ sq. in. Ans.

102. To find the volume of the frustum of a pyramid or cone:

Rule 24.—*Add together the areas of the upper and lower bases and the square root of the product of the two areas; multiply the sum by one-third of the altitude.*

Example 1.—Given, a frustum of a square pyramid (one whose base is a square); each edge of the lower base is 12 inches, each edge of the upper base is 5 inches, and its altitude is 16 inches; what is its volume?

Solution.—Area of upper base = $5 \times 5 = 25$ sq. in.; area of lower base = $12 \times 12 = 144$ sq. in.; the square root of the product of the two areas = $\sqrt{25 \times 144} = 60$. Adding these three results, and multiplying by one-third the altitude, $25 + 144 + 60 = 229$; $229 \times \frac{16}{3} = 1{,}221\frac{1}{3}$ cu. in. = the volume. Ans.

Example 2.—How many gallons of water will a round tank hold which is 4 feet in diameter at the top, 5 feet in diameter at the bottom, and 8 feet deep?

Solution.—There are 231 cubic inches in a gallon, and the volume of the tank should be found in cubic inches. The tank is in the shape of the frustum of a cone. The upper diameter $= 4 \times 12 = 48$ inches; the lower diameter $= 5 \times 12 = 60$ inches, and the depth $= 8 \times 12 = 96$ inches. Area of upper base $= 48^2 \times .7854 = 1{,}809.56$ sq. in.; area of lower base $= 60^2 \times .7854 = 2{,}827.44$ sq. in.; $\sqrt{1{,}809.56 \times 2{,}827.44} = 2{,}261.95$.

Whence, $1{,}809.56 + 2{,}827.44 + 2{,}261.95 = 6{,}898.95$; $6{,}898.95 \times \frac{96}{3} = 220{,}766.4$ cu. in. = contents. Now, since there are 231 cu. in. in 1 gallon, the tank will hold $220{,}766.4 \div 231 = 955.7$ gal., nearly. Ans.

EXAMPLES FOR PRACTICE.

1. Find the convex surface of the frustum of a square pyramid, one edge of whose lower base is 15 inches long, one edge of whose upper base is 14 inches long, and whose slant height is 1 inch. Ans. 58 sq. in.

2. Find the volume of the above frustum, supposing its altitude to be 3 inches. Ans. 631 cu. in.

3. Find the volume of the frustum of a cone whose altitude is 12 feet and the diameters of whose upper and lower bases are 8 and 10 feet, respectively. Ans. 766.55 cu. ft.

4. If a tank had the dimensions of example 3, how many gallons would it hold? Ans. 5,734.2 gal., nearly.

THE SPHERE AND CYLINDRICAL RING.

103. A **sphere** (Fig. 59) is a solid bounded by a uniformly curved surface, every point of which is equally distant from a point within, called the center.

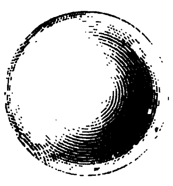


Fig. 59.

The word **ball,** or **globe,** is generally used instead of sphere.

104. To find the area of the surface of a sphere:

Rule 25.—*Square the diameter and multiply the result by 3.1416.*

Example.—What is the area of the surface of a sphere whose diameter is 14 inches?

Solution. — Diameter squared $\times 3.1416 = 14^2 \times 3.1416 = 14 \times 14 \times 3.1416 = 615.75$ sq. in. Ans.

From this it will be seen that the surface of a sphere equals the circumference of a great circle multiplied by the diameter, a rule often used; a *great circle* of a sphere is the intersection of its surface with a plane passing through its center; for instance, the *great circle* of a sphere 6 inches diameter is a circle of 6 inches diameter. Any number of *great circles* could be described on a given sphere.

105. To find the volume of a sphere:

Rule 26.—*Cube the diameter and multiply the result by .5236.*

EXAMPLE.—What is the weight of a lead ball 12 inches in diameter, a cubic inch of lead weighing .41 pound?

SOLUTION.—Diameter cubed × .5236 = 12 × 12 × 12 × .5236 = 904.78 cu. in., or the volume of the ball. The weight, therefore, = 904.78 × .41 = 370.96 lb. Ans.

106. To find the convex area of a cylindrical ring:

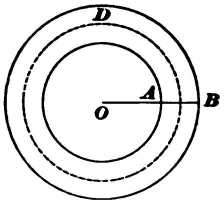


FIG. 60.

A **cylindrical ring** (Fig. 60) is a cylinder bent to a circle. The **altitude** of the cylinder before bending is the same as the length of the dotted center line *D*. The **base** will correspond to a cross-section on the line *A B* drawn from the center *O*. Hence, to find the convex area:

Rule 27.—*Multiply the circumference of an imaginary cross-section on the line A B by the length of the center line D.*

EXAMPLE.—If the outside diameter of the ring is 12 inches and the inside diameter is 8 inches, what is its convex area?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters $= \frac{12 + 8}{2} = 10$, and 10 × 3.1416 = 31.416 in., the length of the center line.

The radius of the inner circle is 4 inches; of the outside circle, 6 inches; therefore, the diameter of the cross-section on the line *A B* is 2 inches. Then, 2 × 3.1416 = 6.2832 in., and 6.2832 × 31.416 = 197.4 sq. in., the convex area. Ans.

107. To find the volume of a cylindrical ring:

Rule 28.—*The volume will be the same as that of a cylinder whose altitude equals the length of the dotted center line D* (Fig. 61) *and whose base is the same as a cross-section of the ring on the line A B drawn from the center O. Hence, to find the volume of a cylindrical ring, multiply the area of an imaginary cross-section on the line A B by the length of the center line D.*

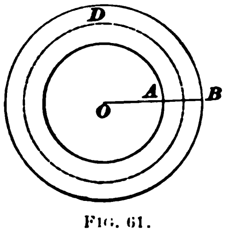


FIG. 61.

EXAMPLE.—What is the volume of a cylindrical ring whose outside diameter is 12 inches and whose inside diameter is 8 inches?

SOLUTION.—The diameter of the center circle equals one-half the sum of the inside and outside diameters $= \frac{12 + 8}{2} = 10$.

$10 \times 3.1416 = 31.416$ inches, the length of the center line.

The radius of the outside circle $= 6$ inches; of the inside circle $= 4$ inches; therefore, the diameter of the cross-section on the line $A\,B$ $= 2$ inches.

Then, $2^2 \times .7854 = 3.1416$ sq. in., the area of the imaginary cross-section.

And $3.1416 \times 31.416 = 98.7$ cu. in., the volume. Ans.

EXAMPLES FOR PRACTICE.

1. What is the volume of a sphere 30 inches in diameter?
Ans. 14,137.2 cu. in.

2. How many square inches in the surface of the above sphere?
Ans. 2,827.44 sq. in.

3. Required the area of the convex surface of a circular ring, the outside diameter of the ring being 10 inches and the inside diameter $7\frac{1}{2}$ inches. Ans. 107.95 sq. in.

4. Find the cubical contents of the ring in the last example.
Ans. 33.73 cu. in.

5. The surface of a sphere contains 314.16 square inches. What is the volume of the sphere? Ans. 523.6 cu. in.

PRINCIPLES OF MECHANICS.

MATTER AND ITS PROPERTIES.

DEFINITION OF MECHANICS.

1. Mechanics is that science which treats of the action of forces on bodies and the effects that they produce; it treats of the laws that govern the movement and equilibrium of bodies and shows how they may be applied.

MATTER.

2. Matter is anything that occupies space. It is the substance of which all bodies consist. Matter is composed of *molecules* and *atoms*.

3. A **molecule** is the smallest portion of matter that can exist without changing its nature.

4. An **atom** is an indivisible portion of matter.

Atoms unite to form molecules, and a collection of molecules forms a mass or body.

A drop of water may be divided and subdivided until each particle is so small that it can only be seen by the most powerful microscope, but each particle will still be water.

Now, imagine the division to be carried on still further, until a limit is reached beyond which it is impossible to go without changing the nature of the particle. The particle of water is now so small that, if it be divided again, it will

For notice of the copyright, see page immediately following the title page.

cease to be water, and will be something else; we call this particle a *molecule*.

If a molecule of water be divided, it will yield 2 atoms of hydrogen gas and 1 of oxygen gas. If a molecule of sulphuric acid be divided, it will yield 2 atoms of hydrogen, 1 of sulphur, and 4 of oxygen.

5. Bodies are composed of collections of molecules. Matter exists in three conditions or forms: *solid*, *liquid*, and *gaseous*.

6. A **solid body** is one whose molecules change their relative positions with great difficulty; as iron, wood, stone, etc.

7. A **liquid body** is one whose molecules tend to change their relative positions easily. Liquids readily adapt themselves to the shape of vessels that contain them, and their upper surface always tends to become perfectly level. Water, mercury, molasses, etc., are liquids.

8. A **gaseous body**, or gas, is one whose molecules tend to separate from one another; as air, oxygen, hydrogen, etc.

Gaseous bodies are sometimes called **aeriform** (air-like) **bodies.** They are divided into two classes: the so-called "*permanent*" *gases* and *vapors*.

9. A **permanent gas** is one that remains a gas at ordinary temperatures and pressures.

10. A **vapor** is a body that at ordinary temperatures is a liquid or solid, but when heat is applied, becomes a gas, as steam.

One body may, under different conditions, exist in all three states; as, for example, mercury, which at ordinary temperatures is a liquid, becomes a solid (freezes) at 40° below zero, and a vapor (gas) at 600° above zero. By means of great cold, all gases, even hydrogen, have been liquefied, and many solidified.

By means of heat, all solids have been liquefied, and a great many vaporized. It is probable that, if we had the means of producing sufficiently great extremes of heat and cold, all solids might be converted into gases, and all gases into solids.

11. Every portion of matter possesses certain qualities called *properties*. Properties of matter are divided into two classes: *general* and *special*.

12. **General properties of matter** are those that are common to all bodies. They are as follows: *Extension*, *impenetrability*, *weight*, *indestructibility*, *inertia*, *mobility*, *divisibility*, *porosity*, *compressibility*, *expansibility*, and *elasticity*.

13. **Extension** is the property of occupying space. Since all bodies must occupy space, it follows that extension is a general property.

14. By **impenetrability** we mean that no two bodies can occupy exactly the same space at the same time.

15. **Weight** is the measure of the earth's attraction upon a body. All bodies have weight. In former times it was supposed that gases had no weight, since, if unconfined, they tend to move away from the earth, but, nevertheless, they will finally reach a point beyond which they cannot go, being held in suspension by the earth's attraction. Weight is measured by comparison with a standard. The standard is a bar of platinum weighing 1 pound, owned and kept by the Government.

16. **Inertia** means that a body cannot put itself in motion nor bring itself to rest. To do either it must be acted upon by some force.

17. **Mobility** means that a body can be changed in position by some force acting upon it.

18. **Divisibility** is that property of matter by virtue of which a body may be separated into parts.

19. **Porosity** is the term used to denote the fact that there is space between the molecules of a body. The molecules of a body are supposed to be spherical, and, hence, there is space between them, as there would be between peaches in a basket. The molecules of water are larger than those of salt; so that when salt is dissolved in water, its molecules wedge themselves between the molecules of the water, and, unless too much salt is added, the water will occupy no more space than it did before. This does not prove that water is penetrable, for the molecules of salt occupy the space that the molecules of water did not.

Water has been forced through iron by pressure, thus proving that iron is porous.

20. Compressibility is a natural consequence of porosity. Since there is space between the molecules, it is evident that by means of force (pressure) they can be brought closer together, and thus the body be made to occupy a smaller space.

21. Expansibility is the term used to denote the fact that the molecules of a body will, under certain conditions (when heated, for example), move farther apart, and so cause the body to *expand*, or occupy a greater space.

22. Elasticity is that property of matter which enables a body when distorted within certain limits to resume its original form when the distorting force is removed. Glass, ivory, and steel are very elastic, clay and putty in their natural state being very slightly so.

23. Indestructibility is the term used to denote the fact that we cannot *destroy* matter. A body may undergo thousands of changes, be resolved into its molecules, and its molecules into atoms, which may unite with other atoms to form other molecules and bodies entirely different in appearance and properties from the original body, but the same number of atoms remain. The whole number of atoms in the universe is exactly the same now as it was millions of years ago, and will always be the same. *Matter is indestructible.*

24. Special properties are those that are not possessed by all bodies. Some of the most important are as follows: *hardness*, *tenacity*, *brittleness*, *malleability*, and *ductility*.

25. Hardness.—A piece of copper will scratch a piece of wood, steel will scratch copper, and tempered steel will scratch steel in its ordinary state. We express all this by saying that steel is *harder* than copper, and so on. Emery and corundum are extremely hard, and the diamond is the hardest of all known substances. It can only be polished with its own powder.

26. Tenacity is the term applied to the power with which some bodies resist a force tending to pull them apart. Steel is very tenacious.

27. Brittleness. — Some bodies possess considerable power to resist either a pull or a pressure, but they are easily broken when subjected to shocks or jars; for example, good glass will bear a *greater* compressive force than most woods, but may be easily broken when dropped upon a hard floor; this property is called *brittleness*.

28. Malleability is that property which permits of some bodies being hammered or rolled into sheets. Gold is the most malleable of all substances.

29. Ductility is that property which enables some bodies to be drawn into wire. Platinum is the most ductile of all substances.

MOTION AND VELOCITY.

DEFINITIONS.

30. Motion is the opposite of rest and indicates a changing of position in relation to some object which is for that purpose regarded as being fixed. If a large stone is rolled down hill, it is in motion in relation to the hill.

If a person is on a railway train and walks in the opposite direction from that in which the train is moving, and with the same speed, he will be in motion as regards the train, but at rest with respect to the earth, since, until he gets to the end of the train, he will be directly over the spot at which he was when he started to walk.

31. The **path of a body in motion** is the line described by a certain point in the body called its center of gravity. No matter how irregular the shape of the body may be, nor how many turns and twists it may make, the line that indicates the direction of this point for every instant that it is in motion is the path of the body.

32. **Velocity** is rate of motion. It is measured by a unit of space passed over in a unit of time. When equal spaces are passed over in equal times, the velocity is said to be **uniform.**

If the flywheel of an engine keeps up a constant speed of a certain number of revolutions per minute, the velocity of any point is uniform. A railway train having a constant speed of 40 miles per hour moves 40 miles every hour, or $\frac{40}{60} = \frac{2}{3}$ mile every minute, and since equal spaces are passed over in equal times, the velocity is uniform.

33. **Variable Velocity.**—When a body moves in such a way that the spaces passed over in equal periods of time are unequal, its velocity is said to be **variable.**

34. The rate of motion of a body with a variable velocity may increase or decrease at a uniform rate. When the velocity varies in either of these ways, the body is said to have a **uniformly varying velocity.**

The most familiar example of uniformly varying velocity is a falling weight. Suppose a stone is dropped from a high bridge. It starts from a state of rest with no velocity, but its velocity constantly increases until it strikes. Its increase in velocity during any equal units of time is nearly the same. Thus, at the end of the first second its velocity will have increased from 0 to a rate of 32.16 feet per second, nearly; during the next second the velocity will have increased by the same amount, making the velocity at the end of the second second 64.32 feet. At the end of the third second a like increase will have taken place, and the velocity will then be 3 × 32.16 = 96.48 feet per second.

35. The change in the velocity of a body during a period of time is called its **acceleration** for that period. Thus, in the case of the falling weight just considered, the change of 32.16 feet per second that takes place in its velocity during each second is its acceleration in feet per second for each second considered. If the period of time considered had been 2 seconds, the acceleration would have been the increase in velocity during this time, that is, 64.32 feet per second for each 2 seconds considered.

36. The change in velocity may be from a higher to a lower rate. Thus, when a stone is thrown upwards, it leaves the hand with a given velocity; its upward motion is constantly resisted by the force of gravity and the resistance of the air, and in consequence of these resistances, it moves slower and slower until it finally stops and begins to return to the earth. A change in velocity of this kind is sometimes called **retardation.**

37. The **mean** or **average velocity** of a body moving with a variable velocity can only be given for a stated period of time, and is numerically equal to the uniform velocity that will take the body over the same distance in the same time.

RULES FOR VELOCITY PROBLEMS.

38. Uniform and Average Velocity.—

Let d = distance;
t = time;
v = velocity.

Rule 1.—*To find the uniform or the average velocity that a body must have to pass over a certain distance or space in a given time, divide the distance by the time.*

Or, $$v = \frac{d}{t}.$$

Example 1.—The piston of a steam engine travels 3,000 feet in 5 minutes; what is its average velocity in feet per minute?

Solution.—Here 3,000 feet is the distance, and 5 minutes is the time. Applying the rule, 3,000 ÷ 5 = 600 ft. per min. Ans.

Caution.—Before applying the above or any of the succeeding rules, care must be taken to reduce the values given to the denominations required in the answer. Thus, in the above example, if the velocity is required in feet per second instead of in feet per minute, the 5 minutes must be reduced to seconds before dividing. The operation will then be: 5 minutes = 5 × 60 = 300 seconds. Applying the rule, 3,000 ÷ 300 = 10 ft. per sec. Ans.

If the velocity is required in inches per second, it is necessary to reduce the 3,000 feet to inches and the 5 minutes to seconds, before dividing. Thus, 3,000 feet × 12 = 36,000 inches. 5 minutes × 60 = 300 seconds. Now applying the rule, 36,000 ÷ 300 = 120 in. per sec. Ans.

Example 2.—A railroad train travels 50 miles in 1½ hours; what is its average velocity in feet per second?

Solution.—Reducing the miles to feet and the hours to seconds, 50 miles × 5,280 = 264,000 feet. 1½ hours × 60 × 60 = 5,400 seconds. Applying the rule, 264,000 ÷ 5,400 = $48\frac{8}{9}$ ft. per sec. Ans.

39. If the uniform velocity (or the average velocity) and the time are given, and it is required to find the distance that a body having the given velocity will travel in the given time:

Rule 2.—*Multiply the velocity by the time.*

Or, $$d = v\,t.$$

Example 1.—The velocity of sound in still air is 1,092 feet per second; how many miles will it travel in 16 seconds?

Solution.—Reducing the 1,092 feet to miles, 1,092 ÷ 5,280 = $\frac{1092}{5280}$. Applying the rule, $\frac{1092}{5280} \times 16 = 3.31$ mi., nearly. Ans.

Example 2.—The piston speed of an engine is 11 feet per second, how many miles does the piston travel in 1 hour and 15 minutes?

Solution.—1 hour and 15 minutes reduced to seconds = 4,500 seconds = the time. 11 feet reduced to miles = $\frac{11}{5280}$ mile = velocity in miles per second. Applying the rule, $\frac{11}{5280} \times 4{,}500 = 9.375$ mi. Ans.

40. If the distance through which a body moves is given, and also its average or uniform velocity, and it is desired to know how long it takes the body to move through the given distance:

Rule 3.—*Divide the distance, or space passed over, by the velocity.*

Or, $$t = \frac{d}{v}.$$

Example 1.—Suppose that the radius of the crank of a steam engine is 15 inches and that the shaft makes 120 revolutions per minute; how long will it take the crankpin to travel 18,849.6 feet?

Solution.—Since the radius, or distance from the center of the shaft to the center of the crankpin, is 15 inches, the diameter of the circle it moves in is 15 inches × 2 = 30 inches = 2.5 feet. The circumference of this circle is 2.5 × 3.1416 = 7.854 feet. 7.854 × 120 = 942.48 feet = distance that the crankpin travels in 1 minute = velocity in feet per minute. Applying the rule, 18,849.6 ÷ 942.48 = 20 min. Ans.

Example 2.—A point on the rim of an engine flywheel travels at the rate of 150 feet per second; how long will it take it to travel 45,000 feet?

Solution.—Applying the rule,

45,000 ÷ 150 = 300 sec. = 5 min. Ans.

EXAMPLES FOR PRACTICE.

1. A locomotive has drivers 80 inches in diameter. If they make 293 revolutions per minute, what is the velocity of the train in (*a*) feet per second? (*b*) miles per hour?

Ans. { (*a*) 102.277 ft. per sec.
{ (*b*) 69.734 mi. per hr.

2. Assuming the velocity of steam as it enters the cylinder to be 900 feet per second, how far can it travel, if unobstructed, during the time the flywheel of an engine revolves 7 times, if the number of revolutions per minute are 120? Ans. 3,150 ft.

3. The average speed of the piston of an engine being 528 feet per minute, how long will it take the piston to travel 4 miles?

Ans. 40 min.

4. A speed of 40 miles per hour equals how many feet per second?

Ans. 58⅔ ft.

5. The earth turns around once in 24 hours. If the diameter be taken as 8,000 miles, what is the velocity of a point on the equator in miles per minute? Ans. 17.45⅓ mi. per min

6. The stroke of an engine is 28 inches. If the engine makes 11,400 strokes per hour, (*a*) what is its speed in feet per minute? (*b*) how far will this piston travel in 11 minutes?

Ans. { (*a*) 443⅓ ft. per min.
{ (*b*) 4,876 ft. 8 in.

FORCE.

GENERAL PRINCIPLES.

41. Force is known only by the effects it produces on matter. Forces cannot, therefore, be compared in the same way that quantities of matter or distances are compared; the only way in which forces can be examined in reference to one another is by noting their relative effects in the production of motion or a change of state in matter.

The most familiar conception of force is that of a push or pull that tends to produce or destroy motion. If the force is great enough, its effect is seen in a change in the state of motion of the body on which it acts; that is, it either produces motion in the body or destroys some or all of the motion already existing. If, however, the body acted on by a force is so situated that the force applied to it is opposed by a resisting force of equal magnitude, no change in motion is produced. In this case the force is not perceivable, unless some other force is introduced whose effects will reveal the existence of the first force.

42. Forces are called by various names according to the ways in which they manifest themselves. Manifestations of force are: *attraction*, *repulsion*, *cohesion*, *adhesion*, *acceleration*, *retardation*, *resistance*, etc., and the forces producing these manifestations are called *attractive*, *repulsive*, *cohesive*, *adhesive*, *accelerating*, *retarding*, *resisting*, etc. forces.

43. Comparison of Forces.—In considering the effects of a force on a body, some standard of comparison must be used. The standard most commonly adopted in English-speaking countries is the **pound**, which is the force required to raise a standard mass of matter from the ground under certain specified conditions.

In practice, force is always regarded as a pressure; that is, a force is considered the equivalent of the pressure exerted by a weight. For example, the effect of a force of

20 pounds acting upon a body is the same as the pressure of 20 pounds exerted by a weight of 20 pounds.

44. In order that we may compare the effect of a force on a body with that of another force on another body, it is necessary that the following *three conditions* be fulfilled in regard to both bodies:

1. The point of application, or point at which the force acts upon the body, must be known.
2. The direction of the force, or, what is the same thing, the straight line along which the force tends to move the point of application, must be known.
3. The magnitude or value of the force in comparison with the given standard must be known.

45. Reduction of Forces.—When a number of forces act upon a body and produce a certain effect, it is often necessary to find a *single* force that, when substituted for the given forces, will produce the *same* effect. In order to find the point of application, the direction, and the magnitude of this single force, it is necessary to know the above three conditions of every one of the given forces.

NEWTON'S LAWS OF MOTION.

46. The fundamental principles of the relations between force and motion were first stated by Sir Isaac Newton, and are called *Newton's Three Laws of Motion.* They are as follows:

1. *All bodies continue in a state of rest, or of uniform motion in a straight line, unless acted upon by some external force that compels a change.*
2. *Every motion or change of motion is proportional to the acting force, and takes place in the direction of the straight line along which the force acts.*
3. *To every action there is always opposed an equal reaction.*

These laws in their accepted form, as just given, have been more or less indirectly derived from experiment, but they are so comprehensive as to defy complete experimental verification.

47. Exemplification of the First Law.—In the first law of motion, it is stated that a body once set in motion by any force, no matter how small, will move forever in a straight line, and always with the same velocity, unless acted upon by some other force that compels a change. It is not possible to actually verify this law, on account of the earth's attraction for all bodies, but, from astronomical observations, we are certain that the law is true. This law is often called the **law of inertia.**

48. The word **inertia** is so abused that a full understanding of its meaning is important. Inertia is not a force, although it is often so called. If a force acts upon a body and puts it in motion, the effect of the force is stored in the body, and a second body, in stopping the first, will receive a blow equal in every respect to the original force, assuming that there has been no resistance of any kind to the motion of the first body.

It is dangerous for a person to jump from a fast-moving train, for the reason that, since his body has the same velocity as the train, it has the same force stored in it that would cause a body of the same weight to take the same velocity as the train, and the effect of a sudden stoppage is the same as the effect of a blow necessary to give the person that velocity.

By "bracing" himself and jumping in the same direction that the train is moving, and running, he brings himself gradually to rest, and thus reduces the danger. If a body is at rest, it must be acted upon by a force in order to be put in motion, and, no matter how great the force may be, it cannot be *instantly* put in motion.

The resistance thus offered to being put in motion is commonly, but erroneously, called the *resistance of inertia.* It should be called the *resistance due to inertia.*

49. Exemplification of the Second Law.—From the second law, we see that if two or more forces act upon a body, their final effect on the body will be in proportion to their magnitudes and to the directions in which they act.

Thus, if the wind is blowing due west with a velocity of 50 miles per hour, and a ball is thrown due north with the same velocity, or 50 miles per hour, the wind will carry the ball west while the force of the throw is carrying it north, and the combined effect will be to cause it to move northwest.

The amount of departure from due north will be proportional to the force of the wind and independent of the velocity due to the force of the throw.

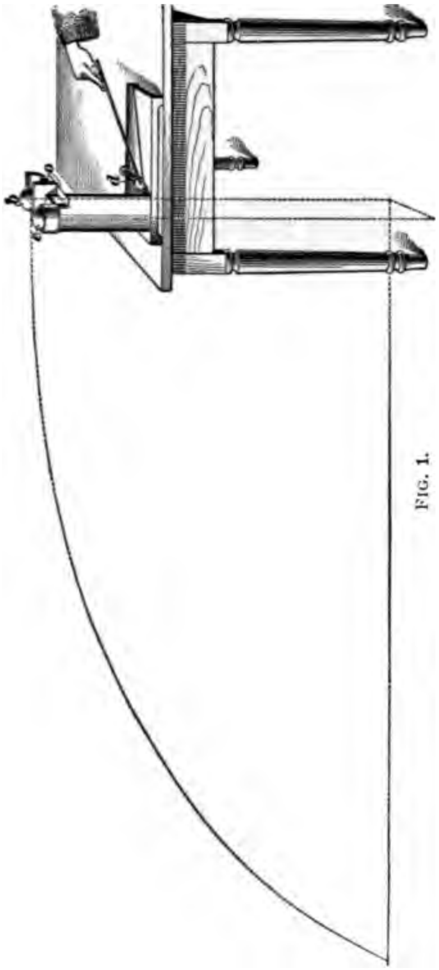


FIG. 1.

50. In Fig. 1 a ball *c* is supported in a cup, the bottom of which is attached to the lever *o* in such a manner that *o* will swing the bottom horizontally and allow the ball to drop. Another ball *b* rests in a horizontal groove that is provided with a slit in the bottom. A swinging arm is actuated by the spring *d* in such a manner that, when drawn back, as shown, and then released, it will strike the lever *o* and the ball *b* at the same time. This gives *b* an impulse in a horizontal direction, and swings *o* so as to allow *e* to fall.

On trying the experiment, it is found that *b* follows a path shown by the curved dotted line, and reaches the floor at the same instant as *c*, which drops vertically. This shows that the force that gave the first ball its horizontal movement had no effect on the vertical force that compelled both balls to fall to the floor; the vertical force produces the same effect as if the horizontal force had not acted. The second law may also be stated as follows: *A force has the same effect in producing motion, whether it acts upon a body at rest or in motion, and whether it acts alone or with other forces.*

51. Exemplification of the Third Law.—The third law states that action and reaction are equal. A man cannot lift himself by his boot straps for the reason that he presses downwards with the same force that he pulls upwards; the downward reaction equals the upward action, and is opposite to it.

In springing from a boat, we must exercise caution or the reaction will drive the boat from the shore. When we jump from the ground, we tend to push the earth from us, while the earth reacts and pushes us from it.

EXAMPLE.—Two men pull on a rope in opposite directions, each exerting a force of 100 pounds; what is the force that the rope resists?

SOLUTION.—Imagine the rope to be fastened to a tree, and that one man pulls with a force of 100 pounds. The rope evidently resists 100 pounds. According to Newton's third law, the reaction of the tree is also 100 pounds. Now, suppose the rope to be slacked, but that one end is still fastened to the tree; the second man then takes hold of the rope near the tree, and pulls with a force of 100 pounds, the first man pulling as before. The resistance of the rope is 100 pounds, as before, since the second man merely takes the place of the tree. He is obliged to exert a force of 100 pounds to keep the rope from slipping through his fingers. If the rope is passed around the tree, and each man pulls an end with a force of 100 pounds in the same and parallel directions, the stress in the rope is 100 pounds, as before, but the tree must resist the pull of both men, or 200 pounds.

52. **Dynamics,** also called **kinetics,** is that branch of mechanics that treats of forces and their effects when they *produce a change in motion* in the bodies on which they act.

53. **Statics** is that branch of mechanics that treats of forces and their effects when they do *not produce a change in motion* in the bodies on which they act.

GRAVITATION AND WEIGHT.

54. Every body in the universe exerts a certain attractive force on every other body, which tends to draw the two together. To scientists, this attractive force is known as **gravitation.**

If a body is held in the hand, a downward pull is felt, and if the hold is loosened, the body will fall to the ground. This pull, which we commonly call **weight,** is the attraction between the earth and the body.

55. The attraction between the earth and bodies at or near its surface is denoted by the term **force of gravity.** This attraction is generally considered as acting along the line joining the center of gravity of the body and the center of the earth. By **center of gravity** is meant that point of a body at which its whole weight may be assumed to be concentrated.

56. The weight of a body is directly proportional to the force of gravity. From this it follows that the weight of a body can only be uniform everywhere if the force of gravity is uniform. As a matter of fact, the force of gravity varies in different locations; consequently, the weight of the body is not the same at all points on the surface of the earth. This has been conclusively shown by sensitive spring balances.

ACCELERATING AND RETARDING FORCES.

57. According to the first law of motion, if a body is set in motion by a force and the force then ceases to act, the body will continue to move at the rate it had at the instant the action of the force was discontinued, unless acted upon by some other force.

If, however, a force acts upon a body for a given period of time, say 1 second, and imparts to it a certain amount of motion, and then, instead of ceasing to act, acts with the same intensity during the next second, it will impart to the body an increase in velocity equal to the velocity imparted during the first second. Then, the velocity at the end of the second second will be twice that at the end of the first. During the third second a like increase in velocity will be produced, making the velocity at the end of the third second three times as great as at the end of the first. This uniform increase in velocity will continue as long as the constant force continues to act on the body. A constant force when producing a constant acceleration is called a **constant accelerating force.**

58. If a constant force is applied to a body in motion in such a manner that it opposes the motion, its effect will be to reduce the motion by a certain amount, which will be the same for each second during which it acts. In this case, the force is called a **constant retarding force.**

We thus see that the effect of a constant force acting upon a body in motion is to produce a uniform acceleration or retardation in the velocity of motion of the body, it being assumed that the motion of the body is not opposed by varying resisting forces.

59. Acceleration Due to the Force of Gravity.—If a body falls freely under the action of the force of gravity, its velocity will increase at a uniform rate; in other words, it will be accelerated. Since the force of gravity varies in different localities, it follows that the acceleration produced by it is not everywhere the same. The greatest range in the acceleration due to the force of gravity in the United States is from a minimum of about 32.089 feet per second up to a maximum of about 32.186 feet per second for each second. In the latitude of Scranton, Pa., and at the level of the sea, the acceleration is nearly 32.16 feet per second;

this value will be used in all calculations in this Course that involve the use of acceleration due to the force of gravity. In accordance with the practice of most scientific writers, we will denote the acceleration due to the force of gravity by the letter g.

60. Mass.—If the weight of a body at any place, as determined by a spring balance, is divided by the acceleration due to the force of gravity at that place, a numerical value will be obtained that, for the same body, will be the same wherever it may be weighed. This quotient is called the **mass** of the body, and is generally designated by the letter m.

Rule 4.—*To find the mass of a body, divide its weight by the acceleration due to the force of gravity.*

Let W = the weight of a body;
g = acceleration due to gravity;
m = mass of the body.

Then,
$$m = \frac{W}{g}.$$

EXAMPLE.—What is the mass of a body weighing 96.48 pounds?

SOLUTION.—Applying the rule, $m = \frac{96.48}{32.16} = 3$. Ans.

61. Law of Gravitation.—The attractive force by which one body tends to draw another body toward it is directly proportional to its mass and inversely proportional to the square of the distance between their centers of gravity.

62. Laws of Weight.—

1. *Bodies weigh most at the surface of the earth. Below the surface, the weight decreases directly as the distance to the center of the earth decreases.*

2. *Above the surface, the weight decreases inversely as the square of the distance.*

63. Change in Motion of a Body.—A change in the motion of a body cannot take place without the action of an accelerating or retarding force. The force required to produce a given acceleration or retardation in a body is given by the following rule, where

f = force in pounds;
a = acceleration or retardation in feet per second.

Rule 5.—*Multiply the mass of the body by the acceleration, or retardation, in feet per second.*

Or, $$f = m a.$$

Since $m = \frac{W}{g}$ (see rule 4), this may also be written $f = \frac{W}{g} a$.

EXAMPLE.—What force will be required to give a body weighing 90 pounds an acceleration of 5 feet per second?

SOLUTION.—Applying rule **5**, we get

$$f = \frac{90}{32.16} \times 5 = 13.99+ \text{ lb., say } 14 \text{ lb. Ans.}$$

64. According to the first law of motion, a body in motion not acted upon by any external force will continue its motion without any further application of a force. In practice, however, the motion of a body is always opposed by some resisting force or forces. According to the third law of motion, the force required to overcome the resistance is equal to the resistance.

The opposing forces are usually constant, or nearly so. Taking the opposing forces into account, the actual force required to accelerate a body meeting with resistance will be the sum of the accelerating force and the opposing forces.

ILLUSTRATION.—Imagine a weight of 321.6 pounds to be lying on a smooth plane surface. Assume that it has been determined experimentally that a force of 100 pounds is required to be exerted continually to overcome the friction between the weight and the surface. What force will be required to produce an acceleration of 2 feet per second?

By rule **5**, the accelerating force is $\frac{321.6}{32.16} \times 2 = 20$ pounds. As a force of 100 pounds must be exerted continually to overcome the resistance due to friction, a force of $100 + 20 = 120$ pounds will be required to produce the required acceleration.

65. The question is often asked, what force is required to start a flywheel and keep it going at a stated number of revolutions per minute? This question, or similar questions, cannot be answered without knowing the time in which the flywheel is to attain the given speed. An accelerating force depending on the mass of the wheel, its diameter, the distribution of the material of which it is made, the time of acceleration, and the constant resistances, is required to bring the wheel up to its speed. When this speed has been attained, the force required to keep it going will be that required to overcome the frictional and air resistances.

MOMENTUM.

66. Experience teaches us that the same force acting upon bodies of different weights produces different effects. For example, if a given force imparts a velocity of 10 feet per second in a certain time to a body weighing 1 pound, we know from experience and observation that it cannot impart the same velocity in the same time when acting upon a body weighing 1,000 pounds.

Scientists have shown that the velocity imparted to a body in a given time by a force *varies directly as the force* and *inversely as the mass* of the body. Hence, forces may be compared with one another by comparing their effects in imparting velocities to bodies whose masses are known.

67. The product obtained by multiplying the mass of a body by its velocity in feet per second is called the **momentum** of the body; it represents the magnitude of the force that will produce the given velocity in the body in 1 second. Hence, we may call momentum the **time effect** of a force.

68. According to the third law of motion, action and reaction are equal to each other. Consequently, if the force required to produce a stated momentum in a given time is known, it is likewise known what force is required to *destroy* this momentum, or to bring the body to rest, in an equal period of time.

When a liquid body is flowing in a stream, as from a nozzle, the weight to be considered in problems involving momentum is the weight of the liquid discharged in 1 second. For example, let it be required to estimate the force with which a man must hold the nozzle of a fire hose to prevent its slipping through his hands when a stream of water issues from it with a velocity of 20 feet per second, the area of the opening in the nozzle being 1 square inch and the weight of a cubic inch of water .0361 pound. The volume of water discharged in 1 second is $1 \times 12 \times 20 = 240$ cubic inches, and since the weight of 1 cubic inch is .0361 pound, the weight discharged per second is $240 \times .0361 = 8.664$ pounds. The momentum of the stream is $\frac{8.664}{32.16} \times 20 = 5.38$ pounds. This is the constant force required to give a body of water weighing 8.664 pounds a velocity of 20 feet per second, in 1 second; it also represents the magnitude of the reaction on the nozzle, and the man must hold the nozzle with a force equal to the reaction, or 5.38 pounds, in order to prevent its slipping through his hands.

WORK, POWER, AND ENERGY.

WORK.

69. **Work** is the overcoming of resistance continually occurring along the path of motion.

Motion in itself is not work; a force must overcome a resistance in order that work may be done.

70. **Unit of Work.**—The unit by which the work done by a force is measured is the work done in overcoming a resistance of 1 pound through a space of 1 foot; this unit is called a **foot-pound.** According to the definition, it may be considered as the force required to raise 1 pound 1 foot vertically. All work is measured by this standard.

A horse going up a hill does an amount of work equal to its own weight, plus the weight of the wagon and its contents, plus the frictional resistances reduced to an equivalent weight, multiplied by the vertical height of the hill. Thus, if the horse weighs 1,200 pounds, the wagon and contents 1,200 pounds, and the frictional resistances equal 400 pounds, then if the vertical height of the hill is 100 feet, the work done is equal to (1,200 + 1,200 + 400) × 100 = 280,000 foot-pounds.

Rule 6.—*In all cases the force (or resistance) multiplied by the distance through which it acts equals the work. If a weight is raised, the weight multiplied by the vertical height of the lift equals the work.*

71. The total amount of work done in overcoming a given resistance through a given distance is independent of time; that is, it is immaterial whether it takes 1 minute or 1 year in which to do it; but in order to compare the *rate* at which work is done by different machines with a common standard, time must be considered. If one machine does a certain amount of work in 10 minutes and another machine does exactly the same amount of work in 5 minutes, the second machine can do twice as much work as the first in an equal period of time.

POWER.

72. **Power** is a term used to denote the rate at which work is done.

73. The common unit used for expressing the rate at which work is done is the **horsepower.**

One horsepower is 33,000 foot-pounds of work per minute; in other words, it is 33,000 pounds raised vertically 1 foot in 1 minute, or 1 pound raised vertically 33,000 feet in 1 minute, or any combination that will, when multiplied together, give 33,000 foot-pounds in 1 minute. Thus, the work done in raising 110 pounds vertically 5 feet in 1 second is a horsepower; for, since in 1 minute there are 60 seconds, 110 × 5 × 60 = 33,000 foot-pounds in 1 minute.

EXAMPLE.—In a steam engine the force impelling the piston forwards and backwards is 10,000 pounds. This force overcomes the resistance due to the load at the rate of 600 feet per minute; that is, it moves the piston back and forth at that rate. What is the horsepower of the engine?

SOLUTION.—According to rule 6, the work done is $10{,}000 \times 600 = 6{,}000{,}000$ foot-pounds per minute. Then, as 33,000 foot-pounds per minute represent a horsepower, the horsepower of the engine is $6{,}000{,}000 \div 33{,}000 = 181.818$. Ans.

ENERGY.

74. Energy is a term used to express the ability of an agent to do work.

75. Kinetic Energy.—If we have a body at rest, a certain amount of force must be exerted and a certain amount of work must be done to set it in motion. A part of this force will be required to overcome those resistances outside of the body, such as friction and the resistance of the air, that oppose the motion of all bodies with which we have to do; another part acts to overcome the inertia of the body, to start it from its state of rest, and give it motion (see Newton's first law). The force that overcomes the resistance due to the inertia of the body does work, and the work so performed is stored in the body; in being brought to rest, the body is capable of overcoming a resistance and of doing an amount of work exactly equal to the work expended in giving it motion. The ability that the moving body has to do work while being brought to rest is called the **kinetic energy** of the body.

76. Rule for the Energy of a Moving Body.—

Let w = weight of body in pounds;
v = its velocity in feet per second;
E = kinetic energy in foot-pounds.

Then, the kinetic energy of a moving body may be found as follows:

Rule 7.—*Multiply the weight of the body by the square of its velocity and divide the product by twice the acceleration due to the force of gravity.*

Or, $E = \frac{w v^2}{64.32}$.

Thus, if a weight is raised a certain height, an amount of work is done equal to the product of the weight and the vertical height. If a weight is suspended at a certain height and allowed to fall, it will do the same amount of work in foot-pounds that was required to raise the weight to the height through which it fell.

EXAMPLE 1.—If a body weighing 25 pounds falls from a height of 100 feet, how much work can it do?

SOLUTION.—Work $= w h = 25 \times 100 = 2,500$ ft.-lb. Ans.

It requires the same amount of work or energy to stop a body in motion within a certain time as it does to give it that velocity in the same length of time.

EXAMPLE 2.—A body weighing 50 pounds has a velocity of 100 feet per second; what is its kinetic energy?

SOLUTION.—Kinetic energy $= \frac{w v^2}{64.32} = \frac{50 \times 100^2}{64.32} = 7,773.63$ ft.-lb. Ans.

EXAMPLE 3.—In the last example, how many horsepower will be required to give the body this amount of kinetic energy in 3 seconds?

SOLUTION.— 1 horsepower = 33,000 pounds raised 1 foot in 1 minute.

If 7,773.63 foot-pounds of work are done in 3 seconds, in 1 second there will be done $\frac{7,773.65}{3} = 2,591.21$ foot-pounds of work. 1 horsepower = 33,000 foot-pounds per minute = 33,000 ÷ 60 = 550 foot-pounds per second.

The number of horsepower required will be

$$\frac{2,591.21}{550} = 4.\dot{7}113 \text{ H. P.} \quad \text{Ans.}$$

77. Potential energy *is latent energy; it is the energy that a body at rest is capable of giving out under certain conditions.*

If a stone is suspended by a string from a high tower, it has potential energy. If the string is cut, the stone will fall to the ground, and during its fall its potential energy will change into kinetic energy, so that at the instant it strikes the ground its potential energy is wholly changed into kinetic energy

At a point equal to one-half the height of the fall, the potential and kinetic energies are equal. At the end of the first quarter the potential energy is three-fourths and the kinetic energy one-fourth; at the end of the third quarter the potential energy is one-fourth and the kinetic energy three-fourths.

A pound of coal has a certain amount of potential energy. When the coal is burned, the potential energy is liberated and changed into kinetic energy in the form of heat. The kinetic energy of the heat changes water into steam, which thus has a certain amount of potential energy. The steam acting on the piston of an engine causes it to move through a certain space, thus overcoming a resistance, changing the potential energy of the steam into kinetic energy, and thus doing work.

Potential energy, *then, is the energy stored within a body that may be liberated and produce motion, thus generating kinetic energy and enabling work to be done.*

78. The principle of **conservation of energy** teaches that energy, like matter, can never be destroyed. If a clock is put in motion, the potential energy of the spring is changed into kinetic energy of motion, which turns the wheels, thus producing friction.

The friction produces heat, which dissipates into the surrounding air, but still the energy is not destroyed—it merely exists in another form.

79. Work of Acceleration and Retardation.—The theoretical amount of work that must be done in order to start a body from a state of rest and accelerate it until it reaches a given velocity is equal to the kinetic energy of the body at the given velocity. Likewise, the theoretical amount of work that must be done on a moving body to retard it and finally bring it to rest is equal to the kinetic energy the moving body possessed at the moment retardation began. The work that must be done in changing the velocity of a body is equal to the difference in the kinetic energies at the initial and final velocities. Since the motion of all bodies is opposed by some resisting force or forces, the actual amount of work required to give a body the given velocity will be the sum of the work of acceleration and the work required to overcome the outside resisting forces.

EXAMPLE 1.—A body weighing 1,000 pounds is started from rest and is to attain a velocity of 88 feet per second in 2 minutes, passing over a distance of 5,280 feet in that time. If a constant force of 120 pounds must be exerted to overcome the frictional resistances, what work must be done?

SOLUTION.—According to this article, the work required to accelerate the body is $\frac{1{,}000 \times 88^2}{64.32} = 120{,}398$ foot-pounds. As a constant force of 120 pounds must act through a distance of 5,280 feet to overcome the frictional resistances, the work done in overcoming friction is $5{,}280 \times 120 = 633{,}600$ foot-pounds. Then, the total amount of work done is $120{,}398 + 633{,}600 = 753{,}998$ ft.-lb. Ans.

EXAMPLE 2.—What horsepower will be required to do the work calculated in the last example?

SOLUTION.—As the work is done in 2 minutes, the horsepower is $\frac{753{,}998}{2 \times 33{,}000} = 11.424$ H. P., nearly. Ans.

FORCE OF A BLOW.

80. The questions are frequently asked, with what force will a falling hammer strike, or with what force will a projectile fired from a gun strike an object? These questions cannot be answered directly, as they are based on a misconception. A moving body possesses kinetic energy, or ability to do work, which ability can only be expressed in foot-pounds, but not in pounds of force, since the work done by the hammer or projectile in coming to rest is not a manifestation of force, but of energy.

Work is the product of force into distance; hence, if the amount of work a body has done or is capable of doing is known, the force can be determined for each case if, by some means, it is possible to determine exactly the distance in which the work is done. This distance depends on various resistances, such as that due to moving the object struck, the resistance to penetration, friction, the resistance to shearing or deformation of the body, etc. The distance through which these resisting forces act is generally indeterminate, and since the average of the resisting forces varies generally with the distance, this average resisting force is also indeterminate; hence, the force that, acting through a distance, will absorb all the kinetic energy of the hammer or projectile cannot be determined for the practical reasons given.

COMPOSITION AND RESOLUTION OF FORCES.

81. According to Art. **44**, in order that forces may be compared with one another, three conditions must be fulfilled. These conditions may all be represented by a line; hence, we may represent forces by lines. Thus, in Fig. 2, let *A* be the point of application of the force, let the length of the line *A B* represent its magnitude, and let the arrowhead indicate the direction in which the force acts; then, the line *A B* fulfils the three conditions and the force is fully represented.

FIG. 2.

COMPOSITION OF FORCES.

82. When two or more forces act upon a body at the same time along lines that meet in a common point, their combined effect on the body may be obtained by an application of the principle of the **triangle of forces.**

In Fig. 3 (*a*), let *A* and *B* be two forces having the magnitudes and directions represented by the two lines. To find

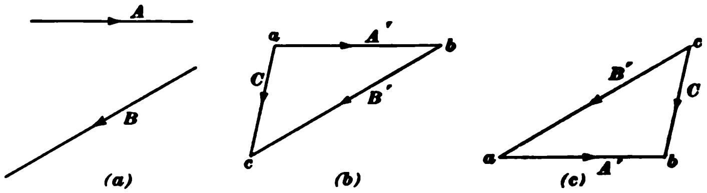


FIG. 3.

the effect due to the combined action of these two forces, draw in any convenient location a line parallel to either of the lines representing the two forces, making it equal in length, to some scale, to the magnitude of the force. Mark upon it an arrowhead pointing in the same direction as the arrowhead on the line representing the force. Then, from one extremity of the line just drawn draw a line parallel to the line representing the second force and equal in length, to the same scale, to the magnitude of the second force, and mark the arrowhead upon it. It is essential that the second line be so drawn that when passing over the two lines with a pencil, commencing at the beginning of either force, the arrowheads will both point in the same direction in reference to the direction of motion of the pencil.

The second line may be drawn from either end of the first line, but its direction must be made to fulfil the above absolutely essential condition. Thus, in Fig. 3 (*b*), the line B' has been drawn from the right-hand extremity of A'. Starting at *a* with a pencil and moving toward *b*, the pencil will move in the direction in which the arrowhead on A' points. Passing over B' from *b* to *c*, the pencil will move in the direction in which the arrowhead on B' points; we thus see that the lines are drawn correctly in reference to each other.

In Fig. 3 (*c*), the line B' has been drawn from the left-hand extremity of A'. Starting at *c* and following up the lines, it is seen that in this case the arrowheads both point in the same direction relative to the direction of motion of the pencil, thus showing the lines to be located correctly.

The two lines having been drawn, complete the triangle by drawing the line *C*. This line, called the **resultant**, represents the combined effect of the two forces; it gives the direction along which the two forces will act when combined. The magnitude of their combined effect is found by measuring this line by the scale with which A' and B' were laid off. The resultant will always have a direction opposite in sense to that of the forces; that is, if we pass a pencil around the triangle in the direction in which the arrowheads on the lines A' and B' point, the arrowhead on *C*, representing the direction of action of the resultant, must point in a direction opposite to that in which the pencil moves.

83. In practice it is often desired to find not only the magnitude and direction, but also the *actual location of the resultant*, the magnitudes and lines of action of the two forces being known. (By location of the forces and resultant is meant the location of the lines along which the forces actually act.) This can readily be done by producing the lines giving the location of the forces until they meet and then drawing the triangle of forces with their point of intersection as the starting point.

ILLUSTRATION.—In Fig. 4 is shown a head-frame erected at the mouth of a deep, vertical shaft. The hoisting rope that leads to the hoisting engine passes over a sheave at the top of the head-frame. A weight of 1,500 pounds hangs at the end of the rope. Neglecting the weight of the rope and sheave, what is the total pressure on the bearings of the sheave?

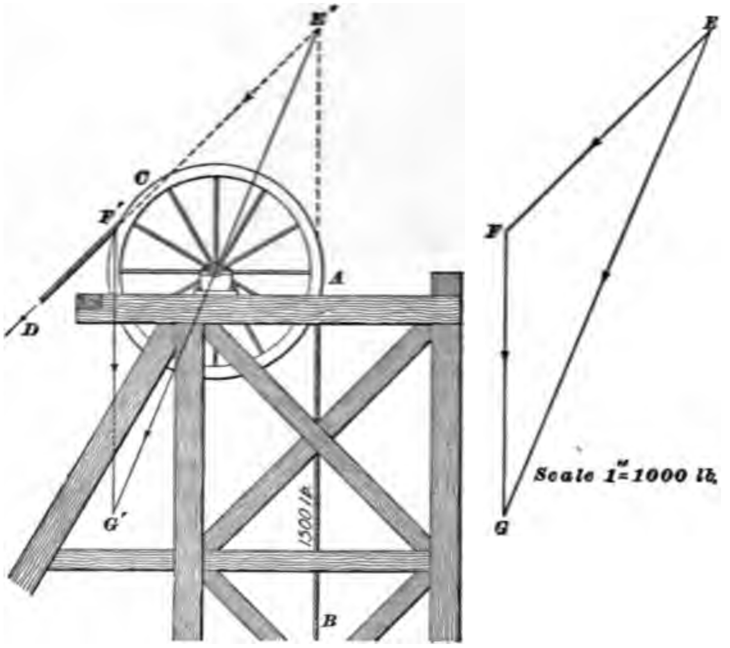


FIG. 4.

According to Art. **51**, the stress in both parts of the hoisting rope is 1,500 pounds. The part $A B$ of the rope supports the weight;

consequently, the force acting along $A\,B$ is downwards. The force acting along $C\,D$ is the pull exerted by the engine and is toward the engine. To find the resultant of these two forces, draw the triangle of forces. Choosing a scale of 1 inch per 1,000 pounds, draw the line $E\,F$ parallel to $C\,D$, making it $\frac{1500}{1000} = 1.5$ inches long. From F draw $F\,G$ parallel to $A\,B$ and $\frac{1500}{1000} = 1.5$ inches long. Join E and G; then $E\,G$ will be the resultant whose magnitude is measured by the same scale. Measuring $E\,G$, it is found to be 2.75 inches long. As each inch represents 1,000 pounds, the magnitude of the resultant is 2.75 × 1,000 = 2,750 pounds.

To find the actual location of the resultant in reference to $A\,B$ and $C\,D$, produce both lines, as shown in dotted lines, until they intersect at E'. Starting the triangle of forces at E', lay off $E'\,F'$ = 1.5 inches. From F' draw $F'\,G' = 1.5$ inches and parallel to $A\,B$. Join $E'\,G'$. Upon measurement it is found to be 2,750 pounds. As inspection shows that the resultant passes through the center of the bearings, the total pressure on the bearings is 2,750 pounds. Ans.

RESOLUTION OF FORCES.

84. Since two forces can be combined to form a single resultant force, we may also treat a single force as if it were the resultant of two forces whose action upon a body will be the same as that of a single force. Thus, in Fig. 5, the force $O\,A$ may be resolved into two forces $O\,B$ and $B\,A$. If the force $O\,A$ acts upon a body moving or at rest upon a horizontal plane, and the resolved force $O\,B$ is vertical and $B\,A$ horizontal, $O\,B$, measured to the same scale as $O\,A$, is the magnitude of that part of $O\,A$ that pushes the body *downwards*, while $B\,A$ is the magnitude of that part of the force $O\,A$ that is exerted in pushing the body in a *horizontal* direction. $O\,B$ and $B\,A$ are called the **components** of the force $O\,A$, and when these components are vertical and horizontal, as in the present case, they are called the *vertical component* and the *horizontal component* of the force $O\,A$.

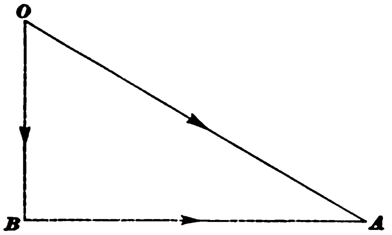


FIG. 5.

85. It frequently happens that the position, magnitude, and direction of a certain force is known, and that it is desired to know the effect of the force in some direction other than that in which it acts. Thus, in Fig. 5, suppose that OA represents, to some scale, the magnitude, direction, and line of action of a force acting upon a body at A, and that it is desired to know what effect OA produces in the direction BA. Now BA, instead of being horizontal, as in the figure, may have any direction. To find the value of the component of OA which acts in the direction BA, we use the following rule:

Rule 8.—*From one extremity of the line representing the given force, draw a line parallel to the direction in which it is desired that the component shall act; from the other extremity of the given force, draw a line perpendicular to the component first drawn, and intersecting it. The length of the component, measured from the point of intersection to the intersection of the component with the given force, will be the magnitude of the effect produced by the given force in the required direction.*

Thus, suppose OA, Fig. 5, represents a force acting upon a body resting upon a horizontal plane, and it is desired to know what *vertical pressure* OA produces on the body. Here the desired direction is vertical; hence, from one extremity, as O, draw OB parallel to the desired direction (vertical in this case), and from the other extremity draw AB perpendicular to OB and intersecting OB at B. Then OB, when measured to the same scale as OA, will be the value of the vertical pressure produced by OA.

86. Tangential Pressure.—One of the most familiar applications of the principle of the resolution of forces occurring in steam engineering is the case of the connecting-rod and crank. When the piston is at the end of its stroke and the crankpin is in a line drawn through the center of the cylinder and crank-shaft, a position that is expressed by saying the engine is "on the center," the pressure of the connecting-rod on the crankpin acts directly against the bearings of the shaft and there is no turning effect on the pin. After the pin leaves the center, the pressure exerted on it by the connecting-rod may be resolved into two components: One of these components acts in the direction of the center line PO of the crank (see Fig. 6) and merely exerts a pressure on the bearings of the shaft; the other component acts along the line AB at right angles to the center line PO and *tangent* to the circle described by the crankpin. This is the force that tends to turn the crank and is called the **tangential pressure** on the crankpin.

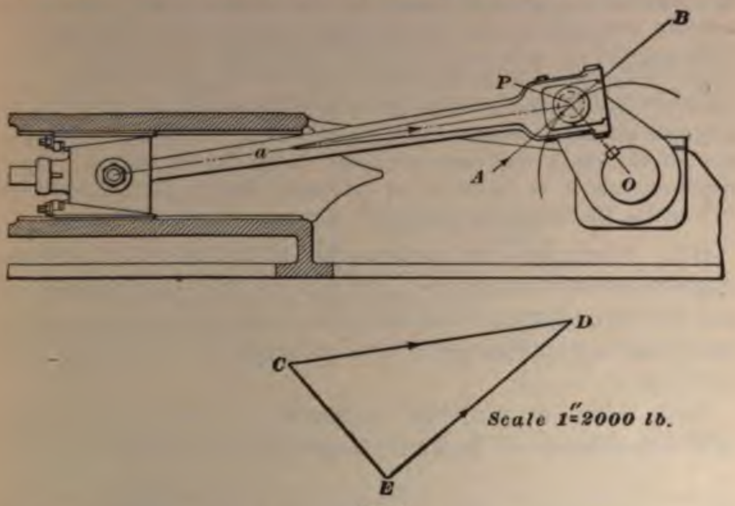


FIG. 6.

When the crank is on the center, there is no tangential pressure on the pin and no tendency to turn the crank. The tangential pressure gradually increases as the pin leaves the center and becomes greatest at the point where the connecting-rod and crank are at right angles to each other; it then decreases until the next center is reached. At the position where the connecting-rod and crank are at right angles to each other, the tangential pressure on the pin is equal to the total pressure exerted on it by the connecting-rod, and there is no component in the direction of the center line PO of the crank.

Example.—If a force of 3,000 pounds acts along the connecting-rod *a* in the direction of the arrow (see Fig. 6), what is the tangential pressure on the crankpin?

Solution.—As the tangential pressure is the pressure perpendicular to the crank, draw *A B* through the crankpin center at right angles to the center line of the crank; *A B* then represents the line along which the tangential pressure acts. Then, in any convenient location, draw *C D* parallel to the connecting-rod. Choosing a scale of 2,000 pounds = 1 inch, make the line *C D* 3,000 ÷ 2,000 = 1½ inches long. From *D* draw an indefinite line *D E* parallel to *A B*, and draw *C E* perpendicular to *D E*. Now *E D*, measured to the scale adopted, will be the magnitude of the tangential pressure for the position of the crank and connecting-rod shown in the figure. Upon measurement, it is found to be 1.3 inches long. Then 1.3 × 2,000 = 2,600 lb., the tangential pressure. Ans.

87. When the total pressure on the piston rod of a steam engine is known, the force acting along the connecting-rod and the force acting upon the guides can be determined by the following application of the principle of the resolution of force:

Draw a line, as *A B*, Fig. 7, parallel to the line of motion of the crosshead *a*. Make its length, to some scale, equal in magnitude to the force impelling the piston. From one extremity of *A B* draw a line *B C* parallel to the center line of the connecting-rod *b*. From the other extremity of *A B* draw a line at a right angle to *A B*, producing it until it intersects the line *B C* at *D*. Then, *B D* represents the force acting along the connecting-rod and *A D* represents the force acting upon the guides.

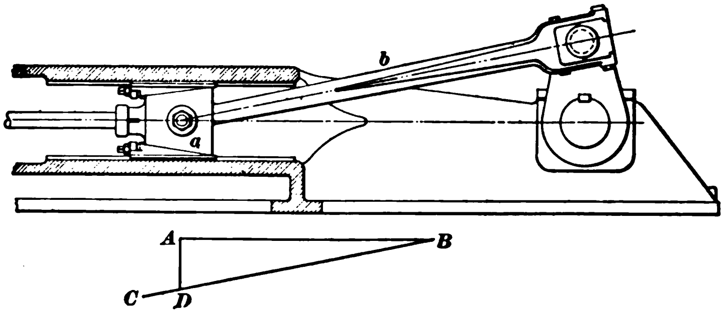


Fig. 7.

FRICTION.

88. Friction is the resistance that a body meets with from the surface upon which it moves.

89. The ratio between the *resistance* to the motion of a body due to friction and the *perpendicular* pressure between the surfaces is called the **coefficient of friction.**

If a weight W, as in Fig. 8, rests upon a horizontal plane, and has a cord fastened to it passing over a pulley a, from which a weight P is suspended, then, if P is just sufficient to start W, the ratio of P to W, or $\frac{P}{W}$, is the coefficient of friction between W and the surface it slides upon. The weight W is the perpendicular pressure, and P is the force necessary to overcome the resistance to the motion of W due to friction. If $W = 100$ pounds and $P = 10$ pounds, the coefficient of friction for this particular case would be $\frac{P}{W}$, $= \frac{10}{100} = .1$.

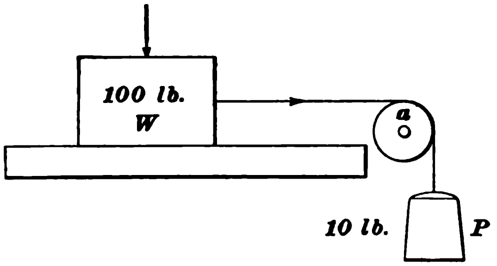


FIG. 8.

90. Laws of Friction.—

1. *Friction is directly proportional to the perpendicular pressure between the two surfaces in contact.*
2. *Friction is independent of the extent of the surfaces in contact when the total perpendicular pressure remains the same.*
3. *Friction increases with the roughness of the surfaces.*
4. *Friction is greater between surfaces of the same material than between those of different materials.*

5. *Friction is greatest at the beginning of motion.*

6. *Friction is greater between soft bodies than between hard ones.*

7. *Rolling friction is less than sliding friction.*

8. *Friction is diminished by polishing or lubricating the surfaces.*

91. Law 1 shows why the friction is so much greater on journals after they begin to heat than before. The heat causes the journal to expand, thus increasing the pressure between the journal and its bearing, and, consequently, increasing the friction.

Law 2 states that, no matter how small may be the surface that presses against another, if the perpendicular pressure is the same the friction will be the same. Therefore, large surfaces are used where possible, not to reduce the friction, but to reduce the wear and diminish the liability of heating.

For instance, if the perpendicular pressure between a journal and its bearing is 10,000 pounds, and the coefficient of friction is .2, the amount of friction is $10,000 \times .2 = 2,000$ pounds. Suppose that the area receiving the pressure is 80 square inches, then the amount of friction for each square inch is $2,000 \div 80 = 25$ pounds.

If the area receiving the pressure had been 160 square inches, the friction would have been the same, that is, 2,000 pounds; but the friction per square inch would have been $2,000 \div 160 = 12\frac{1}{2}$ pounds, just one-half as much as before, and the wear and liability to heat would be one-half as great also.

92. The values of the coefficient of friction given in the following tables are average values determined by General Morin, many years ago. Under certain conditions the coefficient may be considerably less than is given in the tables; it varies greatly, but the variation depends on so many conditions and the numerous experiments that have been made have given such contradictory results that no definite rules have yet been derived for determining the exact values under any condition. The student is, therefore, advised to use the values given in the tables, except where careful experiments have been made that give reliable values for the particular case under consideration. To find the force, in pounds, necessary to overcome friction, the coefficient taken from the table is multiplied by the perpendicular pressure, in pounds, on the surface considered. If the force acts at an angle to the surface, the perpendicular force can be found by resolving the given force into two components, one perpendicular and the other parallel to the surface.

TABLE I.

COEFFICIENTS OF FRICTION FOR PLANE SURFACES.

(Reduced from M. Morin's Data.)

Description of Surfaces in Contact.	State of the Surfaces.	Coefficient of Friction.
Wrought iron on cast iron.	Slightly greasy	.18
Wrought iron on bronze..	Slightly greasy	.18
Cast iron on cast iron....	Slightly greasy	.15
Cast iron on bronze......	Slightly greasy	.15
Bronze on bronze........	Dry	.20
Bronze on cast iron......	Dry	.22
Bronze on wrought iron..	Slightly greasy	.16
Cast iron, wrought iron, steel, and bronze sliding on one another or sliding on themselves.	Ordinary lubrication with lard, tallow, and oil	.07-.08
Cast iron, wrought iron, steel, and bronze sliding on one another or sliding on themselves.	Continuous lubrication	.05

TABLE II.

COEFFICIENTS OF FRICTION FOR JOURNAL FRICTION.

(*Reduced from M. Morin's Data.*)

Journals.	Bearings.	Lubricant.	Coefficient of Friction.	
			Ordinary Lubrication.	Continuous Lubrication.
Cast iron	Cast iron	Lard, olive oil, tallow	.07-.08	.03-.054
Cast iron	Bronze	Lard, olive oil, tallow	.07-.08	.03-.054
Wrought iron	Cast iron	Lard, olive oil, tallow	.07-.08	.03.-054
Wrought iron	Bronze	Lard, olive oil, tallow	.07-.08	.03-.054
Wrought iron	Lignum vitæ	Lard, olive oil	.11	
Bronze	Bronze	Oil	.10	
Bronze	Bronze	Lard	.09	

93. In the case of a weight sliding along a horizontal plane surface, the pressure is equal to the weight. When the surface is inclined, the weight acts vertically downwards, and the pressure perpendicular to the surface can be found by the principle of the resolution of forces. In many cases the pressure on the surfaces is due to the combined action of several forces that must be combined into one common resultant force.

94. The *work* that must be done in overcoming the resistance of friction depends on the distance through which the resistance is overcome. It may be calculated by the following rule:

Rule 9.—*Multiply the total pressure in pounds by the distance in feet and by the coefficient of friction.*

Let W = work in foot-pounds;
f = coefficient of friction;
p = total pressure in pounds;
d = distance in feet.

Then, $$W = p\,d\,f.$$

EXAMPLE.—The average perpendicular pressure on the guide of a steam engine due to the force impelling the piston is 2,500 pounds. The pressure due to the weight of the crosshead and connecting-rod is 400 pounds. The crosshead moves at the rate of 500 feet per minute; what horsepower is required to overcome the friction on the guides?

SOLUTION.—The total perpendicular pressure is 2,500 + 400 = 2,900 pounds. Since the lubrication is usually intermittent, the coefficient of friction, for a brass slipper working on a cast-iron guide, may be taken as .08. The resistance being overcome through a distance of 500 feet each minute, the work done in overcoming friction is 2,900 × 500 × .08 = 116,000 foot-pounds per minute. Then, the horsepower is 116,000 ÷ 33,000 = 3.52 H. P., nearly. Ans.

95. In the case of a shaft rotating in a bearing, the distance through which friction is overcome each minute is found by multiplying the circumference of the journal by the number of revolutions per minute. For a shaft, or any other body rotating in a bearing, the force required to overcome friction, as calculated by multiplying the pressure by the coefficient of friction, is the force that must be applied *at the surface of the journal.*

96. Allowable Pressures.—It has been found by experience that when the pressure per unit of area exceeds a certain amount, the lubricant will be forced out and the bodies rubbing on each other will heat and, finally, seize.

The pressures that can safely be allowed on the bearings of steam engines, on guides, thrust bearings, crankpins, crosshead pins, etc. vary considerably, being dependent to a large extent on the character of the workmanship, the degree of finish, the variation of the pressure, the character of the lubrication, and the quality of the lubricant. With fair workmanship, the following pressures per square inch represent average practice in steam-engine work:

TABLE III.

PRESSURES PER SQUARE INCH ALLOWABLE IN STEAM-ENGINE WORK.

Engine Bearing.	Slow-Speed Stationary Engines. Pounds.	High-Speed Stationary and Marine Engines. Pounds.
Crankpins, iron......	800	400 to 600
Crankpins, steel......	1,200	
Wristpin.............	1,200	600 to 800
Main bearings.......	200	200
Guides	100	100
Thrust bearings......		60

97. For crankpins, wristpins, and guides, the allowable pressures given represent the pressures corresponding to the maximum load, which in the case of a wristpin and crankpin occurs when the crank, connecting-rod, and piston rod are in a straight line, and in the case of guides, when the connecting-rod and crank are at right angles to each other. In the case of pins and journals, the area to be considered in calculating the pressure on the bearing is the **projected area,** which is found by multiplying the length of the journal by its diameter.

EXAMPLES FOR PRACTICE.

1. A body weighs 90 pounds; what is its mass? Ans. 2.799, nearly.

2. What force will be required to accelerate a body at the rate of 2 feet per second, the body weighing 450 pounds, and the frictional resistances being equal to 10 per cent. of the weight of the body?
Ans. 72.99 lb., nearly.

3. What work is done in raising 950 pounds 17 feet?
Ans. 16,150 ft.-lb.

4. If an engine does 205,000 foot-pounds of work per minute, what is its horsepower? Ans. 6.21 H. P., nearly.

5. What is the kinetic energy of a shell fired from a cannon with a velocity of 1,800 feet per second, the shell weighing 1,000 pounds?
Ans. 50,373,135 ft.-lb., nearly.

6. Taking the coefficient of friction at .15, what horsepower will be required to pull 100 pounds at a uniform speed of 5 feet per second along a level surface? Ans. .136 H. P.

CENTER OF GRAVITY.

98. *The center of gravity of a body is that point at which the body may be balanced, or it is the point at which the whole weight of a body may be considered as concentrated.*

This point is not always *in* the body; in the case of a horseshoe or a ring it lies outside of the substance of, but within the space enclosed by, the body.

In a moving body, the line described by its center of gravity is always taken as the path of the body. In finding the distance that a body has moved, the distance that the center of gravity has moved is taken.

The definition of the center of gravity of a body may be applied to a system of bodies if they are considered as being connected at their centers of gravity.

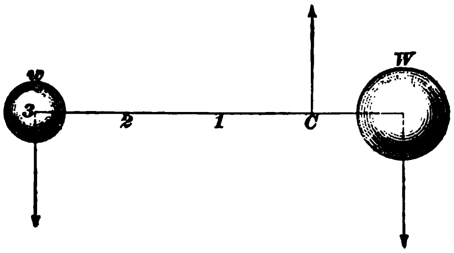


FIG. 9.

If w and W, Fig. 9, are two bodies of known weight, their center of gravity will be at C. This point C may be readily determined as follows:

Rule 10.—*The distance of the common center of gravity from the center of gravity of the large weight is equal to the weight of the smaller body multiplied by the distance between the centers of gravity of the two bodies, and this product divided by the sum of the weights of the two bodies.*

EXAMPLE.—In Fig. 9, $w = 10$ pounds, $W = 30$ pounds, and the distance between their centers of gravity is 36 inches; where is the center of gravity of both bodies situated?

SOLUTION.—Applying the rule, $10 \times 36 = 360$. $10 + 30 = 40$. $360 \div 40 = 9$ in., distance of center of gravity from center of large weight. Ans.

99. It is now very easy to extend this principle to find the center of gravity of any number of bodies, when their weights and the distances apart of their centers of gravity are known, by the following rule:

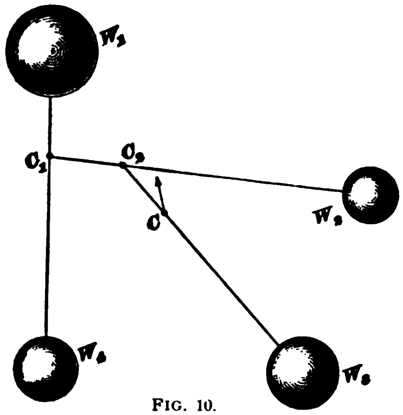


FIG. 10.

Rule 11.—*Find the center of gravity of two of the bodies, as W_1 and W_4 in Fig. 10. Assume that the weight of both bodies is concentrated at C_1, and find the center of gravity of this combined weight at C_1 and the weight of W_2. Let it be at C_2; then find the center of gravity of the combined weights of W_1, W_4, W_2 (concentrated at C_2), and W_3. Let it be at C; then C will be the center of gravity of the four bodies.*

100. To find the center of gravity of **any parallelogram:**

Rule 12.—*Draw the two diagonals, Fig. 11, and their point of intersection C will be the center of gravity.*

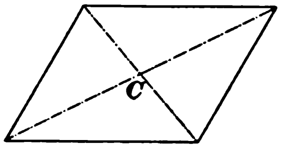


FIG. 11.

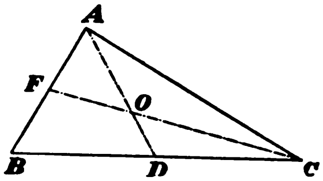


FIG. 12.

101. To find the center of gravity of a **triangle,** as $A\,B\,C$, Fig. 12:

Rule 13.—*From any vertex, as A, draw a line to the middle point D of the opposite side $B\,C$. From one of the other vertexes, as C, draw a line to F, the middle point of the opposite side $A\,B$; the point of intersection O of these two lines is the center of gravity.*

It is also true that the distance $D\,O = \frac{1}{3}\,D\,A$ and that $F\,O = \frac{1}{3}\,F\,C$; the center of gravity could have been found by drawing from any vertex a line to the middle point of the opposite side and measuring back from that side $\frac{1}{3}$ of the length of the line.

The center of gravity of **any regular plane figure** is the same as the center of the inscribed or circumscribed circle.

102. To find the center of gravity of **any irregular plane figure,** but of uniform thickness throughout, divide one of the parallel surfaces into triangles, parallelograms, circles, ellipses, etc., according to the shape of the figure; find the area and center of gravity of each part separately, and combine the centers of gravity thus found in the same

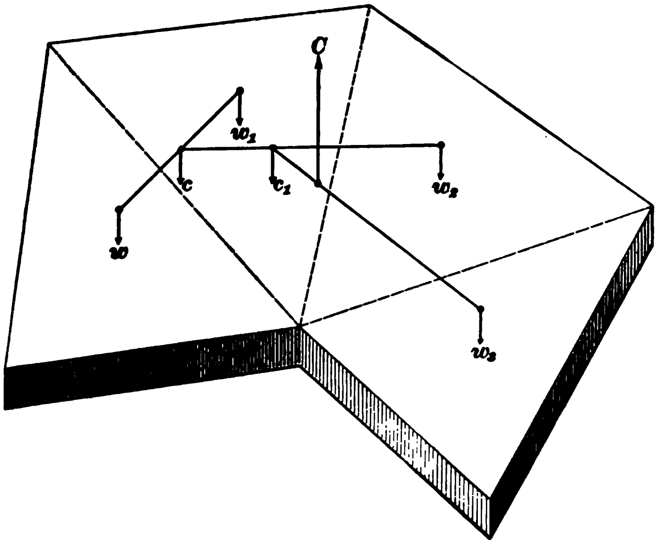


FIG. 13.

manner as in rule **11**, in this case, however, dealing with the *area* of each part instead of its weight. See Fig. 13.

103. Center of Gravity of a Solid.—In a body free to move, the center of gravity will lie in a vertical plumb-line drawn through the point of support. Therefore, to find the position of the center of gravity of an irregular solid, as the crank, Fig. 14, suspend it at some point, as *B*, so that it will move freely. Drop a plumb-line from the point of suspension, and mark its direction. Suspend the body at another point, as *A*, and repeat the process. The intersection *C* of the two lines will be directly over the center of gravity.

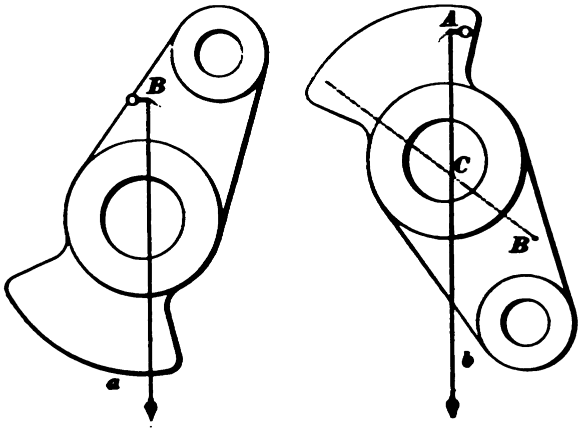


FIG. 14.

It is often desired to find the horizontal distance of the center of gravity from a given point of the body. In many cases this can readily be done by balancing the body on a knife edge, and then measuring, horizontally, the distance between the knife edge and the given point.

Since the position of the center of gravity depends wholly on the shape and weight of a body, it may be without the body, as in the case of a circular ring, whose center of gravity is the same as the center of the circumference of the ring.

By considering the symmetry of form, the position of the center of gravity of homogeneous solids may often be determined without analysis, or it may be limited to a certain plane, line, or point. Thus, the center of gravity of a sphere, or any other regular body, is situated at its center; of a cylinder, in the middle of its axis; of a thin plate having the form of a circle or regular polygon, in the center of the figure; of a straight wire of uniform cross-section, in the middle of its length.

EXAMPLES FOR PRACTICE.

1. A spherical shell has a wrought-iron handle attached to it. The shell is 10 inches in diameter and weighs 20 pounds. The handle is 1½ inches in diameter, and the distance from the center of the shell to the end of the handle is 4 feet. Where is the center of gravity? Take the weight of a cubic inch of wrought iron as .278 pound.
Ans. 13.612 in. from center of shell.

2. The distance between the centers of two bodies is 51 inches. The weights of the bodies being 20 and 73 pounds, where is the center of gravity? Ans. 10.968 in. from the center of the larger weight.

3. Weights of 5, 9, and 12 pounds lie in one straight line in the order named. Distance from the 5-pound weight to the 9-pound weight is 22 inches, and from the 9-pound weight to the 12-pound weight is 18 inches. Where is the center of gravity?
Ans. 13.923 in. from 12-lb. weight.

CENTRIFUGAL FORCE.

104. If a body is fastened to a string and whirled, so as to give it a circular motion, there will be a pull on the string that will be greater or less, according as the velocity increases or decreases. The cause of this pull on the string will now be explained.

Suppose that the body is revolved horizontally, so that the action of gravity upon it will always be the same. According to the first law of motion, a body put in motion tends to move in a straight line unless acted upon by some other force, causing a change in the direction. When the body moves in a circle, the force that causes it to move

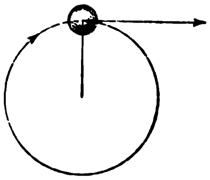


FIG. 15.

in a circle instead of in a straight line is exactly equal to the tension of the string. If the string were cut, the pulling force that drew it away from the straight line would be removed, and the body would then "fly off at a tangent"; that is, it would move in a straight line tangent to the circle, as shown in Fig. 15.

Since, according to the third law of motion, every action has an equal and opposite reaction, we call the force that acts as an equal and opposite force to the pull of the string the **centrifugal force,** and it acts *away* from the center of motion.

105. The other force, or tension, of the string is called the **centripetal force,** and it acts *toward* the center of motion. It is evident that these two forces, acting in opposite directions, tend to pull the string apart, and, if the velocity be increased sufficiently, the string will break. It is also evident that no body can revolve without generating centrifugal force.

106. To Find the Centrifugal Force of Any Revolving Body.—The value of the centrifugal force, expressed in pounds, of any revolving body is calculated by the following rule:

Rule 14.—*The centrifugal force equals the continued product of .00034, the weight of the body in pounds, the radius in feet (taken as the distance between the center of gravity of the body and the center about which it revolves), and the square of the number of revolutions per minute.*

Let F = centrifugal force in pounds;
W = weight of revolving body in pounds;
R = radius in feet of circle described by center of gravity of revolving body;
N = revolutions per minute of revolving body.

Then, $$F = .00034\ W R N^2.$$

EXAMPLE.—What is the tension in the string of Fig. 15, if the ball weighs 2 pounds and is revolved around at the rate of 500 revolutions per minute? The string is 2 feet long.

SOLUTION.—Applying the rule just given, we get

$$F = .00034 \times 2 \times 2 \times 500^2 = 340 \text{ lb. Ans.}$$

107. In flywheels, belt wheels, and pulleys the centrifugal force tends to tear the rim asunder; this tendency is resisted by the tenacity of the material of which the wheel is composed. Since the centrifugal force increases as the square of the number of revolutions, it will be seen that an apparently slight increase in the number of revolutions per minute may be sufficient to burst the wheel.

108. For solid cast-iron wheels and for built-up wheels of cast iron where the strength of the joint is equal to the strength of the rim, the greatest number of revolutions per minute that practice has indicated to be safe may be found by the following rule:

Rule 15.—*Divide 1,930 by the diameter of the wheel.*

Or. $$N = \frac{1,930}{d};$$

where d = diameter of the wheel in feet;
N = number of revolutions per minute.

EXAMPLE.—What is the maximum number of revolutions allowable for a cast-iron flywheel 27 feet 6 inches in diameter?

SOLUTION.—Applying rule **15**, we get

$$N = \frac{1,930}{27.5} = 70 \text{ rev. per min. Ans.}$$

EQUILIBRIUM.

109. When a body is at rest, all the forces that act upon it balance one another and are said to be in **equilibrium**. The most important force to be considered is the attraction of the earth, which acts upon every particle of a body. There are three states of equilibrium: *stable*, *unstable*, and *neutral*.

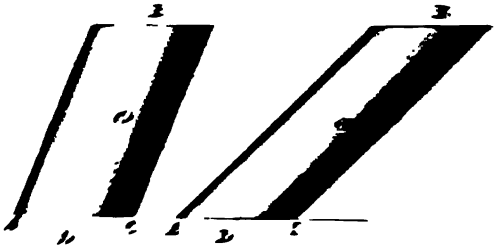
110. A body is in **stable equilibrium** when, if slightly rotated about its support in such manner as to change the

[illegible]

[illegible] **unstable equilibrium** [illegible]

[illegible] **neutral equilibrium** [illegible]

[illegible]



[illegible]

[illegible] whose base is oblique to [illegible] the centre of gravity of the cylinder. So long as the vertical line drawn through *O* falls between *A* and *C*, the cylinder will stand, but the instant it falls without the base, the cylinder will fall.

114. The center of gravity of a body has a tendency to always seek the lowest possible position.

2. That part of the lever $F\,b$ between the fulcrum and the weight is called the **weight arm,** and the part $F\,c$ between the fulcrum and the force is called the **force arm.**

In order that the lever will be in equilibrium (balance), *the force multiplied by the force arm must equal the weight multiplied by the weight arm;* that is, $P \times F\,c$ must equal $W \times F\,b$.

3. If F is taken as the center of a circle, and arcs are described through b and c, it will be seen that if the weight arm is moved through a certain angle, the force arm will move through the same angle. Since in the same or equal angles the lengths of the arcs are proportional to the radii with which they were described, it is seen that the force arm is proportional to the distance through which the force acts, and the weight arm is proportional to the distance through which the weight moves. Hence, instead of writing $P \times F\,c = W \times F\,b$, we might have written $P \times$ (distance through which P acts) $= W \times$ (distance through which W moves). This is the general law of all machines, and can be applied to any mechanism from the simple lever up to the most complicated arrangement. When stated in the form of a rule it is as follows:

Rule 1.—*The force multiplied by the distance through which it acts equals the weight multiplied by the distance through which it moves.*

4. In the above rule, it will be noticed that there are four requirements necessary for a complete knowledge of the lever, viz.: the force, the weight, the force arm, or distance through which the force acts, and the weight arm, or distance through which the weight moves. If any three are given, the fourth may be found by letting x represent the requirement that is to be found, and multiplying the force by the force arm and the weight by the weight arm; then, dividing the product of the two known numbers by the number by which x is multiplied, the result will be the requirement that is to be found.

EXAMPLE.—If the weight arm of a lever is 6 inches long and the force arm is 4 feet long, how great a weight can be raised by a force of 20 pounds at the end of the force arm?

SOLUTION.—In this example, the weight is unknown; hence, representing it by x, we have, after reducing the 4 feet to inches, $20 \times 48 = 960$ = force multiplied by the force arm, and $x \times 6$ = weight multiplied by the weight arm. Dividing the 960 by 6, the result is 160 pounds, the weight. Ans.

5. If the distance through which the force acted or the weight had moved had been given instead of the force arm or weight arm, and it were required to find the force or weight, the process would have been exactly the same, using the given distance instead of the force arm or weight arm.

EXAMPLE.—If, in the above example, the weight had moved $2\frac{1}{2}$ inches, through what distance would the force have acted?

SOLUTION.—In this example, the distance through which the force acts is required. Let x represent the distance. Then, $20 \times x$ = distance multiplied by force, and $2\frac{1}{2} \times 160 = 400$ = distance multiplied by the weight. Hence, $x = \frac{400}{20} = 20$ inches = distance through which the force arm moves. Ans.

The ratio between the weight and the force is $160 \div 20 = 8$. The ratio between the distance through which the weight moves and the distance through which the force acts is $2\frac{1}{2} \div 20 = \frac{1}{8}$. This shows that while a force of 1 pound can raise a weight of 8 pounds, the 1-pound weight must move through 8 times the distance that the 8-pound weight does. It will also be noticed that the ratio of the lengths of the two arms of the lever is also 8, since $48 \div 6 = 8$.

6. The law that governs the straight lever also governs the bent lever, but care must be taken to determine the true lengths of the lever arms, which are, in every case, *the perpendicular distances from the fulcrum to the line of direction of the weight or force.*

Thus, in Figs. 4, 5, 6, and 7, $F c$ in each case represents the force arm and $F b$ the weight arm.

7. A **compound lever** is a series of single levers arranged in such a manner that when a force is applied to the first it is communicated to the second, and from that to the third, and so on.

Fig. 8 shows a compound lever. It will be seen that when a force is applied to the first lever at *P* it will be communicated to the second lever at *P*, from this to the third lever at *P*, and thus raise the weight *W*.

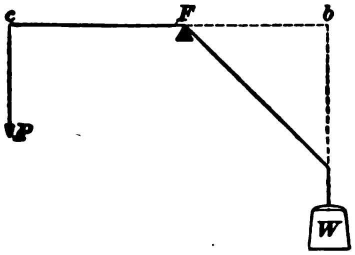


FIG. 4.

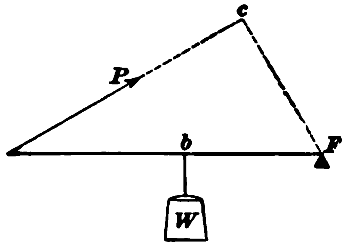


FIG. 5.

The weight that the force applied to the first lever could raise acts as the force of the second, and the weight that this force could raise through the second lever acts as the force of the third lever, and so on, no matter how many single levers make up the compound lever.

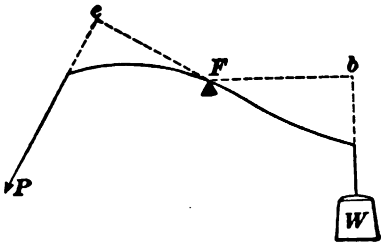


FIG. 6.

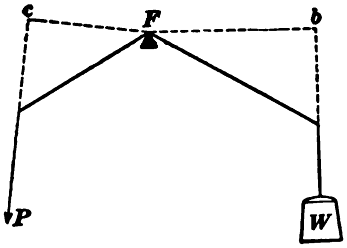


FIG. 7.

In this case, as in every other, the force multiplied by the distance through which it acts equals the weight multiplied by the distance through which it moves.

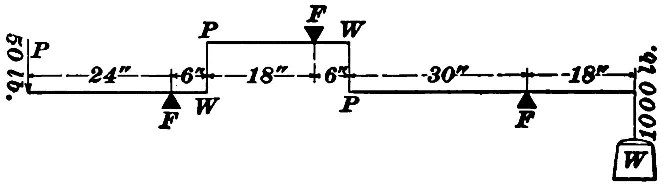


FIG. 8.

Hence, if we move the P end of the lever, say 4 inches, and the end carrying the weight W moves $\frac{1}{5}$ inch, we know that the ratio between P and W is the same as the ratio between $\frac{1}{5}$ and 4; that is, 1 to 20, and, hence, that 10 pounds at P will balance 200 pounds at W, without measuring the lengths of the different lever arms. If the lengths of the lever arms are known, the ratio between P and W may be readily obtained from the following rule:

Rule 2.—*The continued product of the force and each force arm equals the continued product of the weight and each weight arm.*

EXAMPLE.—If, in Fig. 8, $P F = 24$ inches, 18 inches, and 30 inches, respectively, and $W F = 6$ inches, 6 inches, and 18 inches, respectively, how great a force at P will it require to raise 1,000 pounds at W? What is the ratio between W and P?

SOLUTION.—Let x represent the force; then, $x \times 24 \times 18 \times 30 = 12{,}960 \times x$ = continued product of the force and each force arm. $1{,}000 \times 6 \times 6 \times 18 = 648{,}000$ = continued product of the weight and each weight arm; and, since $12{,}960 \times x = 648{,}000$,

$$x = \frac{648{,}000}{12{,}960} = 50 \text{ lb.} = \text{the force. Ans.}$$

$$1{,}000 \div 50 = 20 = \text{ratio between } W \text{ and } P. \text{ Ans.}$$

8. The **wheel and axle** consists of *two cylinders of different diameters rigidly connected*, so that they turn

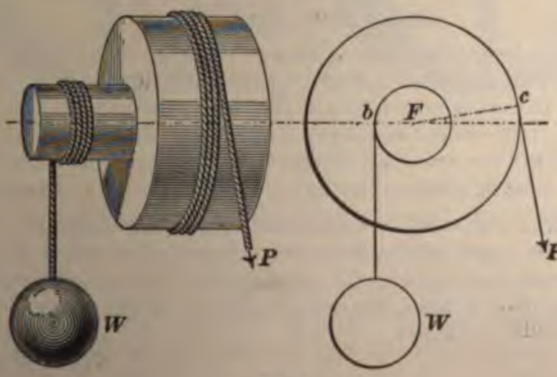


FIG. 9.

together about a common axis, as in Fig. 9. Then, as before, $P \times$ distance through which it acts $= W \times$ distance through which it moves; and, since these distances are proportional to the radii of the force cylinder and weight cylinder, $P \times Fc = W \times Fb$.

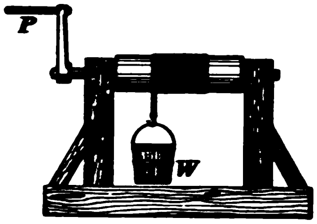


FIG. 10.

It is not necessary that an entire wheel be used; an arm, projection, radius, or anything that the force causes to revolve in a circle, may be considered as the wheel. Consequently, if it is desired to hoist a weight with a windlass, Fig. 10, the force is applied to the handle of the crank, and the distance between the center line of the crank-handle and the axis of the drum corresponds to the radius of the wheel.

EXAMPLE.—If the distance between the center line of the handle and the axis of the drum in Fig. 10 is 18 inches and the diameter of the drum is 6 inches, what force will be required at P to raise a load of 300 pounds?

SOLUTION.— $P \times (18 \times 2) = 300 \times 6$, or $P = 50$. Ans.

EXAMPLES FOR PRACTICE.

1. The lever of a safety valve is of the form shown in Fig. 1, where the force is applied at a point between the fulcrum and the weight lifted. If the distance from the fulcrum to the valve is 5½ inches and from the fulcrum to the weight is 42 inches, what total force is necessary to raise the valve, the weight being 78 pounds and the weight of valve and lever being neglected? Ans. 595.64 lb.

2. If, in Fig. 8, $PF = 10$ inches, 12 inches, 14 inches, and 16 inches, respectively, and $WF = 2$ inches, 3 inches, 4 inches, and 5 inches, respectively, (*a*) how great a weight can a force of 20 pounds raise? (*b*) What is the ratio between W and P? (*c*) If P moves 4 inches, how far will W move?

Ans. $\begin{cases} (a) & 4{,}480 \text{ lb.} \\ (b) & 224. \\ (c) & \frac{1}{56} \text{ in.} \end{cases}$

3. A windlass is used to hoist a weight. If the diameter of the drum on which the rope winds is 4 inches, and the distance from the center of the handle to the axis of the drum is 14 inches, how great a weight can a force of 32 pounds applied to the handle raise?

Ans. 224 lb.

PULLEYS.

9. Pulleys for the transmission of power by belts may be divided into two principal classes: (1) The *solid pulley* shown in Fig. 11, in which the hub, arms, and rim are one entire casting. (2) The *split pulley* shown in Fig. 12, which is cast in halves.

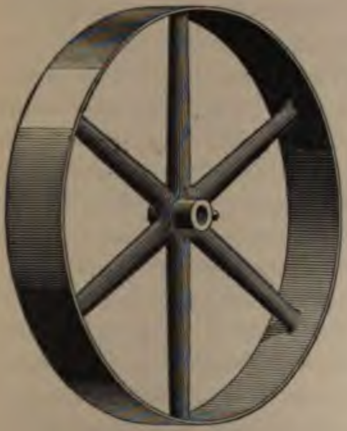


FIG. 11.

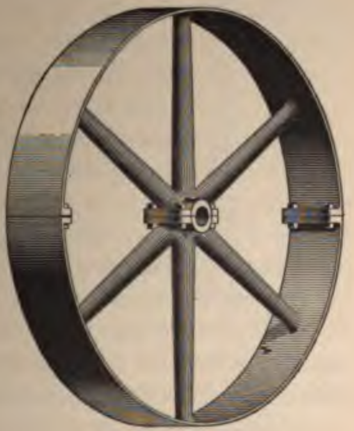


FIG. 12.

The latter style of pulley is more readily placed on and removed from the shaft than the solid pulley. Pulleys are generally cast in halves or parts when they are more than 6 feet in diameter; this is done on account of the shrinkage strain in large pulley castings, which renders them liable to crack as a result of the unequal cooling of the metal.

10. Of late years, wooden pulleys have come into extensive use. They are built of segments securely glued together, maple being the wood used. Wooden split pulleys can be procured that are fitted with removable bushings, thus allowing the same pulley to be readily adapted to various diameters of shafting. They are somewhat lighter than cast-iron pulleys.

11. Crowning Pulley Faces.—In Fig. 13, suppose the shafts *a* and *b* to be parallel. Let the pulley on the shaft *a* be cone-shaped, as shown. The right-hand side of the belt will be pulled ahead more rapidly than the left-hand side, because of the greater diameter and, consequently, greater speed of the right-hand end of the pulley. It has been observed that in this case the belt will leave its normal position, which is indicated by the dotted lines, and climb toward the part of the pulley that has the largest diameter, as shown in the illustration. This tendency of the belt to climb toward the high side is taken advantage of to make the belt stay on a pulley. Suppose the pulley on the shaft *a* is replaced with one formed of two equal cones, with the large diameter in the center. Then, each side of the belt will tend to climb toward the highest point, and the consequence is that the belt will stay in the center of the pulley.

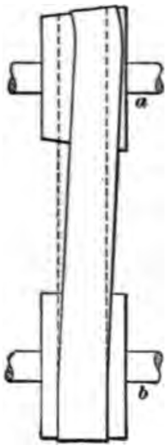


FIG. 13.

A pulley with its surface formed in this way is said to be **crowned.** The surface need not necessarily represent the frustums of two cones; it may simply be curved, as shown in Fig. 14. It is only required that the pulley be larger in diameter in the center.

As to the proper amount of crowning necessary, the practice of the makers of pulleys differs considerably; usually, though, it is from $\frac{3}{16}$ to $\frac{1}{2}$ inch per foot of width of the pulley face.

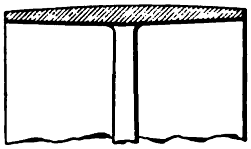


FIG. 14.

12. Balancing Pulleys.—All pulleys that rotate at high speeds should be balanced. If they are not, the centrifugal force that is generated by the rotation of the pulley, is greater on one side than on the other and will cause the pulley shaft to vibrate and shake. Pulleys are usually balanced in the manner shown in Fig. 15. A closely fitting arbor, which is simply a truly cylindrical piece of iron or steel, is driven into the bore of the pulley, which is then placed on the so-called "balancing ways." These are two planed iron or steel bars, preferably planed to a knife edge. These bars are placed on convenient supports far enough apart to allow the pulley to go between them. The bars should be carefully leveled with a spirit level, so that both bars are in the same horizontal plane. When the arbor rests on the ways and the pulley is slightly rotated, the latter will quickly come to rest with the heavy part downwards. Now, either some metal must be removed from the heavy part or some weight added to the light part. For small pulleys, a convenient substance to use for finding the proper location and weight of the counterweight is ordinary glazier's putty. By repeated trials the proper weight of the counterweight is found, and then a mass of metal of convenient shape equal in weight to the putty is fastened to the light part of the pulley in whatever way is safe and convenient. The proper weight and location of the counterweight will have been obtained when the pulley will be at rest on the balancing ways in any position in which it is put. In other words, the pulley is balanced when it is in neutral equilibrium. This balance is called a **standing balance.** The method just explained answers very well for pulleys that are narrow in comparison

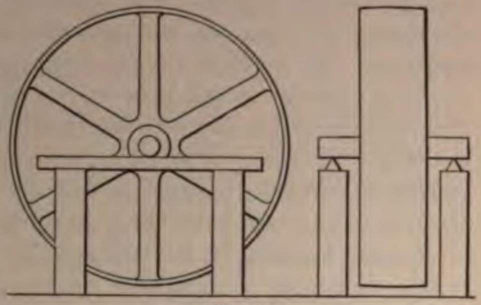


FIG. 15.

to their diameter. When pulleys with a wide face are run at a high speed, it is often found that serious vibrations are set up even when they are in perfect standing balance, thus showing that they do not possess the so-called **running balance.** No method has yet been found that will insure a running balance at all speeds, nor has any method become known by which it can be discovered directly where to apply the counterweight. The usual remedy for pulleys not possessing a running balance is to turn carefully the inside of the rim to run true with the outside of the pulley. Absence of running balance is, in all cases, due to an unequal distribution of metal in reference to the axis of the shaft, this unequal distribution being due to lack of homogeneity of the metal, to poor foundry work, or to poor lathe work.

13. Pulleys should run true in order that the strain, or tension, of the belt will be equal at all parts of the revolution, thus making the transmitting power equal. The smoother the surface of a pulley, the greater is its driving power.

The transmitting power of a pulley can be increased by covering its face with a leather or rubber band; this increases the driving power about one-quarter.

14. Relation Between Speeds of Drivers and Driven Pulleys.—The pulley that imparts motion to the belt is called the **driver**; that which receives the motion is called the **driven.**

The revolutions of any two pulleys over which a belt is run vary in an inverse proportion to their diameters; consequently, if a pulley 20 inches in diameter is driven by one 10 inches in diameter, the 20-inch pulley will make 1 revolution while the 10-inch pulley makes 2 revolutions, or they are in the ratio of 2 to 1. From this fact, the following formulas have been deduced:

Let $D =$ diameter of the driver;
$d =$ diameter of the driven;
$N =$ number of revolutions of the driver;
$n =$ number of revolutions of the driven.

Note.—The words revolutions per minute are frequently abbreviated to R. P. M.

15. To find the diameter of the driving pulley when the diameter of the driven pulley and the number of revolutions per minute of each is given:

Rule 3.—*The diameter of the driving pulley equals the product of the diameter and the number of revolutions of the driven pulley divided by the number of revolutions of the driving pulley.*

Or, $$D = \frac{dn}{N}.$$

Example.—The driving pulley makes 100 revolutions per minute, the driven pulley makes 75 revolutions per minute and is 18 inches in diameter; what is the diameter of the driving pulley?

Solution.—Applying the rule just given and substituting, we have $D = \frac{18 \times 75}{100} = 13\frac{1}{2}$ in. Ans.

16. The diameter and number of revolutions per minute of the driving pulley being given, to find the diameter of the driven pulley, which must make a given number of revolutions per minute:

Rule 4.—*The diameter of the driven pulley equals the product of the diameter and the number of revolutions of the driving pulley divided by the number of revolutions of the driven pulley.*

Or, $$d = \frac{DN}{n}.$$

Example.—The diameter of the driving pulley is $13\frac{1}{2}$ inches and it makes 100 revolutions per minute; what must be the diameter of the driven pulley to make 75 revolutions per minute?

Solution.—Applying rule 4 we have $d = \frac{13\frac{1}{2} \times 100}{75} = 18$ in. Ans.

17. To find the number of revolutions per minute of the driven pulley, its diameter and the diameter and the number of revolutions per minute of the driving pulley being given:

Rule [illegible] *revolutions of the driven pulley* [illegible] *and the number of revolu-* [illegible] *diameter of the driven pulley.*

$$n = \frac{DN}{d}.$$

[illegible] inches in diameter and makes [illegible] revolutions will the driven [illegible] inches in diameter?

[illegible]

[illegible] = 75 R. P. M. Ans.

18. [illegible] revolutions per minute of [illegible] and the diameter and the [illegible] of the driven pulley being [illegible]

Rule [illegible] *of the driving pulley* [illegible] *the number of revolu-* [illegible] *by the diameter of the* [illegible]

[illegible]

[illegible] inches in diameter and makes [illegible] revolutions will the driving [illegible] diameter?

[illegible]

[illegible] R. P. M. Ans.

BELTS.

19. A belt is a flexible connecting band that drives a pulley by its frictional resistance to slipping at the surface of the pulley. Belts are most commonly made of leather, cotton, or rubber, and are united in long lengths by cementing, riveting, or lacing.

20. Leather belts are made single and double. A single belt is one composed of a single thickness of leather; a double belt is one composed of two thicknesses of leather cemented and riveted together the whole length of the belt.

21. Cotton belts are in use to some extent, as are also belts made of a number of layers of duck sewed together and impregnated with a preparation rendering them waterproof. These belts, in accordance with the number of layers or "plys," are called two-ply, three-ply, etc. Four-ply cotton and duck belting is about equal to single leather belting, and eight-ply to double leather belting.

22. Rubber belts are especially adapted for use in damp or wet places; they will endure a great degree of heat or cold without injury, are quite durable, and are claimed to be less liable to slip than leather belts.

CALCULATIONS FOR BELTS.

23. To Find the Length of a Belt.—In practice, the necessary length for a belt to pass around pulleys that are already in their position on a shaft is usually obtained by passing a tape line around the pulleys, the stretch of the tape line being allowed as that necessary for the belt. The lengths of open-running belts for pulleys not in position can be obtained approximately as follows:

Rule 7.—*The length of a belt for open-running pulleys equals $3\frac{1}{4}$ times one-half the sum of the diameters of the pulleys plus 2 times the distance between the centers of the shaft.*

Let D = diameter of one pulley in inches;
d = diameter of the other pulley in inches;
L = distance between the centers of the shafts in inches;
B = length of the belt in inches.

Then,
$$B = 3\tfrac{1}{4}\left(\frac{D + d}{2}\right) + 2\,L.$$

EXAMPLE.—The distance between the centers of two shafts is 9 feet 7 inches; the diameter of the large pulley is 36 inches and the diameter of the small one is 14 inches; what is the necessary length of the belt?

SOLUTION.—Applying the rule just given and substituting the values given, we have, since 9 feet 7 inches = 115 inches,

$$B = 3\tfrac{1}{4}\left(\frac{36 + 14}{2}\right) + 2 \times 115 = 311\tfrac{1}{4} \text{ in., or } 25 \text{ ft. } 11\tfrac{1}{4} \text{ in.} \quad \text{Ans.}$$

The length of crossed belts cannot be determined by any simple calculation, it being a rather difficult mathematical problem.

24. To Find the Width of Belts.—A belt should be wide enough to bear safely and for a reasonable length of time the greatest tension that will be put upon it. This will be the tension of the driving side. The safe tension for single belts may be taken as 60 pounds per inch of width; single belts average $\frac{3}{16}$ inch in thickness. The tension on the driving side, however, does not represent the force tending to turn the pulley. The force tending to turn the pulley, or the **effective pull,** is the difference in tension between the driving side and the slack side of the belt. The tension on the driving side depends on three factors: the *effective pull* of the belt, the *coefficient of friction* between the belt and pulley, and the size of the *arc of contact* of the belt on the smaller pulley.

25. The effective pull that may be allowed per inch of width for single leather belts with different arcs of contact (the arc in which the belt touches the smaller pulley), is given in Table I.

26. To Find the Arc of Contact.—The arc of contact in degrees, or as a fraction of the circumference, can be determined, practically, as follows: Stretch a string over the two pulleys to represent the belt. Then, take another string, wrap it around the small pulley and cut it off so that the ends meet. This represents the circumference of the small pulley. Now take a third string, hold one end at the

TABLE I.

ALLOWABLE BELT PULLS.

Arc Covered by Belt.		Allowable Effective Pull Per Inch of Width in Pounds.
Degrees.	Fraction of Circumference.	
90	.250	23.0
112½	.312	27.4
120	.333	28.8
135	.375	31.3
150	.417	33.8
157½	.437	34.9
180 or over	.500	38.1

beginning of the arc of contact, as shown by the string stretched around both pulleys, wrap it around the smaller pulley, and cut it off at the end of the arc of contact. The length of this last string represents the length of the arc of contact. We now have the proportion: *the length of the string representing the circumference* : *the length of the string representing the arc of contact* :: *360* (*the number of degrees in a circle*) : *the number of degrees in the arc of contact.* Whence, the number of degrees in the arc of contact equals the quotient obtained by dividing the product of the length of the arc of contact and 360 by the circumference of the pulley.

To obtain the fraction of the circumference, divide the length of the string representing the arc of contact by the circumference.

27. To use the table for finding the width of a single leather belt for transmitting a given number of horsepower, we have the following rule,

where C = allowable effective pull, from table;
H = horsepower to be transmitted;
W = width of single belt in inches;
V = velocity of belt in feet per minute.

Rule 8.—*Multiply the horsepower to be transmitted by 33,000, and divide this product by the product of the velocity of the belt and the allowable effective pull, as taken from the table. The quotient will be the width of the belt.*

Or,
$$W = \frac{33{,}000\,H}{VC}.$$

EXAMPLE.—What width of single belt is needed to transmit 20 horsepower, the arc of contact on the small pulley being 135° and the speed of the belt 1,500 feet per minute?

SOLUTION.—According to Table I, the allowable effective pull for 135° is 31.3 pounds. Then, applying rule **8**, we have

$$W = \frac{33{,}000 \times 20}{1{,}500 \times 31.3} = 14 \text{ in., nearly. Ans.}$$

28. To Find the Horsepower of a Belt.—The horsepower that a single belt will transmit is given by the following rule:

Rule 9.—*Multiply together the effective pull taken from the table, the width of the belt in inches, and the speed of the belt in feet per minute. Divide the product by 33,000.*

Or,
$$H = \frac{C\,W\,V}{33{,}000}.$$

29. Speed of Belts.—By applying rule **8** to the same belt running at different velocities, it will be seen that the higher the velocity, the greater is the horsepower that the same belt can transmit, and from rule **9** it will be seen that the higher the speed of the belt, the less may be its width to transmit a given horsepower. From this it follows that a belt should be run at as high a velocity as conditions will permit, the maximum velocity allowable for a laced belt being about 3,500 feet per minute for ordinary single leather and double leather belts. For belts spliced by cementing, where the splice is practically as strong as the belt itself, the velocity may be as high as 5,000 feet per minute. Cases are on record where wide main belts have been run at as high a velocity as 6,000 feet per minute.

30. In choosing a proper belt speed, due regard must be paid to commercial conditions. While a high speed of the belt means a narrow and, consequently, a cheaper belt, the increased cost of the larger pulleys that may be required may offset the gain due to the high belt speed, at least as far as first cost is concerned.

For illustration, let the problem be to transmit 10 horsepower from one shaft to another. Let the revolutions of both the driven and the driving shaft be equal, and let the shafts make 200 revolutions per minute. Choosing a belt speed of 2,000 feet per minute, the width of a single belt to transmit the given horsepower will be, by rule **8**, say, 4½ inches. The diameter of the pulley that at 200 revolutions per minute will give a belt speed of 2,000 feet per minute is $\frac{2,000 \times 12}{200 \times 3.1416} = 38\frac{1}{4}$ inches, nearly. Taking the price of a 38-inch cast-iron pulley, 5-inch face, as $15, we have the price of two pulleys as $30. Let the distance between the pulleys be 20 feet. Then, the length of belt, according to rule **7**, is 50 feet 4 inches. Taking 51 feet as the length of the belt, and the price of a single leather belt 4½ inches wide at 50 cents per foot, the price of the belt will be $25.50. Then, the cost of belt and pulleys, not counting freight or express charges, etc., will be $25.50 + $30 = $55.50.

Choosing a belt speed of 3,500 feet per minute, the width of belt will be 2½ inches, nearly. The proper diameter of the pulley is $\frac{3,500 \times 12}{200 \times 3.1416} = 66\frac{7}{8}$ inches, say, 67 inches. The length of the belt will be 59 feet, about. The price of the belt at 30 cents per foot is $17.70. The price of two 67-inch pulleys 3½-inch face is, say, $80. Then, the total first cost is $80 + $17.70 = $97.70, showing that in this particular case the use of a low belt speed reduces the first cost by \$97.70 — \$55.50 = \$42.20.

The above illustration is not intended, and must not be construed, to be an argument against high belt speed: it simply shows the advisability of considering the commercial features in each and every case. In many cases it will be found that the narrow, high-speed belt is by far the more economical one to use.

31. Double belts are made of two single belts cemented and, usually, riveted together their whole length, and are used where much power is to be transmitted. As the effective pull for single belts, as given in Table I, is based primarily on the strength through the lace holes, a double belt, which is twice as thick, should be able to transmit twice as much power as a single belt, and in fact more than this, where, as is quite common, the ends of the belt are cemented instead of laced.

Where double belts are used on small pulleys, however, the contact with the pulley face is less perfect than it would be if a single belt were used, owing to the greater rigidity of the former. More work is, also, required to bend the belt as it runs over the pulley than in the case of the more pliable single belt, and the centrifugal force tending to throw the belt from the pulley also increases with the thickness. Moreover, in practice, it is seldom that a double belt is put on with twice the tension of a single belt. For these reasons, the width of a double belt required to transmit a given horsepower is generally assumed to be seven-tenths the width of a single belt to transmit the same power. On this basis, rules 8 and 9 become for double belts, by multiplying rule 8 by $\frac{7}{10}$ and dividing rule 9 by $\frac{7}{10}$, as follows:

Rule 10.—*To find the width of a double belt, multiply the horsepower to be transmitted by 23,100. Divide this product by the product of the velocity of the belt and the allowable effective pull, as taken from the table.*

Or,
$$W = \frac{23,100\,H}{V\,C}.$$

Rule 11.—*To find the horsepower that a double belt can transmit, multiply together the effective pull taken from the table, the width of the belt, and its velocity. Divide the product by 23,100.*

Or,
$$H = \frac{C\,W\,V}{23,100}.$$

EXAMPLE 1.—What width of double belt is required to transmit 20 horsepower, the arc of contact on the smaller pulley being 135° and the speed of the belt 1,500 feet per minute?

SOLUTION.—According to Table I, the effective pull is 31.3 pounds. Then, by rule **10**,

$$W = \frac{23,100 \times 20}{1,500 \times 31.3} = 10 \text{ in., nearly. Ans.}$$

EXAMPLE 2.—What horsepower can be transmitted by a 6-inch double belt running at 400 feet per minute with an arc of contact of 180°?

SOLUTION.—According to Table I, the effective pull is 38.1 pounds. Applying rule **11**, we have

$$H = \frac{38.1 \times 6 \times 400}{23,100} = 4 \text{ H. P., nearly. Ans.}$$

THE CARE AND USE OF BELTS.

32. Leather Belts.—It is a much disputed question as to which side of the belt should be run next to the pulley. The more common practice, it is believed, is to run the belt with the hair or grain side nearest the pulley. This side is harder and more liable to crack than the flesh side. By running it on the inside the tendency is to cramp or compress it as it passes over the pulley, while if it ran on the outside, the tendency would be for it to stretch and crack. The flesh side is the tougher side, but for the reason given above the life of the belt will be longer if the wear comes upon the grain side. The lower side of the belt should be the driving side, the slack side running from the top of the driving pulley. The sag of the belt will then cause it to encompass a greater length of the circumference. Long belts, running in any other direction than vertical, work better than short ones, as their weight holds them more firmly to their work.

It is bad practice to use rosin to prevent slipping. It gums the belt, causes it to crack, and prevents slipping for only a short time. If a belt, in good condition, persists in slipping, a wider belt should be used. Sometimes larger pulleys on the driving and driven shafts are of advantage, as they increase the belt speed and reduce the stress on the belt. Belts may be kept soft and pliable by oiling them once a month with castor oil or neatsfoot oil.

33. The Flapping of Belts.—One of the most annoying troubles experienced with belting of all kinds is the violent flapping of the slack side. Flapping may be due to any one of several causes, or to a combination of them. The most usual cause is that one or both of the pulleys run out of true. The belt is then alternately stretched and released, and while this may not cause flapping at one speed, it will usually do so at a higher speed. If the belt is rather slack, tightening it somewhat may cure or alleviate the flapping. The most obvious and best remedy, but the most expensive, is to turn the pulleys to run true.

Pulleys being out of line with each other are another prolific source of flapping, especially when one or both run out of true. First, bring the pulleys in line; if this fails, tighten the belt if it is rather loose. If no improvement is noticed and it is not possible to turn the pulleys, try to lower the belt speed a little, either by the substitution of smaller pulleys or by changing the speed of the driven shaft, according to circumstances.

With belts running at speeds above 4,000 feet per minute, flapping may occur when the pulleys are perfectly true and in line with each other, even when the belt has the proper tension. This is believed to be due to air becoming entrapped between the face of the pulley and the belt. At any rate, it has been observed that perforating the belt with a series of small holes will cure this trouble. Perforated belts may now be bought in the market.

Lack of steadiness in running, due either to sudden variations in the speed of the engine, or sudden changes in the load of the machines driven by the belt, will produce a flapping that it is almost impossible to cure. The only known cure is to take such steps as will insure steady running, as for instance, increasing the weight of the flywheel on the engine, or placing a flywheel instead of a pulley on the driven machine.

The belt not being joined square will also cause flapping, especially when the belt is running at a rather high speed. The remedy is to unlace or unfasten the joint and make it square.

Too great a distance between the pulleys may also cause flapping. In general, the distance between the pulleys should not exceed 15 feet for belts up to 4 inches in width; 20 feet for belts above 4 and below 12 inches; 25 feet for belts above 12 inches and below 18 inches ; and 30 feet for larger belts. The distances here given are occasionally exceeded considerably, but as no experiments have ever been made public that would enable a fairly correct formula to be deduced for the distance between pulleys when the belt speed, width of belt, and effective pull are known, they must be taken as representing average practice.

A horizontal belt is, by many, considered to have the proper tension when it has about 1 inch of sag, while in motion, for every 8 feet between the pulleys. For belts other than horizontal, this should be less, there being no sag at all for vertical belts.

JOINING THE ENDS OF BELTS.

34. Lacing.—The ends of a belt may be joined by *lacing*, *sewing*, *riveting*, or *cementing*. Many ways of **lacing** belts are used. A very satisfactory method for belts up to 3 inches in width is shown in Fig. 16. Cut the ends of the belt square, using a sharp knife and a try square. Punch a row of holes according to the width of the belt, punching corresponding holes exactly opposite each other in each end of the belt, using 3 holes in belts up to 2 inches wide, and 5 holes in belts between 2 and 3 inches wide. The number of holes in the row should always be uneven for the style of lacing shown. In the figure, *A* is the outside of the belt and *B* the side running nearest the pulley. The lacing should be drawn halfway through one of the middle holes from the under side, as at *1*; before going any further, it is well to see to it that the belt has no twists in it, or, in the case of a crossed belt, that it has not been given a wrong twist. The same side of the belt should run over both pulleys, which will be the case with a crossed belt if it has been twisted correctly. Having made sure that the belt is fair, pass the end of the lace on the upper side of the belt through *2* under the belt and up through *3*, back again through *2* and *3*, through *4* and up through *5*, where an incision is made in one side of the lacing, which forms a barb that will prevent the end from pulling through. Lace the right-hand side in the same manner. The lacing may advantageously be carried on at once to the right and left alternately.

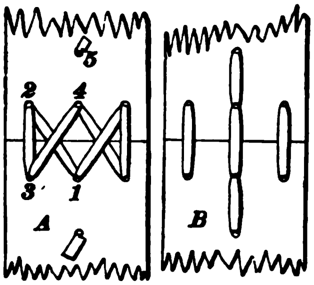


FIG. 16.

35. For belts wider than 3 inches, the lacing shown in Fig. 17 is a good one. As will be observed, there are two rows of holes. The number of holes in the row nearest the joint should exceed by one the number of holes in the second row. For belts up to 4½ inches wide, use 3 holes in the row nearest the joint and 2 holes in the second row. For belts up to 6 inches wide, use 4 and 3 holes, respectively. For larger belts, make the total number of holes in each end either one or two more than the number of inches of width, with the object of getting an odd total number of holes. For example, for a 10-inch belt, the total number of holes should be $10 + 1 = 11$. For a 13-inch belt, it should be $13 + 2 = 15$ holes. The outside holes of the first row should not be nearer the edges of the belt than $\frac{3}{4}$ inch, nor should the first row be nearer the joint than $\frac{7}{8}$ inch. The second row should be at least $1\frac{3}{4}$ inches from the end. In the figure, *A* is the outside and *B* the side nearest the pulley. Begin at one of the center holes in the outside row, as *1*, and continue through *2*, *3*, *4*, *5*, *6*, *7*, *4*, *5*, *2*, *3*, etc.

Another method is to begin the lacing on one side instead of in the middle. This method will give the rows of lacing on the under side of the belt the same thickness all the way across. The lacing should not be crossed on the side of the belt that runs next to the pulley.

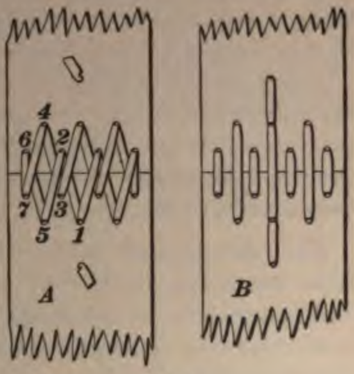


FIG. 17.

Lacing affords convenient means of *shortening belts* when they stretch and of *increasing their tension*. If the belts are at all large, the ends of the belt should be drawn together by belt clamps.

36. Cementing.—Cementing makes probably the best kind of a joint ever devised. It has the serious disadvantage, however, that the stretch of the belt cannot be readily taken up, and, hence, tightening pulleys must be used when the center-to-center distance of the pulleys is not adjustable.

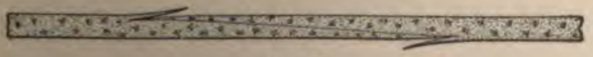


FIG. 18.

In dynamo driving, where endless belts are used to the exclusion of all others, the dynamo is usually mounted on a slide, so that the tension of the belt can be adjusted. For a cemented joint, the ends of the belt should be pared down with a very sharp knife to the form shown in Fig. 18, which shows a form that is recommended by belt manufacturers.

Example.—If the radius of the pulley A, Fig. 19, is 20 inches, of C 15 inches, and of E 24 inches, and the radius of the drum F is 4 inches, of the pinion D 5 inches, and of the pinion B 4 inches, how great a weight will a force of 1 pound applied at P raise?

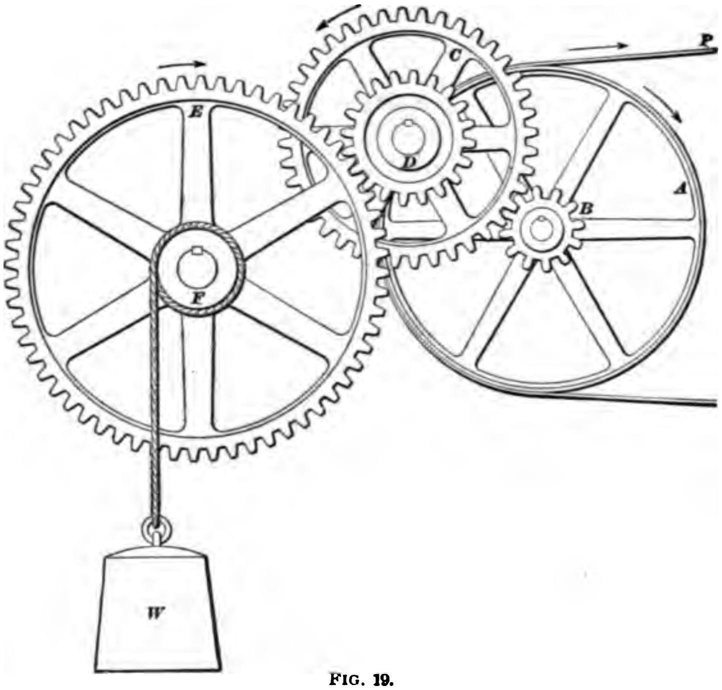


Fig. 19.

Solution.—Applying rule **12**, we have

$$1 \times 20 \times 15 \times 24 = W \times 4 \times 5 \times 4,$$

or

$$W = \frac{7,200}{80} = 90 \text{ lb. Ans.}$$

40. Although the combination of wheels in this example enables the lifting of a weight 90 times as great as the force applied, it has been necessary to exert the force through a distance 90 times the height through which the weight was raised. It is a universal law in the application of machines that *whenever there is a gain in power without a corresponding increase in the initial force, there is a loss in speed;* this is true of any machine.

In the example, if P were to move the entire 90 inches in 1 second, W would move only 1 inch in the same period of time.

41. Instead of using the diameter or radius of a gear, as in the last example, the number of teeth may be used when computing the weight that can be raised, or the velocity.

EXAMPLE.—The radius of the pulley A, Fig. 19, is 40 inches and that of F is 12 inches. The number of teeth in B is 9, in C 27, in D 12, and in E 36. If the weight to be lifted is 1,800 pounds, how great a force at P is it necessary to apply to the belt?

SOLUTION.—Let P represent the force; then, by the rule

$$P \times 40 \times 27 \times 36 = 1{,}800 \times 12 \times 9 \times 12,$$

or

$$P \times 38{,}880 = 2{,}332{,}800.$$

Hence,

$$P = \frac{2{,}332{,}800}{38{,}880} = 60 \text{ lb.},$$

= the amount of force necessary to apply to the belt. Ans.

GEAR-WHEELS.

42. A wheel that is provided with teeth that mesh with similar teeth on another wheel is called a **gear-wheel,** or **gear.** In Fig. 20 is shown a **spur gear.** On spur gears

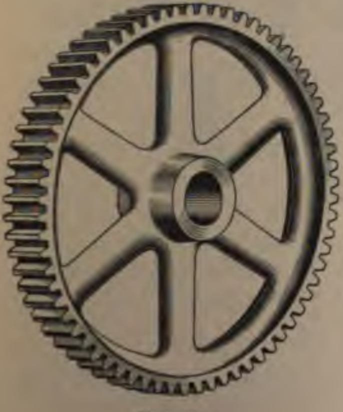


FIG. 20.

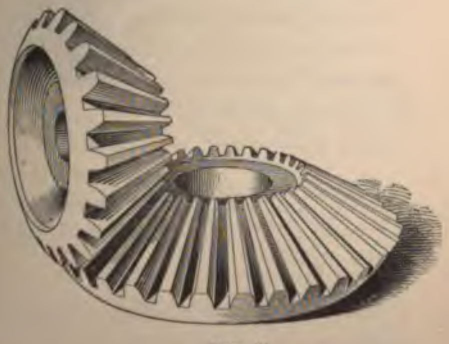


FIG. 21.

the teeth are usually parallel to the axis of the wheel or to its shaft.

Gears are said to be *in mesh* when the teeth of two wheels, respectively, engage each other or interlock.

43. In Fig. 21 is shown a pair of **bevel gears** in mesh. Of the two wheels shown, one is smaller than the other; when both wheels of a pair of bevel gears are of the same diameter they are called **miter gears.** In Fig. 22 is shown a pair of miter gears in mesh. It is obvious that the angle that the teeth of these gears make with the axis of the shaft must be 45°.

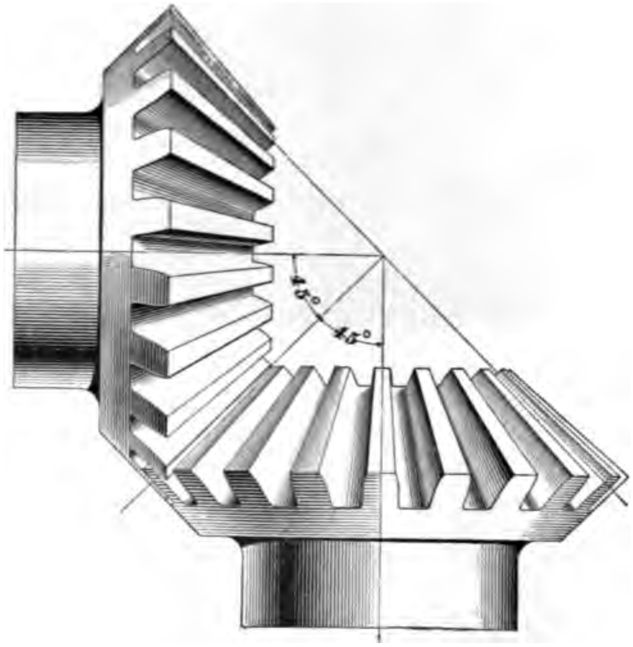


Fig. 22.

44. Of a pair of gear-wheels (either spur or bevel) having different diameters, the smaller is called a **pinion.**

In Fig. 23 is shown a revolving screw, or **worm,** as it is called, that meshes with a **worm-wheel.** It is used to transmit motion from one shaft to another at right angles to it.

As the worm is nothing else than a screw, each revolution given to it will rotate the wheel a distance equal to the pitch of the worm; consequently, if there are 40 teeth in the worm-wheel, a single-threaded worm must make 40 revolutions in order to turn the wheel once.

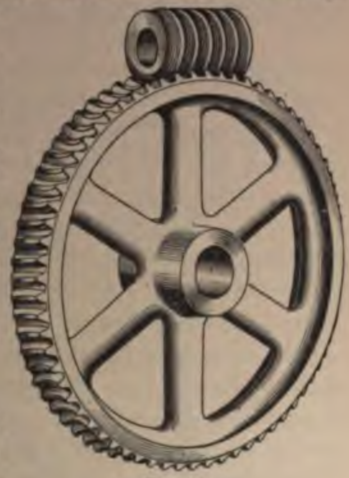


FIG. 23.

45. A **rack** is a straight bar that has gear-teeth cut on it. It may be considered as part of a gear-wheel in which the diameter is infinitely large. The teeth of racks are proportioned by the same rules as those of gear-wheels.

TEETH OF GEAR-WHEELS.

46. The object in designing the teeth of gear-wheels should be to so shape them that the motion transmitted will be exactly the same as with a corresponding pair of wheels, or cylinders, without teeth and running in contact without slipping. Such cylinders are called **pitch cylinders,** and are always represented on the drawing of a gear-wheel by a line called the **pitch circle** (see Fig. 24). The pitch circle is also called the **pitch line.**

The diameter of the pitch circle is called the **pitch diameter.** When the word "diameter" is applied to gears, it is always understood to mean the *pitch diameter*, unless especially stated as the "diameter over all" or "diameter at the root."

47. Circular and Diametral Pitch.—The distance from a point on one tooth to a corresponding point on the next tooth, measured along the pitch circle, is the **circular**

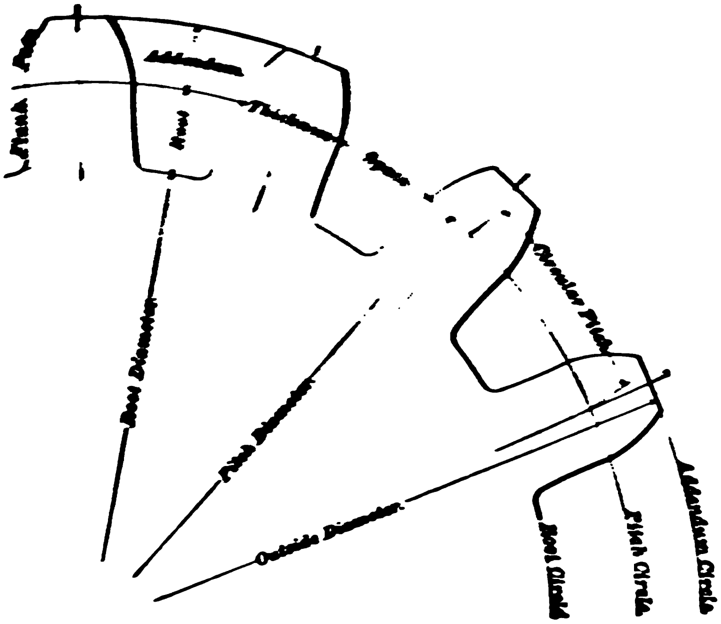


Fig. 24

pitch. This is obtained by dividing the circumference (pitch circle) by the number of teeth, and is used in laying out the teeth of large gears and, also, when calculating their strength.

It would be very convenient to have the circular pitch expressed in manageable numbers like 1-inch, ¾ inch, etc.; but as the circumference of a gear is 3.1416 times its diameter, this requires awkward numbers for the diameters. Thus, a wheel of 40 teeth, 1-inch pitch, would have a circumference on the pitch circle of 40 inches and a diameter of 12.732 inches. Of the two it is more convenient, in the great majority of cases, to have the diameters expressed in numbers that can be easily handled. In order, however, to have the pitch in a convenient form also, a pitch has been devised that is *expressed in terms of the diameter* and called the **diametral pitch.**

The diametral pitch is not a measurement like the circular pitch, but a *ratio. It is the ratio of the number of teeth in the gear to the number of inches in the diameter; or, it is the number of teeth on the circumference of the gear for 1 inch diameter of the pitch circle.* It is obtained by dividing the number of teeth by the diameter.

A gear, for example, has 60 teeth and is 10 inches in diameter. The diametral pitch is the ratio of 60 to $10 = \frac{60}{10} = 6$; this gear would be called a 6-pitch gear. From the definition it follows that teeth of any particular diametral pitch are of the same size and have the same width on the pitch line, whatever may be the diameter of the gear. Thus, if a 12-inch gear has 48 teeth, it will be 4 pitch. A 24-inch gear to have teeth of the same size will have twice 48, or 96 teeth, and as $96 \div 24 = 4$, has the same diametral pitch as before.

48. Other Definitions.—The other necessary definitions applying to the parts of a gear can be readily understood from Fig. 24. The thickness of the tooth and the width of the space are measured on the pitch circle. A tooth is composed of two parts, the **addendum,** or part outside of the pitch circle, and the **root,** which is inside.

A line through the outside end of the addendum is called the **addendum circle,** or **addendum line,** and one through the inside part of the root is called the **root circle,** or **root line.** The amount by which the width of the space is greater than the thickness of the tooth is called the **backlash,** or **side clearance.**

49. Proportions for Gear-Teeth.—With gears of large size, and often with cast gears of all sizes, the circular-pitch system is used. In these cases, it is usual to have the addendum, whole depth, and thickness of the tooth conform to arbitrary rules based on the circular pitch.

The usual proportions are for cast gears: Addendum $= .3 \times$ circular pitch. Root $= .4 \times$ circular pitch. Thickness of tooth $= .48 \times$ circular pitch.

The gears most often met with are the cut gears of small and medium size like those, for example, on machine tools, which are almost invariably diametral-pitch gears. The teeth are cut from the solid with standard milling cutters, proportioned with the diametral pitch as a basis. This system is also coming into use for cast gearing. In all diametral-pitch gears, the addendum, in inches, is made equal to 1 divided by the diametral pitch, and the working depth to twice the addendum. The end clearance is usually taken equal to $\frac{1}{8}$ the addendum for cut gears, though The Brown & Sharpe Manufacturing Company use $\frac{1}{10}$ the thickness of the tooth on the pitch line as the clearance. The side clearance, or "backlash," is barely enough to give a good working fit, and seldom exceeds $\frac{1}{50}$ the pitch.

Using the above proportions, a 4-pitch gear will have the addendum $= 1 \div 4 = \frac{1}{4}$ inch; the working depth will be $2 \times \frac{1}{4} = \frac{1}{2}$ inch; and the clearance, if made $\frac{1}{8}$ the addendum, $\frac{1}{8} \times \frac{1}{4} = \frac{1}{32}$ inch. The whole length of the tooth will be $\frac{1}{2} + \frac{1}{32} = \frac{17}{32}$ inch. The thickness of the tooth will be one-half the circular pitch, nearly. In a 10-pitch wheel, the addendum will be $\frac{1}{10}$ inch and the length of the tooth $\frac{17}{80}$ inch; in a $2\frac{1}{2}$-pitch, it will be $1 \div 2\frac{1}{2} = \frac{2}{5}$ inch and the length $\frac{17}{20}$ inch.

FORMS OF GEAR-TEETH.

50. The forms of teeth used in ordinary practice form part of certain curves known as the *epicycloid*, *hypocycloid*, and *involute*.

51. The Epicycloidal Tooth.—In the so-called epicycloidal tooth, which more properly is called a *cycloidal* tooth, the face of the tooth is part of an epicycloid, and the flank, part of a hypocycloid.

An **epicycloidal curve** is the path described by any point of a circle rolling, without slipping, on the outside of another circle. A **hypocycloid** is the path described by any point of a circle rolling, without slipping, on the inside of another circle.

Epicycloidal teeth can always be recognized by their appearance; they are formed by two curves that, commencing at the pitch circle, curve in opposite directions. Fig. 25 clearly exhibits the characteristic tooth form.

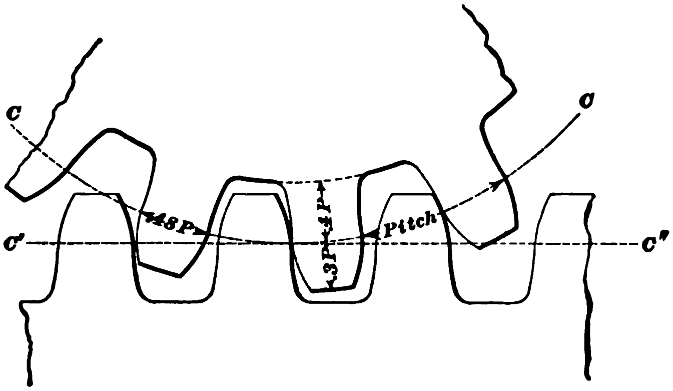


FIG. 25.

The use of epicycloidal teeth is being gradually abandoned, as they possess some practical defects, the chief defect being that the center-to-center distance of the two gears must be practically perfect in order to insure a uniform velocity of the driven gear.

52. Involute Teeth.—In Fig. 26 is shown the involute form of tooth, which is composed of but one curve.

The **involute** is the path described by any point of a string that is being wound on or off a cylinder, the cylinder being stationary. In the involute system, the sides of the teeth of the rack are straight lines, as shown in Fig. 26.

Involute teeth have two great advantages over epicycloidal teeth: (1) *They are stronger for the same pitch, as they are thicker at the root.* (2) *The gears may be spread* *slightly apart so that their pitch circles do not run tangent to*

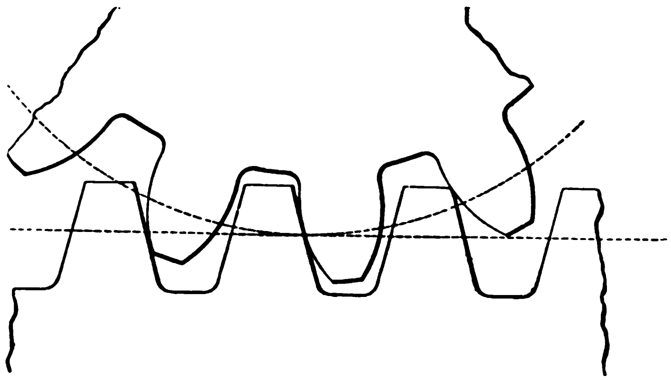


FIG. 26.

one another, without affecting the perfect action of the teeth to an appreciable extent.

GEAR CALCULATIONS.

53. The Circular-Pitch System.—For calculating the pitch diameter, number of teeth, etc. of gear-wheels, we have, for the circular-pitch system, the following rules,

where P = circular pitch in inches;
T = number of teeth;
D = pitch diameter of the gear in inches.

54. To find the pitch diameter of a gear-wheel in inches, when the pitch and number of teeth are given:

Rule 13.—*The pitch diameter equals the product of the pitch and the number of teeth divided by 3.1416.*

Or,
$$D = \frac{P\,T}{3.1416}.$$

EXAMPLE.—What is the diameter of the pitch circle of a gear-wheel that has 75 teeth and whose pitch is 1.675 inches?

SOLUTION.—Applying rule **13**, we have

$$D = \frac{1.675 \times 75}{3.1416} = 40 \text{ in. Ans.}$$

55. To find the number of teeth in a gear-wheel when the diameter and pitch are given:

Rule 14.—*The number of teeth equals the product of 3.1416 and the diameter divided by the pitch.*

Or,
$$T = \frac{3.1416\,D}{P}.$$

Example.—The diameter of a gear-wheel is 40 inches and the pitch of the teeth is 1.675 inches; how many teeth are there in the wheel?

Solution.—Applying the rule just given, we have

$$T = \frac{3.1416 \times 40}{1.675} = 75 \text{ teeth. Ans.}$$

56. To find the pitch of a gear-wheel when the diameter and the number of teeth are given:

Rule 15.—*The pitch of the teeth equals the product of 3.1416 and the diameter divided by the number of teeth.*

Or,
$$P = \frac{3.1416\,D}{T}.$$

Example.—The diameter of a gear-wheel is 40 inches and it has 75 teeth; what is the pitch of the teeth?

Solution.—By rule **15**, we have

$$P = \frac{3.1416 \times 40}{75} = 1.675 \text{ in. pitch. Ans.}$$

57. The Diametral-Pitch System.—The diameter of the gear-wheel, the number of teeth, etc. are given by the following rules, where P_d = diametral pitch; D_o = outside diameter; N = number of teeth; and the other letters have the same meaning as in the three preceding rules.

58. To find the pitch diameter of the gear-wheel when the number of teeth and the pitch are given:

Rule 16.—*Divide the number of teeth by the diametral pitch.*

Or,
$$D = \frac{N}{P_d}.$$

Example.—A wheel is to have 40 teeth, 4 pitch; what is its pitch diameter?

Solution.—By applying rule **16**, we have

$$D = \frac{40}{4} = 10 \text{ in. Ans.}$$

59. To find the diameter over all, that is, the diameter of the blank from which the gear-wheel is cut, the number of teeth and the diametral pitch being given:

Rule 17.—*Add 2 to the number of teeth and divide by the diametral pitch.*

Or, $$D_o = \frac{N+2}{P_d}.$$

Example.—In the last example, what is the diameter over all the blank?

Solution.—Applying the rule just given, we get

$$D_o = \frac{40+2}{4} = 10\tfrac{1}{2} \text{ in. Ans.}$$

60. The number of teeth and the outside diameter of the gear-wheel being known, to find the diametral pitch:

Rule 18.—*Add 2 to the number of the teeth and divide by the outside diameter.*

Or, $$P_d = \frac{N+2}{D_o}.$$

Example.—A gear-wheel has 60 teeth and is 6⅕ inches in diameter over all; what is the diametral pitch?

Solution.—By applying rule **18**, we get

$$P_d = \frac{60+2}{6\frac{1}{5}} = 10. \text{ Ans.}$$

61. To find the diametral pitch, the number of teeth and the pitch diameter being known:

Rule 19.—*Divide the number of teeth by the pitch diameter.*

Or, $$P_d = \frac{N}{D}.$$

Example.—A wheel has 90 teeth and its pitch diameter is 30 inches; what is the diametral pitch?

Solution.—By rule **19**, we get

$$P_d = \frac{90}{30} = 3. \quad \text{Ans.}$$

62. The pitch diameter and the diametral pitch being given, to obtain the number of teeth:

Rule 20.—*Multiply the pitch diameter by the diametral pitch.*

Or, $$N = D P_d.$$

Example.—How many teeth are there in a gear-wheel having a pitch diameter of 36 inches and a diametral pitch of 5?

Solution.—Applying the rule just given, we get

$$N = 36 \times 5 = 180 \text{ teeth.} \quad \text{Ans.}$$

63. The diameter over all and the diametral pitch being given, to find the number of teeth:

Rule 21.—*Subtract 2 from the product of the outside diameter and the diametral pitch.*

Or, $$N = D_o P_d - 2.$$

Example.—How many 10-pitch teeth has a gear-wheel having an outside diameter of 8⅕ inches?

Solution.—By rule **21**, we get

$$N = 8\tfrac{1}{5} \times 10 - 2 = 80 \text{ teeth.} \quad \text{Ans.}$$

64. The standard diametral pitches used for cut gears are as follows: 2, 2¼, 2½, 2¾, 3, 3½, 4, 5, 6, 7, 8, 9, 10, 11, 12, 14, 16, 18, 20, 22, 24, 26, 28, 30, 32, 36, 40, 48. Gears having a diametral pitch differing from those given here are, usually, either very large or very small.

65. Diameters and Distances Between Centers.—The distance between the centers of two gear-wheels being known and the ratio of their speeds, the diameter of the pitch circle of the smaller wheel is given by the following rule,

where A = center-to-center distance;
R = revolutions per minute of large gear;
r = revolutions per minute of small gear;
d = pitch diameter of small gear.

Rule 22.—*Multiply twice the center-to-center distance by the number of revolutions of the large gear, and divide the product by the sum of the revolutions of the large and small gear.*

Or,
$$d = \frac{2AR}{R + r}.$$

EXAMPLE.—Given, the distance between centers = $5\frac{1}{2}$ inches; the small gear is to make 24 revolutions for every 8 revolutions of the large gear. What is the diameter of each gear?

SOLUTION.—By rule **22**,

$$d = \frac{2 \times 5\frac{1}{2} \times 8}{8 + 24} = 2\tfrac{3}{4} \text{ in., the diameter of the small wheel.}$$

Then, as the center distance is equal to the sum of the radii of the two wheels, the radius of the large wheel is $5\frac{1}{2} - \frac{2\frac{3}{4}}{2} = 4\frac{1}{8}$ in., and its diameter is $4\frac{1}{8} \times 2 = 8\frac{1}{4}$ in. Ans.

66. Speed and Number of Teeth.—To calculate the number of teeth or the speed of one of two gear-wheels that are to gear together:

Let N = number of revolutions per minute of the driving wheel;
n = number of revolutions per minute of the driven wheel;
T = number of teeth in the driving wheel;
t = number of teeth in the driven wheel.

Rule 23.—*The number of teeth in the driving wheel equals the product of the number of teeth and number of revolutions of the driven wheel divided by the number of revolutions of the driving wheel.*

Or,
$$T = \frac{tn}{N}.$$

EXAMPLE.—The driven wheel has 27 teeth and will make 66 revolutions per minute; if the driving wheel makes 99 revolutions per minute, how many teeth are there in the driving wheel?

SOLUTION.—Applying rule **23**, we have

$$T = \frac{27 \times 66}{99} = 18 \text{ teeth. Ans.}$$

67. The number of revolutions per minute of the driving wheel and the driven wheel and the number of teeth in the driving wheel being given, to find the number of teeth in the driven wheel:

Rule 24.—*The number of teeth in the driven wheel equals the product of the number of teeth and revolutions per minute of the driving wheel divided by the number of revolutions per minute of the driven wheel.*

Or, $$t = \frac{TN}{n}.$$

EXAMPLE.—The driving wheel has 18 teeth and makes 99 revolutions per minute, and the driven wheel must make 66 revolutions per minute; how many teeth must there be in the driven wheel?

SOLUTION.—Applying the rule just given, we have

$$t = \frac{18 \times 99}{66} = 27 \text{ teeth. Ans.}$$

68. The number of teeth in the driving wheel and the driven wheel and the number of revolutions per minute of the driving wheel being given, to find the number of revolutions per minute of the driven wheel:

Rule 25.—*The number of revolutions per minute of the driven wheel equals the product of the number of teeth and number of revolutions of the driving wheel divided by the number of teeth of the driven wheel.*

Or, $$n = \frac{TN}{t}.$$

EXAMPLE.—There are 18 teeth in the driving wheel and it makes 99 revolutions per minute; how many revolutions per minute will the driven wheel make if it has 27 teeth?

SOLUTION.—Applying rule **25**, we have

$$n = \frac{18 \times 99}{27} = 66 \text{ R. P. M. Ans.}$$

69. The number of teeth in the driving wheel and the driven wheel and the number of revolutions per minute of the driven wheel being given, to find the number of revolutions per minute of the driving wheel:

Rule 26.—*The number of revolutions of the driving wheel equals the product of the number of teeth and revolutions of the driven wheel divided by the number of teeth of the driving wheel.*

Or, $$N = \frac{t\,n}{T}.$$

EXAMPLE 1.—If there are 27 teeth in the driven wheel and if it makes 66 revolutions per minute, how many revolutions per minute will the driving wheel make if it has 18 teeth?

SOLUTION.—Applying the rule just given, we have

$$N = \frac{27 \times 66}{18} = 99 \text{ R. P. M. Ans.}$$

EXAMPLE 2.—In Fig. 27, the crank-shaft makes 60 revolutions per minute; the governor pulley is 4 inches in diameter; the bevel gear on the governor pulley shaft has 19 teeth; the bevel gear that meshes with it and drives the governor has 30 teeth. The governor is to make 95 revolutions per minute; what should be the size of the pulley on the crank-shaft?

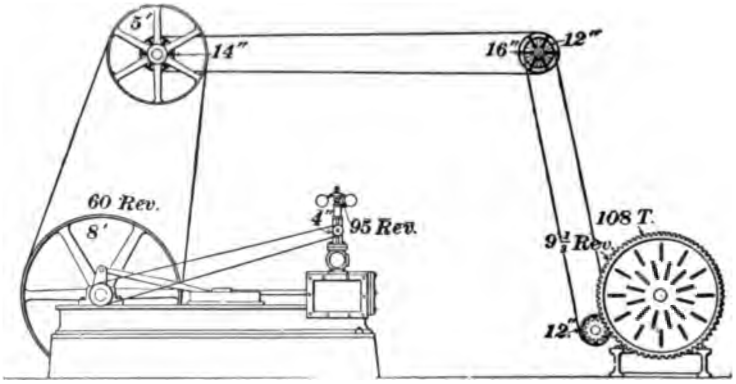


FIG. 27.

SOLUTION.—First determine the number of revolutions of the 4-inch pulley in order that the governor shall turn 95 times per minute. Applying rule 26, $N = \frac{t\,n}{T} = \frac{30 \times 95}{19} = 150$ revolutions of gear on pulley shaft = revolutions of governor pulley. Now, applying rule 3, the diameter of the pulley on the crank-shaft $= \frac{d\,n}{N} = \frac{4 \times 150}{60} = 10$ in. Ans.

EXAMPLE 3.—In Fig. 27, the flywheel is 8 feet in diameter and drives a 5-foot pulley on the main shaft. A 14-inch pulley on the main shaft drives a 16-inch pulley on the countershaft. A 12-inch pulley on the countershaft drives a 12-inch pulley on a shaft on which is a pinion that meshes into a large gear attached to the face plate of a large lathe, and which has 108 teeth. How many teeth must the pinion have in order that the face plate may make 9⅓ revolutions per minute?

SOLUTION.—Applying rule **5**, to find the revolutions per minute of the main shaft, $\frac{8 \times 60}{5} = 96$ R. P. M. Applying the same rule again to find the revolutions of the countershaft, $\frac{14 \times 96}{16} = 84$ R. P. M. Applying it once more to find the revolutions of the pulley that turns the small gear, $\frac{12 \times 84}{12} = 84$ R. P. M. Applying rule **23**, $\frac{108 \times 9\frac{1}{3}}{84} = 12$ teeth in pinion or driver. Ans.

EXAMPLES FOR PRACTICE.

1. The driving pulley makes 110 R. P. M. and is 21 inches in diameter; what should be the size of the driven pulley in order to make 385 R. P. M.? Ans. 6 in.

2. The main shaft of a certain shop makes 120 R. P. M. It is desired to have the countershaft make 150 R. P. M. There are on hand pulleys 16 inches, 24 inches, 28 inches, 35 inches, and 38 inches in diameter. Can two of these be used, or must a new pulley be ordered? Ans. Use the 28-inch and 35-inch pulleys.

3. The pinion (driver) makes 174 R. P. M. and the follower 24 R. P. M.; how many teeth must the pinion have if the follower has 87 teeth? Ans. 12 teeth.

4. If an engine flywheel is 66 inches in diameter and makes 160 R. P. M., what must be the diameter of the pulley on the main shaft to make 128 R. P. M.? Ans. 82½ in.

5. What is the pitch diameter of a gear whose circular pitch is 1¼ inches and has 28 teeth? Ans. 11.14 in.

6. How many teeth are there in a gear whose circular pitch is .7854 inch and which is 23 inches in diameter? Ans. 92 teeth.

7. What is the circular pitch of a gear whose diameter is 20.372 inches and which has 128 teeth? Ans. ½ in.

8. What is the pitch diameter of a gear-wheel having 80 teeth, 7 diametral pitch? Ans. 11$\frac{3}{7}$ in.

9. What is the over-all diameter of a gear-wheel of 9 diametral pitch and 47 teeth? Ans. 5$\frac{4}{9}$ in.

10. What is the diametral pitch of a gear-wheel having 90 teeth and an outside diameter of $5\frac{3}{4}$ inches? Ans. 16 pitch.

11. How many teeth of 20 diametral pitch can be cut in a gear-wheel having an outside diameter of $2\frac{1}{10}$ inches? Ans. 40 teeth.

12. If the center-to-center distance of two gear-wheels is 6 inches and the small gear is to make 4 revolutions for 1 revolution of the large gear, what must be the diameter of the large gear?

Ans. 9.6 in.

13. In a pair of gear-wheels, the driver has 48 teeth and the driven wheel 56 teeth. If the driven wheel makes 98 revolutions per minute, how many revolutions must the driver make? Ans. $114\frac{1}{3}$ R. P. M.

14. In a train of gears, the drivers have 16, 30, 24, and 18 teeth, respectively; the followers have 12, 24, 36, 40 teeth, respectively. If the first driver makes 80 R. P. M., how many R. P. M. will the last follower make? Ans. 40 R. P. M.

FIXED AND MOVABLE PULLEYS.

70. Pulleys are also used for hoisting or raising loads, in which case the frame that supports the axle of the pulley is called the **block.**

71. A **fixed pulley** is one whose block is not movable, as in Fig. 28. In this case, if the weight *W* be lifted by pulling down *P*, the other end of the cord *W* will evidently move the same distance upwards that *P* moves downwards; hence, *P* must equal *W*.

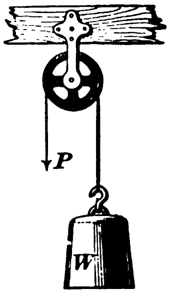


FIG. 28.

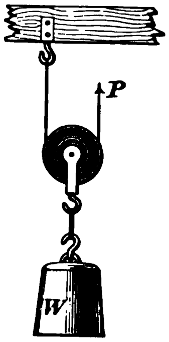


FIG. 29.

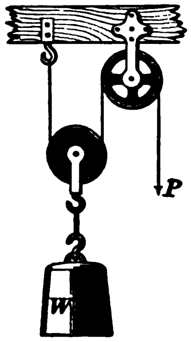


FIG. 30.

72. A movable pulley is one whose block is movable, as in Fig. 29. One end of the cord is fastened to the beam and the weight is suspended from the pulley, the other end of the cord being drawn up by the application of a force P. A little consideration will show that if P acts through a certain distance, say 1 foot, W will move through *half* that distance, or 6 inches; hence, a pull of 1 pound at P will lift 2 pounds at W.

The same would also be true if the free end of the cord were passed over a *fixed pulley*, as in Fig. 30, in which case the fixed pulley merely changes the direction in which P acts, so that a weight of 1 pound hung on the free end of the cord will balance 2 pounds hung from the *movable pulley*.

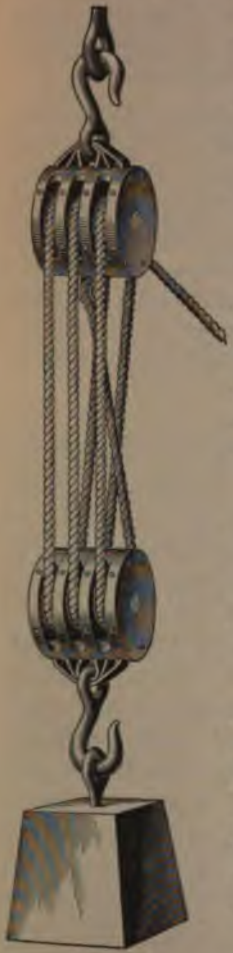


FIG. 31.

73. A combination of pulleys, as shown in Fig. 31, is sometimes used. In this case there are three movable and three fixed pulleys, and the amount of movement of W, owing to a certain movement of P, is readily found.

It will be noticed that there are 6 *parts* of the rope, not counting the free end; hence, if the movable block be lifted 1 foot, P remaining in the same position, there will be 1 foot of slack in each of the 6 parts of the rope, or 6 feet in all. Therefore, P must move 6 feet in order to take up this slack, or P moves 6 times as far as W. Hence, 1 pound at P will support 6 pounds at W, since the *force multiplied by the distance through which it moves equals the weight multiplied by the distance through which it moves.* It will also be noticed that there are three movable pulleys, and that $3 \times 2 = 6$.

LAW OF COMBINATION OF PULLEYS.

74. Rule 27.—*In any combination of pulleys where one continuous rope is used, a load on the free end will balance a weight on the movable block as many times as great as itself as there are parts of the rope supporting the load, not counting the free end, assuming that there are no friction losses.*

The above law is good whether the pulleys are side by side, as in the ordinary block and tackle, or whether they are in line and beneath one another.

EXAMPLE.—In a block and tackle having 5 movable pulleys, how great a force must be applied to the free end of the rope to raise 1,250 pounds, assuming that there are no friction losses?

SOLUTION.—Since there are 5 movable pulleys, there must be 10 parts of the rope to support them. Hence, according to the above law, a force applied to the free end will support a load 10 times as great as itself, or the force $= \frac{1,250}{10} = 125$ lb. Ans.

75. Frictional Losses.—Owing to the friction of the sheaves and the friction and resistance to bending of the rope, it is not possible to move a weight with a block and tackle by applying the theoretical force calculated by the above rule. The actual pull required depends on such factors as the condition of the rope and the friction and number of the sheaves. By experiment it has been determined that under average conditions the load that can be raised with a block and tackle having 4 sheaves will average about 75 per cent. of the theoretical load; with 5 sheaves, about 70 per cent.; with 6 sheaves, about 66 per cent.; with 7 sheaves, about 63 per cent.; and with 8 sheaves, about 60 per cent. No records of tests with a larger number of sheaves have been made public, but it is fair to assume that the decrease in the load that can be lifted will vary in about the same ratio.

Let f_a = the force applied at the free end of the rope;
c = the ratio of the theoretical to the actual load corresponding to the number of sheaves;
m = the number of movable pulleys;
l = the load to be raised.

Then, the force that is actually required to raise a given load, may be found approximately by

Rule 28.—*Divide 100 times the load by the product of twice the number of movable pulleys and the number of per cent. corresponding to the number of sheaves used.*

Or,
$$f_a = \frac{100\, l}{2\, m\, c}.$$

EXAMPLE.—In an ordinary block and tackle having three movable pulleys, as shown in Fig. 31, how great a force must actually be applied to raise 2,000 pounds?

SOLUTION.—According to Art. **75**, c may be taken as 66, there being 6 sheaves. Then applying rule **28**, we get

$$f_a = \frac{100 \times 2{,}000}{2 \times 3 \times 66} = 505 \text{ lb., nearly. Ans.}$$

76. The load that may actually be raised with a block and tackle will be approximately given by the following rule:

Rule 29.—*Multiply the force to be applied to the free end of the rope by twice the number of movable pulleys and by the number of per cent. corresponding to the number of sheaves used. Divide the product by 100.*

Or,
$$l = \frac{f_a\, 2\, m\, c}{100}.$$

EXAMPLE.—If there are enough men pulling on the free end of the rope of a block and tackle having two movable pulleys to exert a force of 250 pounds, what weight may they expect to raise?

SOLUTION.—There being 4 sheaves, by Art. **75**, $c = 75$. Then by rule **29**,

$$l = \frac{250 \times 2 \times 2 \times 75}{100} = 750 \text{ lb. Ans.}$$

77. If the free end of the hauling rope passes first around a stationary sheave, as in Fig. 31, it does not make any difference in what direction in the plane of the sheave the rope is pulled. If it passes first around a movable sheave, however, as in Fig. 29, the pull must be exerted in a line parallel to the line of action of the resistance, or a line joining the centers of the movable and stationary sheaves, in order to obtain the maximum effect. If the rope pulls on the movable sheave at an angle, its effect will be a displacement sideways, instead of straight up.

78. The **Weston differential pulley block,** more commonly known as a **chain hoist,** is shown in Fig. 32. With the help of this device, one man can hoist a load far beyond what can be done with the ordinary block and tackle. There are two pulleys slightly different in size alongside of each other in the stationary block and rigidly connected so as to turn together. An endless chain passes over both pulleys and supports the movable block in one of its bights. The other loop, or bight, is free and forms the hauling part. This kind of a pulley block possesses the valuable property that the load can be stopped anywhere by simply ceasing to haul on the hoisting part of the free loop.

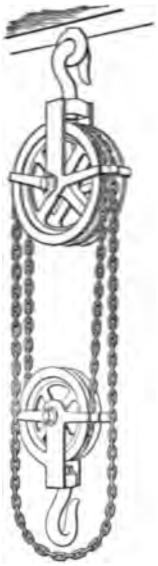


FIG. 32.

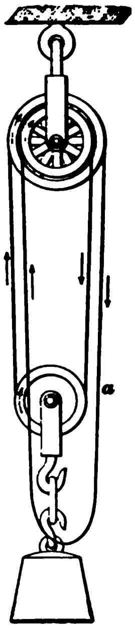


FIG. 33.

In Fig. 33 the differential pulley block is shown in diagrammatic form. Suppose that the part *a* of the free bight is pulled downwards, that is, in the direction of the arrow. Evidently, the upper pulleys will both turn in the direction of the hands of a watch, as indicated by the curved arrow. Then, the left-hand part of the bight supporting the movable pulley will move upwards, and the right-hand part downwards. But as the large and small pulley of the upper block move together, it follows that the left-hand side of the bight supporting the movable sheave will run faster over the large pulley than the right-hand side of the bight will run downwards over the small pulley. The result is that the upward motion of the movable block will be equal to one-half the difference of the distances passed over in the same time by fixed points in both sides of the bight.

79. Differential chain hoists are invariably so constructed that one man can hoist any load up to the maximum capacity of the hoists. This maximum load will usually be found stamped on the upper block, and there is, hence, no need in practice of calculating the force that must be exerted to hoist a load with a chain hoist.

EXAMPLES FOR PRACTICE.

1. In a block and tackle with two movable pulleys, (*a*) what force will be required to lift 300 pounds, assuming that there are no friction losses? (*b*) What force may be expected to be actually required, taking friction into account?

Ans. { (*a*) 75 lb.
(*b*) 100 lb.

2. A man weighing 120 pounds throws his whole weight on the free end of the rope of a block and tackle with 3 movable pulleys. What weight can he raise, (*a*) assuming that there are no friction losses? (*b*) taking friction into account?

Ans. { (*a*) 720 lb.
(*b*) 475 lb., about

THE INCLINED PLANE.

80. An **inclined plane** is a slope, or flat surface, making an angle with another surface or base plane.

81. Three cases may arise in practice with an inclined plane having a horizontal base plane: (1) Where the force acts parallel to the plane, as

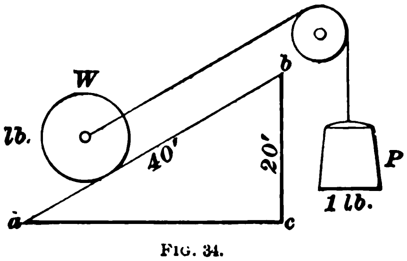


FIG. 34.

in Fig. 34. (2) Where the force acts parallel to the base, as in Fig. 35. (3) Where the force acts at an angle to the plane or to the base, as in Fig. 36.

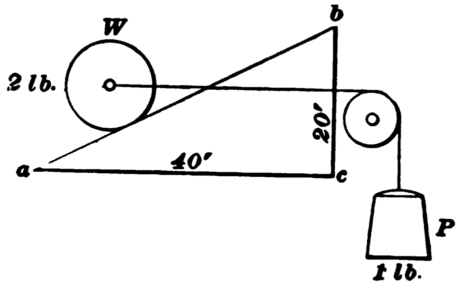


FIG. 35.

82. Relation Between Force and Weight.—In Fig. 34, the relation existing between the force and the weight is easily found. The weight ascends a distance equal to *c b*, or the height of the inclined plane, while the force acts through a distance equal to *a b*, or the length of the inclined plane. Therefore,

Rule 30.—*To find the force, multiply the weight by the height of the plane and divide the product by the length of the plane. To find the weight that can be raised, multiply the force by the length of the plane and divide this product by the height of the plane.*

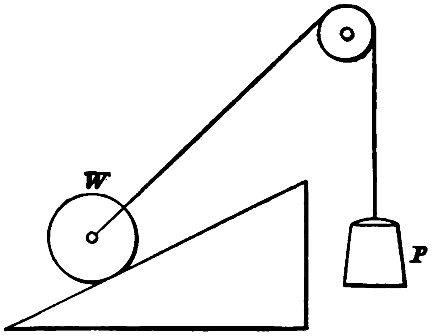


FIG. 36.

Or, let $P =$ force;

$l =$ length of the plane;

$b =$ base of the plane;

$h =$ height of plane;

$W =$ weight.

Then, $$P = \frac{Wh}{l} \text{ and } W = \frac{Pl}{h}.$$

EXAMPLE 1.—The length of an inclined plane is 40 feet and its height is 5 feet; what force is required to sustain a weight of 100 pounds?

Solution.—Applying rule **30**, we have

$$P = \frac{100 \times 5}{40} = 12\tfrac{1}{2} \text{ lb. Ans.}$$

Example 2.—The length of an inclined plane is 8 feet and its height is 15 inches; what weight will a force of 120 pounds support?

Solution.—Applying the rule just given,

$$W = \frac{120 \times 96}{15} = 768 \text{ lb. Ans.}$$

83. In Fig. 35, the force is supposed to act parallel to the base for any position of W; therefore, while W is moving from the level $a\,c$ to b, or through the height $c\,b$ of the inclined plane, P will move through a distance equal to the length of the base $a\,c$. When the force acts parallel to the base, we have

Rule 31.—*To find the force required, multiply the weight by the height of the plane and divide the product by the length of the base of the plane. To find the weight that can be raised, multiply the force by the length of the base of the plane and divide the product by the height of the plane.*

Or, $$P = \frac{Wh}{b} \text{ and } W = \frac{Pb}{h}.$$

Example 1.—With a base 80 feet long and a height of 6 feet, what force will sustain a weight of 75 pounds?

Solution.—By rule **31**,

$$P = \frac{75 \times 6}{30} = 15 \text{ lb. Ans.}$$

Example 2.—A force of 12 pounds sustains a weight on a plane whose base is 6 feet long and whose height is 18 inches; find the weight.

Solution.—Applying the rule just given, we have

$$W = \frac{12 \times 6}{1\tfrac{1}{2}} = 48 \text{ lb. Ans.}$$

For Fig. 36, no rule can be given. The ratio of the power varies as the position of W varies; it may be determined by the principle of the resolution of forces. As W

shifts its position, the angle that its cord makes with the face of the plane varies, and the magnitude of the force P depends on this angle; the smaller the angle the less is P required to be in order to support a given weight on a given plane.

84. Applications of the Inclined Plane.—The wedge is a movable inclined plane that in various modifications serves for many different purposes. In the form of a stake wedge, as shown in Fig. 37, it serves to move heavy weights or to separate closely joined pieces of material. In the form of a cold chisel or knife, it serves to sever substances. In the form of a key, it serves to move the brasses in the different pin-joint connections of a steam engine. Familiar examples of inclined planes about machinery are the keys used for fastening a crank to the crankshaft or a pulley to the shaft; in some designs of crossheads, the cotters securing the crosshead to the piston rod are an example of an inclined plane; as are also the adjustable shoes in many designs of crossheads for steam engines.

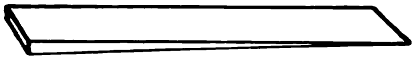


FIG. 37.

85. Taper of Keys.—In keys, cotters, and stake wedges, it is customary to express the inclinations as a certain amount per foot, or taper per foot. Thus, in Fig. 38, the inclination would be 3 − 2 = 1 inch in 12 inches, and the taper would be, as there are 12 inches in a foot, 1 inch per foot. The taper per foot can be ascertained by the following rule,

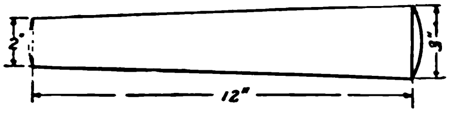


FIG. 38.

where a = the large diameter in inches;
b = the small diameter in inches;
l = the length in inches;
T = taper in inches per foot.

Rule 32.—*Subtract the small diameter from the large diameter, multiply the remainder by 12, and divide the product by the length of the key.*

Or,
$$T = \frac{(a - b)\ 12}{l}.$$

Example.—A key measures 4 inches at the large end and 3 inches at the small end; if its length is 16 inches, what is the taper per foot?

Solution.—Applying the rule just given, we get

$$T = \frac{(4 - 3) \times 12}{16} = \tfrac{3}{4} \text{ in. per ft. Ans.}$$

86. Where tapered keys are used for adjusting brasses, it is often desirable to know how much the brasses will be closed in on the pin by a given movement of the key, or how much to back out a key that has been driven home until the brasses nip the pin, the clearance between the brasses and the pin necessary for satisfactory running having been decided on.

Let T = taper in inches per foot;
d = distance the key is driven in inches;
x = amount the brass is moved in inches.

Rule 33.—*To find the amount the brass has been moved, multiply the taper by the distance the key is driven and divide the product by 12. To find how far to back out a key, multiply the amount the brass is to be moved back by 12 and divide by the taper.*

Or,
$$x = \frac{Td}{12} \text{ and } d = \frac{12x}{T}.$$

Example.—In Fig. 39, the key has been driven home until the brasses nip the pin; it has been determined to run the brasses with a clearance of $\frac{1}{64}$ inch; how much must the key be backed out, its taper being $\frac{3}{4}$ inch per foot?

Solution.—By rule 33,

$$d = \frac{12 \times \frac{1}{64}}{\frac{3}{4}} = \tfrac{1}{4} \text{ in. Ans.}$$

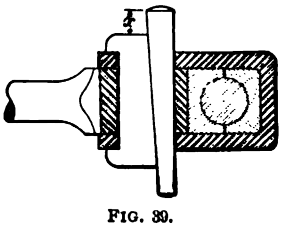


Fig. 39.

A ready way of measuring the distance the key is driven back is to measure from the top surface of the cotter to the upper edge of the key while the key is home. Add the amount the key is to be driven back, and move it until a rule shows it to have been driven the right amount. This method is preferable to making scriber marks on the flat sides of the key, as there is no liability of getting mixed up by the multiplicity of marks sure to be found after years of service.

87. Keys for fastening pulleys to shafts in order to cause them to rotate together belong usually to one of the three classes shown in Fig. 40. The one shown at *a* is a concave key that is hollowed out to fit the shaft. As it holds the pulley by friction alone, it is only suitable for light work. In case such a key slips, its holding power can be increased a little by hollowing out its concave side to a radius smaller than the radius of the shaft. The flat key is shown in Fig. 40 at *b*; a flat surface is cut on the shaft, and, consequently, the key is more effective than the concave key, as far as slipping under a rotative strain is concerned. The sunk key shown in Fig. 40 at *c* is the most effective, since it is impossible for the pulley to turn on the shaft without shearing off the key. Keys for fastening pulleys, etc. are usually given a taper of from $\frac{1}{8}$ to $\frac{1}{4}$ inch per foot.

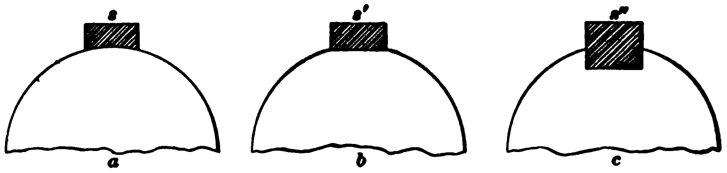


FIG. 40.

EXAMPLES FOR PRACTICE.

1. An inclined plane is 30 feet long and 7 feet high; what force is required to roll a barrel of flour weighing 196 pounds up the plane, the friction being neglected? Ans. 45.7 lb.

2. What is the taper per foot of a stake wedge measuring $\frac{1}{32}$ inch at the small end and $\frac{1}{4}$ inch at the large end, its length being 4 inches?

Ans. $\frac{21}{32}$ in. per ft.

3. Given, a key having a taper of 1 inch per foot. How much must it be backed out to give the brasses a working clearance of $\frac{1}{32}$ inch?

Ans. $\frac{3}{8}$ in.

THE SCREW.

88. A **screw** is a cylinder having a helical projection winding around its circumference. This helix is called the **thread** of the screw. The distance that a point on the helix is drawn back or advanced in the direction of the length of the screw during one turn is called the **pitch** of the screw. The screw in Fig. 41 is turned in a nut *a* by means of a force applied at the end of the handle *P*. For one complete revolution of the handle, the screw will be advanced lengthwise an amount equal to the pitch. If the nut is fixed and a weight placed upon the end of the screw, as shown, it will be raised vertically, by one revolution of the screw, a distance equal to the pitch. During this revolution, the force at *P* will move through a distance equal to the circumference of a circle whose radius is PF. Therefore, $W \times$ *pitch of thread* $= P \times$ *circumference of circle through which P moves.*

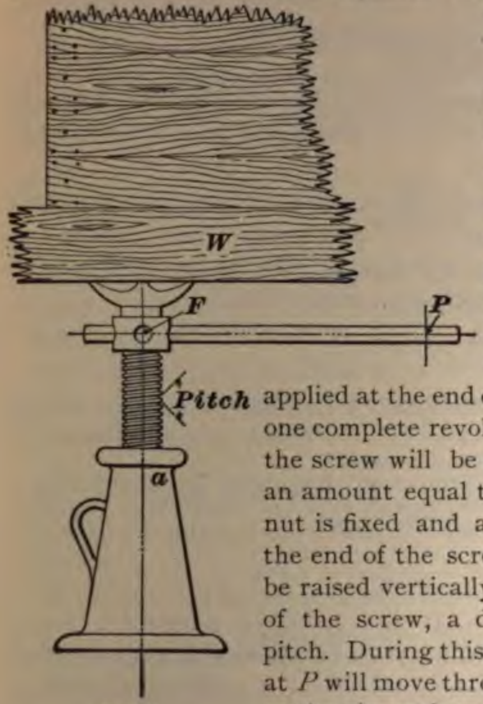


FIG. 41.

89. Owing to the friction between the nut and the screw and between the pivot of the screw and its cap, the weight that can be raised with a screw, or the force that can be exerted by it in the direction of its axis, is far less than that calculated on the assumption that friction does not exist. The friction depends on the pitch, the condition of the thread on the screw and in the nut, and the nature of the lubricant.

From a long series of experiments, Mr. Wilfred Lewis has deduced a formula for determining a factor by which to multiply the weight that could be raised if there were no friction, so as to determine the probable actual weight that can be raised.

Let d = diameter of screw in inches;
p = pitch in inches;
E = the number by which to multiply the theoretical weight in order to obtain the approximate actual weight.

Rule 34.—*Add the diameter of the screw to the pitch and divide the pitch by this sum; the quotient so obtained will be the factor by which to multiply the theoretical weight in order to determine the approximate actual weight.*

Or,
$$E = \frac{p}{p + d}.$$

EXAMPLE.—With a screw having 4 threads to the inch and 2 inches in diameter, what is the factor by which to determine how much of the theoretical load can probably be raised?

SOLUTION.—Since there are 4 threads per inch, the pitch is $1 \div 4$, or $\frac{1}{4}$ inch. Then applying rule **34**, we get

$$E = \frac{\frac{1}{4}}{\frac{1}{4} + 2} = .11. \text{ Ans.}$$

90. The weight that can be lifted when friction is neglected, or the force that can be exerted by a screw when a given force is exerted on the lever, is given by the following rule,

where W = weight or force exerted by the screw;
P = force applied to the end of the lever;
p = pitch;
r = distance from the axis of the screw to the point of application of the force P.

Rule 35.—*Multiply the force applied to the lever, in pounds, by the distance of its point of application from the axis of the screw, in inches, and multiply this product by 6.2832; divide the result by the pitch in inches and the quotient will be the theoretical force that will be exerted by the screw.*

Or, $$W = \frac{6.2832\,P\,r}{p}.$$

91. To find the theoretical force that must be applied to the lever to raise a given weight or to exert a given force:

Rule 36.—*Multiply the weight or given force by the pitch and divide this product by the product of 6.2832 and the distance from the axis of the screw to the point on the lever where the force is applied.*

Or, $$P = \frac{W\,p}{6.2832\,r}.$$

92. To obtain the probable actual weight that may be raised, multiply the result obtained by rule **35** by the value obtained from rule **34.** To obtain the probable force that must be applied to the lever in order to lift a given weight, divide the result obtained from rule **36** by the value obtained from rule **34.**

EXAMPLE 1.—A screw jack has a screw $1\frac{3}{4}$ inches diameter with 5 threads to the inch. If a man pulls at the end of the lever 14 inches from the axis with a force of 40 pounds, what weight may he expect to raise?

SOLUTION.— 5 threads per inch = $\frac{1}{5}$-inch pitch.

By rule **35,** $W = \dfrac{6.2832 \times 40 \times 14}{\frac{1}{5}} = 17{,}593$ pounds, nearly.

By rule **34,** $E = \dfrac{\frac{1}{5}}{\frac{1}{5} + 1\frac{3}{4}} = .1$, nearly.

Then, as just stated, the probable weight that may be raised is

$$17{,}593 \times .1 = 1{,}759.3 \text{ lb.} \quad \text{Ans.}$$

EXAMPLE 2.—The pull that an ordinary man can exert at the end of a lever is usually taken at 40 pounds; would it be possible for one man

to raise a weight of 3 tons with a jack-screw having 4 threads to the inch and 2¼ inches in diameter when the longest lever available is 24 inches?

Solution.—According to rule 36, the theoretical force that must be exerted is

$$\frac{3 \times 2000 \times \frac{1}{4}}{6.2832 \times 24} = 9.9 \text{ pounds, nearly.}$$

By rule 34, $$E = \frac{\frac{1}{4}}{\frac{1}{4} + 2\frac{1}{4}} = .091, \text{ nearly.}$$

Then, as just stated, the probable actual force required is 9.9. $9.9 + .091 = 109$ pounds, about, or more than a single man may be expected to exert steadily. Ans.

SCREWS USED AS FASTENINGS.

93. In machine construction, screws are used more frequently as fastening devices than as a means of transmitting motion or producing pressure.. Screws used as fastening devices may be divided into two general classes, which are distinguishable from each other by the form of thread used: (1) *Metal screws*, or screws intended to be screwed into metals. (2) *Wood screws*, or screws intended for wood.

94. Metal screws have either a sharp **V** thread or the United States standard thread shown in Fig. 42. In the latter, as also in the sharp **V** thread, the sides of the thread form a 60° angle. In the United States standard thread, the thread is flattened at the top and bottom, as shown in

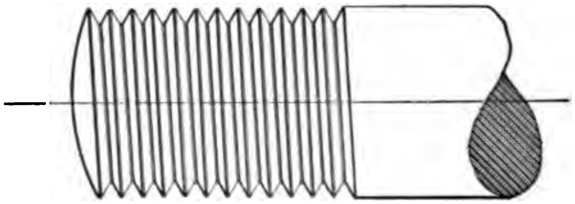


FIG. 42.

the figure. The advantage of this is that it makes a stronger screw for equal pitches and diameter.

95. Metal screws are divided by the trade into *machine bolts*, *capscrews*, and *machine screws.*

96. In **machine bolts,** or **bolts,** for short, one of which is shown in Fig. 43 (*a*), the shank and head are left rough, or just as they come from the bolt-heading machine. Machine bolts are regularly made in lengths varying by half inches from $1\frac{1}{2}$ to 8 inches. Above 8 inches and up to 20 inches the length varies by whole inches. The length of a machine bolt is always measured under the head, as the distance *l* in Fig. 43 (*a*).

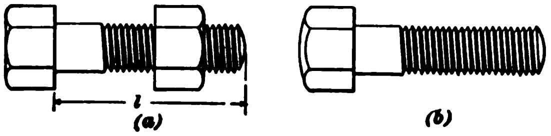


FIG. 43.

97. The standard diameters in which machine bolts are regularly made are $\frac{1}{4}$, $\frac{5}{16}$, $\frac{3}{8}$, $\frac{7}{16}$, $\frac{1}{2}$, $\frac{9}{16}$, $\frac{5}{8}$, $\frac{3}{4}$, $\frac{7}{8}$, and 1 inch. Sizes differing from these are special and can only be obtained on special order. By the *diameter of a bolt* is always meant its diameter over the top of the thread. Machine bolts can be obtained with either hexagonal or square heads and with hexagonal or square nuts.

98. Capscrews, one of which is shown in Fig. 43 (*b*), are bolts with either hexagonal or square heads; they have the shanks turned and the heads finished all over by milling and turning. Unlike machine bolts, they are usually furnished without nuts, unless especially ordered.

99. Machine bolts and capscrews are made with a pitch of thread corresponding to the United States standard thread, as shown in Table II.

TABLE II.

STANDARD THREADS PER INCH.

Diameter. Inches.	Threads per Inch.	Diameter. Inches.	Threads per Inch.
$\frac{1}{4}$	20	$1\frac{1}{4}$	7
$\frac{5}{16}$	18	$1\frac{3}{8}$	6
$\frac{3}{8}$	16	$1\frac{1}{2}$	6
$\frac{7}{16}$	14	$1\frac{5}{8}$	$5\frac{1}{2}$
$\frac{1}{2}$	13	$1\frac{3}{4}$	5
$\frac{9}{16}$	12	$1\frac{7}{8}$	5
$\frac{5}{8}$	11	2	$4\frac{1}{2}$
$\frac{3}{4}$	10	$2\frac{1}{4}$	$4\frac{1}{2}$
$\frac{7}{8}$	9	$2\frac{1}{2}$	4
1	8	$2\frac{3}{4}$	4
$1\frac{1}{8}$	7	3	$3\frac{1}{2}$

The only deviation from this table of threads is in $\frac{1}{2}$-inch square and hexagonal capscrews, where the usual number of threads for a sharp V thread is 12 per inch instead of 13, as called for. Capscrews are usually furnished with a sharp V thread, but can now be obtained with the United States standard thread. Machine bolts usually have the United States standard thread and the number of threads per inch called for in the table.

100. Machine screws are small screws with a slotted head. They are known to the trade as *flat heads*, shown in Fig. 44 (*a*); *round heads*, shown in Fig. 44 (*b*); *button heads*, shown in Fig. 44 (*c*), and *fillister heads*, shown in Fig. 44 (*d*).

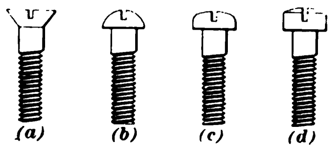


FIG. 44.

101. Screws for uniting metals are made in a variety of forms that differ from those here shown to suit special conditions. Some of these are setscrews, carriage bolts, tire bolts, boiler patch bolts, elevator bolts, etc. Most of these can only be obtained on special order, although some of the more common sizes may be found in stock.

102. Wood screws are either flat headed or round headed, with a slotted head. When wood screws have a square head for a wrench, instead of a slot for a screwdriver, they are known as *lagscrews*, or *coach screws*.

VELOCITY RATIO AND EFFICIENCY.

103. By the term **velocity ratio** is meant the ratio of the distance through which the force acts to the corresponding distance the weight moves. Thus, if the force acts through a distance of 12 inches while the weight moves 1 inch, the velocity ratio is 12 to 1, or 12, that is, P moves 12 times as fast as W.

If the velocity ratio be known, the weight that any machine will raise, under the assumption that there are no frictional resistances, can be found by multiplying the force by the velocity ratio. If the velocity ratio is 8.7 to 1, or 8.7, $W = 8.7 \times P$, since $W \times 1 = 8.7$.

104. In any machine, the force actually required to raise a weight or overcome a resistance *is always greater than the quotient obtained by dividing this weight, or resistance, by the velocity ratio of the machine.*

Thus, if there were no friction, a machine whose velocity ratio was 5 would, by an application of 100 pounds of force, raise a weight of 500 pounds. Now, suppose that the friction in the machine is equivalent to 10 pounds of force; then, it would take 110 pounds of force to raise 500 pounds.

If in the above illustration friction were neglected, we would have 110 lb. × 5 = 550 lb. as the weight that 110 pounds would raise; but owing to the frictional resistance, it only raised 500 pounds. Then, we have for the ratio between the two, $\frac{500}{550} = .91$. That is, $500 : 550 :: .91 : 1$.

105. Efficiency.—The ratio between the weight actually raised and the force multiplied by the velocity ratio is called the **efficiency of the machine.** For example, if the weight actually raised by a machine, say a screw, is 1,600 pounds, and the force multiplied by the velocity ratio is 2,400 pounds, the efficiency of this machine is $\frac{1600}{2400} = .66\frac{2}{3}$, or $66\frac{2}{3}$ per cent.

EXAMPLE.—In a machine having a combination of pulleys and gears, the velocity ratio of the whole is 9.75; a force of 250 pounds just lifts a weight of 1,626 pounds; what is the efficiency of the machine?

SOLUTION.—Efficiency $= \frac{1,626}{250 \times 9.75} = .6671$ or 66.71 per cent. Ans.

Since the total amount of friction varies with the load, it follows that the efficiency will also vary for different loads.

106. If the efficiency of a machine is known, the force actually required to raise a given load may be found by dividing the load by the product of the velocity ratio of the machine and the efficiency. Thus, if a certain machine has a velocity ratio of 10.6 and its efficiency is 60 per cent., the force that must actually be applied to raise a load of 840 pounds is $\frac{840}{10.6 \times .60} = 132.1$ pounds, nearly. If there have been no losses through friction, etc., the force required will be $840 \div 10.6 = 79.25$ pounds, nearly.

107. If the efficiency is known, the weight that a certain force will raise may be found by multiplying together the force, velocity ratio, and the efficiency. Thus, if a certain machine has a velocity ratio of $6\frac{1}{2}$ and an efficiency of 78 per cent., a force of 140 pounds will raise a weight of $140 \times 6\frac{1}{2} \times .78 = 709.9$ pounds.

EXAMPLES FOR PRACTICE.

1. The distance from the axis of a screw to the point on the handle where the force is applied is 12 inches. The screw has 8 threads per inch and is 1 inch in diameter. (*a*) What force is necessary to raise a weight of 1,248 pounds, assuming that there is no friction? (*b*) What probable force will be required?

Ans. { (*a*) 2.07 lb., nearly.
(*b*) 19 lb., about.

2. What weight can actually be raised with a screw jack having a screw 1¼ inches in diameter and 6 threads per inch, when a man is pulling with a force of 40 pounds at the end of a bar 16 inches long?

Ans. 2,838 lb., about.

3. In example 2, what is the velocity ratio? Ans. 603, nearly.

4. In example 2, what is the efficiency? Ans. 11¾ per cent.

ELEMENTS OF ELECTRICITY AND MAGNETISM.

INTRODUCTION.

1. Electricity is the name given to the cause of all electrical phenomena. The word is derived from a Greek word meaning *amber*, that substance having been observed by the Greeks to possess peculiar properties which we now understand to be due to electricity.

Although electrical science has advanced sufficiently far to recognize the fact that the exact nature of electricity is unknown, yet recent research tends to demonstrate that all electrical phenomena are due to a peculiar strain or stress of a medium called *ether;* that when in this condition the *ether* possesses ***potential energy*** or *capacity for doing work*, as is manifested by attractions and repulsions, by chemical decomposition, and by luminous, heating, and various other effects.

In all probability, electricity is not a form of matter because it does not possess most of the ordinary properties of matter. Electricity, itself, is not a form of energy, though energy may be necessary to move it under certain conditions, and electricity in motion is capable of performing work.

NOTE.—This section is the same as that formerly entitled *Dynamos and Motors*, Part 1.

For notice of the copyright, see page immediately following the title page.

Electrical science is founded upon the effects produced by the action of certain forces upon matter, and all knowledge of the science is deduced from these effects. The study of the fundamental principles of electricity is an analysis of a series of experiments and the classification of the results in each particular case under general laws and rules. It is not necessary to keep in mind any hypothesis of the exact nature of electricity; its effects and the laws which govern them are quite similar to those of well-known mechanical and natural phenomena and will be best understood by comparison. The two most essential features, therefore, in acquiring a knowledge of the electrical science are: first, to learn how to develop electrical action; and, second, to determine the effects produced by it.

2. The number of processes for developing electrical action is almost innumerable, but the most important can be classified under one of the following general heads:

(*a*) By the contact of dissimilar substances.
(*b*) By chemical action.
(*c*) By the application of heat.
(*d*) By magnetic induction.

3. The presence of electricity, also, can be detected in many different ways; under certain conditions, it will

(*a*) Cause attractions and repulsions of light particles of matter, such as feathers, pith, gold-leaf, pieces of paper, etc.

(*b*) Decompose certain forms of matter into their various elements and cause other chemical changes.

(*c*) Produce motion in a freely suspended magnetic needle, such as the needle of a compass.

(*d*) Violently agitate the nervous system of all animals, causing a **shock**.

(*e*) Heat the substances through which it acts.

These are the principal effects produced by the action of electricity; others of less importance will appear from time to time during the study of the different branches of the science.

4. Electricity may either appear to reside upon the surface of bodies as a **charge,** under *high pressure* or *tension*, or flow through their substance as a **current,** under comparatively *low pressure* or *tension*.

That branch of the science which treats of charges upon the surface of bodies is termed **electrostatics,** and the charges are said to be **static charges** of electricity.

Electrodynamics is that branch which treats of the action of *electric currents*.

STATIC CHARGES.

5. When a glass rod or a piece of amber is rubbed with silk or fur the parts rubbed will have the property of attracting light particles of matter, such as pieces of silk, wool, feathers, gold-leaf, pith, etc., which, after momentary contact, are repelled. These attractions and repulsions are caused by a static charge of electricity residing upon the surface of those bodies. A body in this condition is said to be **electrified.**

A better experiment for demonstrating this action is to suspend a small pith-ball by a silk thread from a support or bracket, as shown in Fig. 1. If a *static charge* of electricity be developed on a *glass rod*, by rubbing it with *silk*, and the rod be brought near the pith-ball, the ball will be attracted to the rod, but, after momentary contact, will be repelled. By this contact the ball receives a charge of the same nature as that on the glass rod, and as long as the two bodies retain their charges, mutual repulsion will take place whenever they

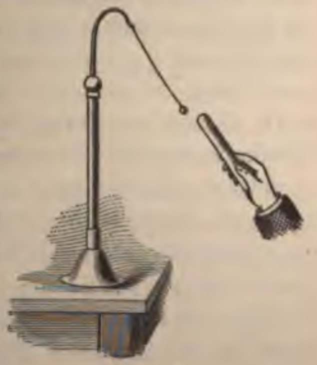


FIG. 1.

are brought near each other. If a stick of *sealing-wax*, electrified by being rubbed with fur, is approached to another pith-ball, the same results will be produced, i. e., the ball will fly towards the sealing-wax, and after contact will be repelled. But the charges respectively developed in these two cases are not in the same condition. For if, after the pith-ball in the first case had been touched with the *glass rod* and repelled, the electrified *sealing-wax* be brought in the vicinity, *attraction* would take place between the ball and the sealing-wax. Conversely, if the pith-ball be charged with the electrified sealing-wax, it will be repelled by the wax and attracted by the glass rod.

An electric charge developed upon *glass* by rubbing it with silk has been termed, for convenience, a **positive** (+) **charge,** and that developed on resinous bodies by rubbing with flannel or fur, a **negative** (—) **charge.**

Neither a positive nor a negative charge is produced alone, for there is always an equal quantity of both charges produced, one charge appearing on the body rubbed and an equal amount of the opposite charge upon the rubber.

The intensity of the charge developed by rubbing the two substances together is independent of the actual amount of friction which takes place between the bodies. For, in order to obtain the highest possible degree of electrification, it is only necessary to bring every portion of one surface into intimate contact with every particle or every portion of the other; when this is done, no extra amount of rubbing can develop any greater charge upon either substance.

6. From the foregoing experiments are derived the following laws:

When two dissimilar substances are placed in contact, one of them always assumes the positive and the other the negative condition, although the amount may sometimes be so small as to render its detection very difficult.

Electrified bodies with similar charges are mutually repellent, while electrified bodies with dissimilar charges are mutually attractive.

7. In the following list, called the **electric series,** the substances are arranged in such order that each receives a *positive* charge when rubbed or placed in contact with any of the bodies following it, and a *negative* charge when rubbed with any of those which precede it:

1. Fur.
2. Flannel.
3. Ivory.
4. Crystals.
5. Glass.
6. Cotton.
7. Silk.
8. The body.
9. Wood.
10. Metals.
11. Sealing-wax.
12. Resins.
13. Sulphur.
14. Gutta-percha.
15. Guncotton.

For example, *glass* when rubbed with *fur* receives a *negative* charge, but when rubbed with *silk* it receives a *positive* charge.

CONDUCTORS AND NON-CONDUCTORS.

8. Only that part of a dry glass rod which has been rubbed will be electrified; the other parts will produce neither repulsion nor attraction when brought near a suspended pith-ball. The same is true of a piece of sealing-wax or resin. These bodies do not readily *conduct* electricity; that is, they *oppose* or *resist* the passage of electricity through them. Therefore, it can only reside as a *charge* upon that part of their surface where it is developed. Experiments show that when a metal receives a charge at any point, the electricity immediately passes or flows through its substance to all parts. Metals, therefore, are said to be **good conductors** of electricity. Bodies have accordingly been divided into two classes, i. e., **non-conductors,** or **insulators,** or those bodies which offer a very high **resistance** to the passage of electricity, and **conductors,** or those bodies which offer a comparatively low resistance to its passage. This distinction is not absolute, for all bodies conduct electricity to some extent, while there is no known substance which does not offer some resistance to the flow of electricity.

In giving the following list and dividing the different substances into two classes, it should be understood that it is done only as a guide for the student. Between these two classes are many substances which might be included in either list, and no hard or fast line can be drawn.

Silver, Copper, Other Metals, Charcoal, Ordinary Water, The Body.	CONDUCTORS.
Paper, Oils, Porcelain, Wood, Silk, Resins, Gutta-percha, Shellac, Ebonite, Paraffin, Glass, Dry Air, etc.	NON-CONDUCTORS OR INSULATORS.

ELECTRODYNAMICS.

9. In dealing with *electric currents*, the word **potential** will be substituted for the general and vague phrase *electrical condition*.

The term *potential*, as used in electrical science, is analogous with *pressure* in gases, *head* in liquids, and *temperature* in heat.

When an electrified body *positively* charged is connected to the earth by a conductor, electricity is said to flow *from* the body *to* the earth; and, conversely, when an electrified body *negatively* charged is connected to the earth in a similar manner, electricity is said to flow *from* the earth *to* that body. This is called the **direction of flow** of an electric current. That which determines the *direction of flow* is the relative *electrical potential*, or *pressure*, of the two charges in regard to the earth.

It is impossible to say with certainty in which direction electricity really flows, or, in other words, to declare which of two points has the higher and which the lower electrical potential, or pressure. All that can be said with certainty is that when there is a *difference of electrical potential*, or *pressure*, electricity tends to flow *from* the point of higher *to* that of the lower *potential*, or *pressure*.

For convenience, it has been arbitrarily assumed and conventionally adopted that that electrical condition called *positive* is at a higher potential, or pressure, than that called *negative*, and that electricity tends to flow *from* a *positively to* a *negatively* electrified body.

The zero or normal level of water is taken as that of the surface of the sea, and the normal pressure of air and gases as that of the atmosphere at the sea level; similarly, there is a *zero potential*, or *pressure*, *of electricity* in the earth itself. The earth may be regarded as a reservoir of electricity of infinite quantity, and its potential, or pressure, may therefore be taken as zero.

The electrical condition called *positive* is assumed to be at a higher potential, or pressure, than the earth, and that called *negative* is assumed to be at a lower potential, or pressure, than the earth.

10. It must be understood that electricity is a *condition of matter*, and not matter itself, for it possesses neither *weight* nor *dimensions*. Consequently, the statement that electricity is *flowing* through a conductor must not be taken too literally; it must not be supposed that any material substance, such as a liquid, is actually passing through the conductor in the same sense as water flows through a pipe. The statement that electricity is flowing through a conductor is only another way of expressing the fact that the conductor and the space surrounding it are in different conditions than usual and that they possess unusual properties. The action of electricity, however, is quite similar in many respects to the flow of liquids, and the study of electric currents is much simplified by the analogy.

11. In order to produce what is called an electric current, *it is first necessary to cause a difference of electrical potential between two bodies or between two parts of the same body.*

It was stated that when two dissimilar substances are simply placed in contact, one always assumes the positive and the other the negative condition; or, in other words, *a difference of electrical potential is developed between the two bodies.*

Placing a piece of copper and zinc in contact will develop a difference of electrical potential which can easily be detected. The same results will follow if the plates are slightly separated from each other and placed in a vessel containing saline or acidulated water, leaving a small portion of one end of each plate exposed. The exposed ends of the zinc and copper are now electrified to different degrees, or, in other words, there is a *difference of electrical potential* between them, one plate being at a higher potential than the other.

When the exposed ends are connected by any conducting material, the potential between the plates tends to *equalize* and a momentary rush or discharge of electricity passes between the exposed ends through the conductor, and also between the submerged ends through the liquid. During its passage through the liquid, the electricity causes certain chemical changes to take place; these chemical changes cause in their turn a fresh *difference of potential* between the plates, which is followed immediately by another equalizing discharge, and that by a further difference, and so on. These changes follow one another with great rapidity—so rapidly, in fact, that it is impossible to distinguish them apart, and they appear absolutely *continuous.* The equalizing flow which is constantly taking place from one plate to the other is known as a **continuous current** of electricity. Consequently, an electric current becomes continuous when the difference of potential is constantly maintained.

By the use of a very delicate instrument, the submerged end of the copper is found to be electrified with a *negative* charge, while the submerged end of the zinc is electrified with a *positive* charge. The direction of the current, therefore, will be *from* the *submerged* end of the zinc through the liquid *to* the *submerged* end of the copper, and *from* the *exposed* end of the copper *to* the *exposed* end of the zinc.

12. A **simple voltaic,** or **galvanic, cell,** Fig. 2, is an apparatus for developing a continuous current of electricity. It consists essentially of a vessel containing saline or acidulated water in which are submerged two plates of dissimilar metals, or one metal and a metalloid (as, for instance, carbon).

Electrolyte is the name given to the liquid, which, as it transmits the current, is decomposed by it.

The two dissimilar metals, when spoken of separately, are called **voltaic,** or **galvanic, elements;** and, when taken collectively, are known as a **voltaic couple.**

A **voltaic,** or **galvanic, battery** is a number of simple cells properly joined together.

Electrodes, or **poles,** of a cell or battery are metallic terminals or connectors attached to the exposed ends of the plates, and are used to connect the cell or battery to any exterior conductor or to another cell or battery.

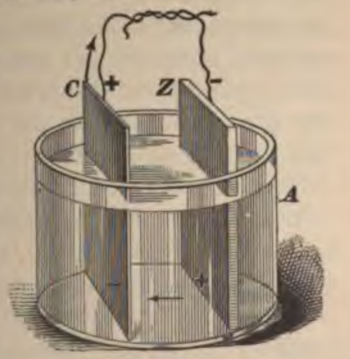


FIG. 2.

It should be remembered that the polarity of the submerged ends of the plates is always of opposite sign to that of their electrodes. For example, in the case of the zinc-and-copper couple, the electrode fastened to the zinc would be spoken of as the *negative* electrode of the cell, while the zinc itself would be the *positive* element of the cell, its submerged end being *positive*.

In any voltaic, or galvanic, couple, the element which is acted upon by the electrolyte will always be the *positive* element and its electrode the *negative* electrode of the cell.

13. The following list of voltaic elements composes what is called the **electromotive series:**

1. Zinc.
2. Cadmium.
3. Tin.
4. Lead.
5. Iron.
6. Nickel.
7. Bismuth.
8. Antimony.
9. Copper.
10. Silver.
11. Gold.
12. Platinum.
13. Graphite.

Any two of these metals form a *voltaic couple* and produce a difference of potential when submerged in saline or acidulated water, the one standing first on the list being the *positive* element or plate, and the other the *negative*. For example, if *nickel* and *graphite* are used, the *nickel* will be acted upon by the liquid and will form the *positive* element; but if *nickel* and *zinc* are used, the *zinc* will be acted upon by the liquid and hence will be the *positive* element.

The difference of potential will be greater in proportion to the distance between the positions of the two substances in the list. For example, the difference of potential developed between *zinc* and *graphite* is much greater than that developed between *zinc* and *nickel;* in fact, the difference of potential developed between zinc and graphite is equal to the difference of potential developed between zinc and nickel *plus* that developed between nickel and graphite.

Electricity flowing as a *current* differs from *static charges* in three important degrees—namely, (1) its *potential* is much lower; (2) its *actual quantity* is greater; and (3) it is *continuous.*

A substance charged from a strong voltaic battery possesses the property of attracting light substances in only the slightest degree; in fact, the attractions can only be detected with the most delicate instruments. The *potential* of a current of electricity is comparatively so small that a voltaic battery composed of a large number of cells is not sufficient to produce a spark more than one or two one-hundredths of an inch long in air, whereas a small, rapidly moving leather belt will sometimes produce static sparks of more than an inch in length. The length of the spark affords a means of estimating potentials, a high potential being capable of producing a longer spark than a low potential; but the length of spark gives us no means of estimating the **current strength** or quantity of electricity flowing. The actual *quantity* of electricity is measured by the amount of water it will decompose. Gauged by this standard, the quantity of electricity produced by a voltaic cell no larger than a thimble would be found greater than that from a large, rapidly moving belt giving static sparks several inches in length.

14. There are three different methods of connecting or grouping the cells in a voltaic battery: *In series; in parallel, or multiple-arc; in multiple-series.*

Cells are connected **in series** when the positive electrode of the first cell is connected to the negative electrode of the second, and the positive electrode of the second is connected to the negative electrode of the third, and so on, as shown in the diagram, Fig. 3. In this we have adopted the usual signs for representing a cell, the short, broad line representing the positive electrode of the cell and the long, narrow line the negative electrode. In this method of connecting or grouping cells, when the negative electrode of the first cell is connected to the positive electrode of the last by some exterior conductor, the total current produced will flow successively through each cell. This method of

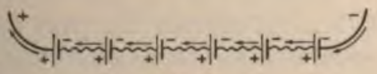


FIG. 3.

grouping is used when there is available a large number of *low*-potential cells and a *high* potential is desired, as in long telegraph lines or any other *high*-resistance circuit.

15. Cells are connected in **parallel,** or **multiple-arc,** when the positive electrodes of all the cells are connected to one main positive conductor and all the negative electrodes are connected to one main negative conductor, as shown by the diagram, Fig. 4. In parallel, or multiple-arc, grouping, only a part of the total current flowing in main conductors will pass through each cell. This method of grouping is used when it is desired to obtain a strong current from a number of cells (when the external resistance is *low*), as in electroplating.

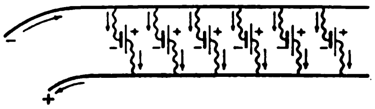


FIG. 4.

16. Cells are connected in **multiple-series** by arranging them in several groups, each group being composed of several cells connected in series, and then connecting all the groups together in parallel, or multiple-arc, as shown in the diagram, Fig. 5. This method is used where both a higher potential and a stronger current are required than any one cell of the group will give.

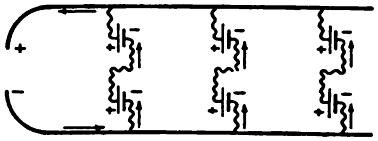


FIG. 5.

CIRCUITS.

17. A **circuit** is a path composed of a conductor or of several conductors joined together, through which an electric current flows from a given point around the conducting path back again to its starting point.

A circuit is **broken,** or **open,** when its conducting elements are disconnected in such manner as to prevent the current from flowing.

A circuit is **closed,** or **complete,** when its conducting elements are so connected as to allow the current to flow.

A circuit in which the earth, or ground, forms part of the conducting path is called an **earth,** or a **grounded,** circuit.

The **external** circuit is that part of a circuit which is outside or external to the electric source.

The **internal** circuit is that part of a circuit which is included within the electric source.

In the case of the simple voltaic cell, the *internal circuit* consists of the two metallic plates, or elements, and the electrolyte; an *external circuit* would be a wire or any conductor connecting the free ends of the electrodes.

18. Conductors are said to be connected **in series** when they are so joined together as to allow the current to pass consecutively through each. For example, Fig. 6 represents a *closed* circuit consisting of a simple voltaic cell *B* and four conductors *a*, *b*, *c*, and *d* connected *in series*.

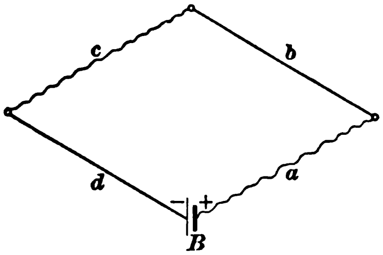


FIG. 6.

A circuit which is divided into two or more branches, each branch transmitting part of the main current, is a **derived,** or **shunt, circuit**, and the separate branches are said to be connected in **parallel,** or **multiple-arc.** An example of a *derived* circuit of two branches in *parallel* is shown in Fig. 7. The main current flows first through the conductor *a*, then divides between the branches *c* and *b*, and finally unites and completes the circuit through the conductor *d*, the two branches *c* and *b* being the conductors which are connected in *parallel*, or *multiple-arc*. The way the current divides and how the amount which will flow through the branches *b* and *c* is determined will be treated of later.

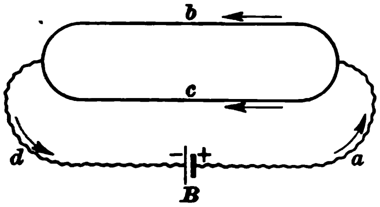


FIG. 7.

MAGNETISM.

19. **Magnets** are substances which have the property of attracting pieces of iron or steel, and the term **magnetism** is applied to the cause of this attraction. *Magnetism* exists in a natural state in an ore of iron, which is known in chemistry as *magnetic oxide of iron*, or *magnetite*. This magnetic ore was first found by the ancients in *Magnesia*, a city in Asia Minor; hence, substances possessing this property have been called magnets.

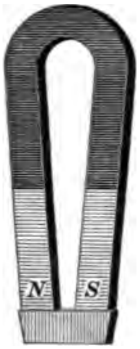


FIG. 8.

It was also discovered that when a small bar of this ore is suspended in a horizontal position by a thread, it has the property of pointing in a north and south direction. From this fact the name **lodestone**—*leading-stone*—was given to the ore.

When a bar or needle of hardened steel is rubbed with a piece of lodestone, it acquires magnetic properties similar to those of the lodestone without the latter losing any of its own force. Such bars are called **artificial** magnets.

Artificial magnets which retain their magnetism for a long time are called **permanent magnets.**

The common form of artificial, or permanent, magnet, Fig. 8, is a bar of steel bent into the shape of a *horseshoe* and then hardened and magnetized. A piece of soft iron, called an **armature**, or a **keeper**, is placed across the two free ends, which helps to prevent the steel from losing its magnetism.

20. If a bar magnet is dipped into iron filings, the filings are attracted toward the two ends and adhere there in tufts, while toward the center of the bar, half way between the two ends, there is no such tendency. (See Fig. 9.) That part of the magnet where there is no apparent magnetic attraction is called the **neutral line**, and the parts around the two ends where the attraction is greatest are called **poles**. An imaginary line drawn through the center of the magnet from end to end, connecting the two poles together, is called the **axis of magnetism.**

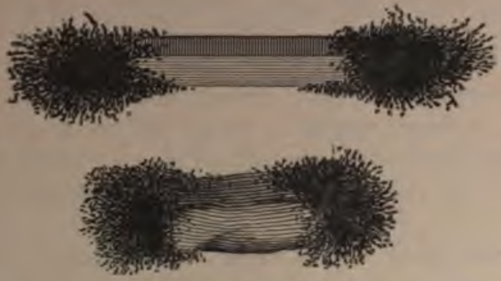


FIG. 9.

A **compass** consists of a magnetized steel needle, Fig. 10, resting upon a fine point, so as to turn freely in a horizontal plane. When not in the vicinity of other magnets or magnetized iron, the needle will always come to rest with one end pointing towards the north and the other towards the south. The end pointing northward is the **north-seeking pole,** or, simply, the *north pole*, and the opposite end is the **south-seeking** or *south pole*. This *polarity* applies as well to all magnets.

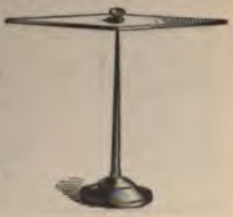


FIG. 10.

If the *north pole* of one magnet is brought near the *south pole* of another magnet, attraction takes place; but if two north poles or two south poles are brought together, they repel each other. In general, *like magnetic poles repel one another; unlike poles attract one another.*

The earth is a great magnet whose magnetic poles coincide nearly, but not quite, with the true geographical north and south poles. A freely suspended magnet, therefore, will always point in an approximately north and south direction.

It is impossible to produce a magnet with only one pole. If a long bar magnet is broken into any number of parts, each part will still be a magnet and have two poles, a north and a south one.

21. Magnetic substances are those substances which, although not in themselves magnets, that is, not possessing poles and neutral lines, are, nevertheless, capable of being attracted by a magnet. In addition to iron and its alloys, the following elements are magnetic substances: *Nickel, cobalt, maganese, oxygen, cerium,* and *chromium.* These, however, possess magnetic properties in a very inferior degree compared with iron and its alloys. All other known substances are called **non-magnetic substances.**

22. The space surrounding a magnet, in which any magnetic substance will be attracted or repelled,is called its **magnetic field,** or, simply, its **field.** Magnetic attractions and repulsions are assumed to act in a definite direction and along imaginary lines called **lines of magnetic force,** or, simply, **lines of force,** and every magnetic field is assumed to be traversed by such *lines of force*—in fact, to exist by virtue of them. Their position in any plane may be shown by placing a sheet of paper over a magnet and sprinkling fine iron filings over the paper. In the case of a bar magnet lying on its side, the iron filings will arrange themselves in curved lines extending from the north to the south pole, as shown in Fig. 11. A view of the magnetic field looking towards either pole of a bar magnet would exhibit merely radial lines, as shown by the filings in Fig. 12.

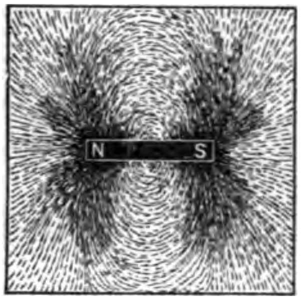


FIG. 11.

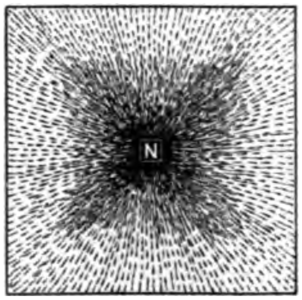


FIG 12.

Every line of force is assumed to pass out from the north pole, make a complete circuit through the surrounding medium, and pass into the south pole, thence through the magnet to the north pole again, as shown in Fig. 13. This is called the *direction of the lines of force*, and the path which they take is called the *magnetic circuit*.

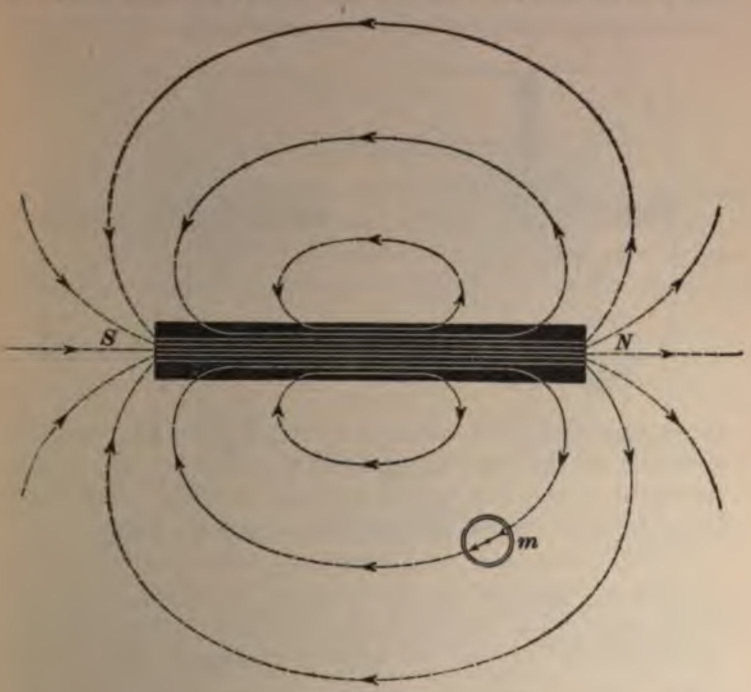


FIG. 13.

23. The *direction of the lines of force* in any magnetic field can be traced by a small, freely suspended magnetic needle, or a small compass such as indicated by *m* in Fig. 13. The north pole of the needle will always point in the direction of the lines of force, the length of the needle lying either parallel or tangent to the lines of force at that point. If the needle be moved bodily in the direction towards which the north pole points, its center or pivot will describe a path coinciding with the direction of the lines of force in that part of the magnetic field.

NOTE.—In all diagrams, the *direction of the lines of force* will be represented by arrowheads upon dotted lines.

Lines of force can never intersect one another; when two opposing magnetic fields are brought together, as indicated by the iron filings in Fig. 14 and Fig. 15, the lines of force from each will be crowded and distorted from their original direction until they coincide in direction with those opposing, and form a resultant field in which the direction of the lines of force will depend upon the relative strengths of the two opposing negative fields. The resulting poles thus formed are called **consequent poles.**

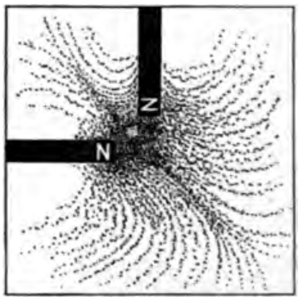


FIG. 14.

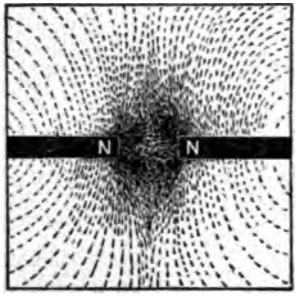


FIG. 15.

In every magnetic field there are certain stresses which produce a *tension* along the lines of force and a *pressure* across them; that is, they tend to *shorten* themselves from end to end and to *repel* one another as they lie side by side.

24. When a magnetic substance is brought into a magnetic field, the lines of force in that vicinity crowd together and all tend to pass through the substance. If the substance is free to move on an axis (but not bodily) towards the magnet pole, it will always come to rest with its greatest extent or length in the direction of the lines of force. The body will then become a magnet, its south pole being situated where the lines of force enter it and its north pole where they pass out. The production of magnetism in a magnetic substance in this manner is called **magnetic induction.** The production of artificial magnetism in a hardened steel needle or bar by contact with lodestone is one case of magnetic induction.

The **amount, or quantity, of magnetism** is expressed by the total number of lines of force contained in a magnetic circuit.

Magnetic density is the number of lines of force passing through a unit area measured perpendicularly to their direction.

ELECTROMAGNETISM.

25. If a conductor be placed parallel to the magnetic axis of a compass needle and a current be passed through the conductor in either direction, the needle will tend to place itself at right angles to the conductor, as shown by arrows in Fig. 16; or, in general, an electric current and a magnet exert a mutual force upon each other. From the definition given in Art. **22**, the space surrounding the conductor is a *magnetic field*. If the conductor is threaded up through a piece of cardboard, and iron filings are sprinkled on the cardboard, they will arrange themselves in concentric circles around the conductor, as represented in Fig. 17. This effect will be observed throughout the entire length of the conductor and is caused entirely by the current. In fact, every conductor conveying a current of electricity can be imagined as completely surrounded by a sort of magnetic *whirl*, the

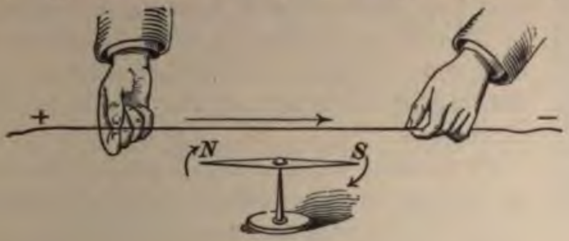


FIG. 16.

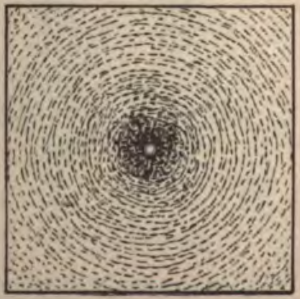


FIG. 17.

magnetic density decreasing as the distance from the current increases. (See Fig. 18.)

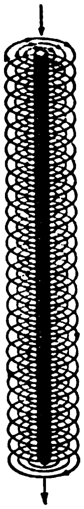


FIG. 18.

26. If the current in a horizontal conductor is flowing *towards* the *north*, and a compass is placed *under* the conductor, Fig. 19, the north pole of the needle will be deflected towards the *west;* by placing the compass *over* the wire, Fig. 20, the north pole of the needle will be deflected towards the *east*. By reversing the direction of the current in the conductor, the needle will point in the opposite direction in each case, respectively.

If the conductor is placed *over* the needle and then bent back *under* it, forming a loop as shown in Fig. 21, the tendency of the current in both top and bottom portions of the wire is to deflect the north pole of the needle in the same direction.

From these experiments, knowing the direction of current in the conductor, the following rule is deduced for the direction of the lines of force around the conductor:

Rule.—*If the current is flowing in the conductor away from the observer, then the direction of the lines of force will be around the conductor in the direction of the hands of a watch.*

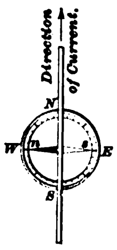


FIG. 19.

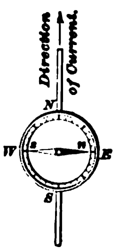


FIG. 20.

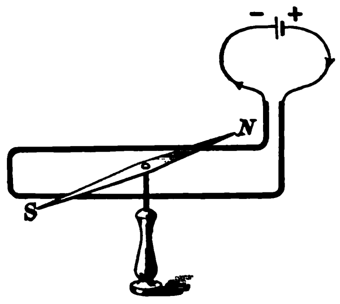


FIG. 21.

The direction of the lines of force around a conductor is indicated in Fig. 22, where the current is assumed to be flowing downwards, that is, piercing the paper.

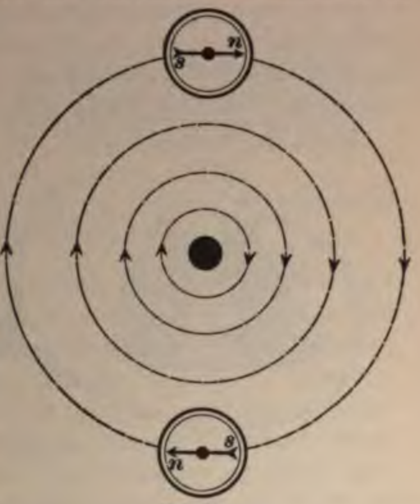


FIG. 22.

27. Two parallel conductors, both transmitting currents of electricity, are either mutually attractive or repellent, depending upon the relative direction of their currents. If the currents are flowing in the *same* direction in both conductors, as represented in Fig. 23, the lines of force will tend to surround both conductors and contract, thus *attracting* the conductors. If, however, the currents are flowing in opposite directions, as in Fig. 24, the lines of force lying between the conductors will have the same direction and therefore *repel* the conductors.

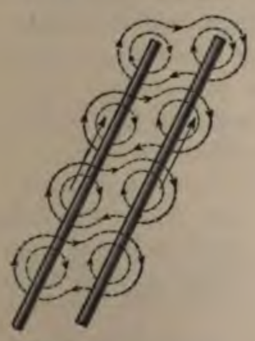


FIG. 23.

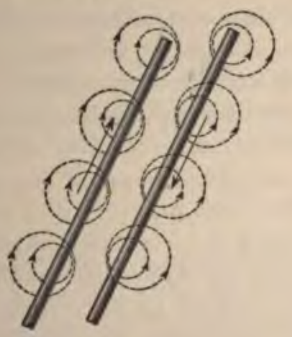


FIG. 24.

28. If the conductor carrying the current is bent into the form of a loop, as in Fig. 25, then all the lines of force around the conductor will thread through the loop in the same direction. By bending the conductor into a long *helix* of several loops, the lines of force around each loop will coincide with those around the adjacent loops, forming several long lines of force which thread through the entire helix, entering at one end and passing out at the other. The same conditions now exist in the helix as exist in a bar magnet, i. e., the lines of force *pass out* from one end and *enter* the other. In fact, the helix possesses a *north* and a *south pole*, a *neutral line*, and all the properties of attraction and repulsion of a magnet. If it is suspended in a horizontal position and free to turn, it will come to rest pointing in a north-and-south direction.

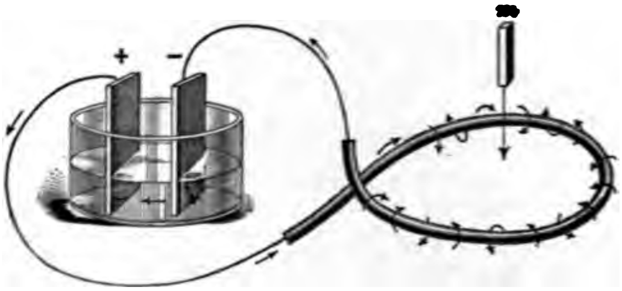


FIG. 25.

A helix made in this manner, around which a current of electricity is circulating, is called a **solenoid.**

29. The **polarity** of a solenoid, that is, the direction of the lines of force which thread through it, depends upon the direction in which the conductor is coiled and the direction of the current in the conductor.

To determine the polarity of a solenoid, knowing the direction of the current:

Rule.—*In looking at the end of the helix, if it is so wound that the current circulates around the helix in the direction of the hands of a watch, that end will be a south pole; if in the other direction, it will be a north pole.*

Fig. 26 represents a conductor coiled in a right-handed helix. If the current starts to flow from the end where the observer stands, that end will be a south pole and the observer will be looking through the helix *in the direction of the lines of force.*

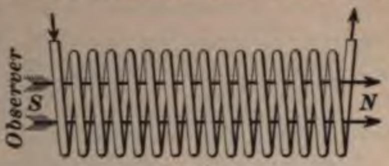


FIG. 26.

The polarity of a solenoid can be changed by reversing the direction of the current in the conductor.

30. It has been stated that when a magnetic substance is brought into a magnetic field, the lines of force in that field crowd together, and all try to pass through that substance; in fact, they will alter their circular shape and extend a considerable distance from their original position in order to pass through it. A magnetic substance, therefore, offers a better path for the lines of force than air or other non-magnetic substances.

The facility afforded by any substance to the passage through it of lines of force is called **magnetic permeability,** or, simply, **permeability.**

The *permeability* of all non-magnetic substances, such as air, copper, wood, etc. is taken as 1, or unity. The permeability of soft iron may be as high as 2,000 times that of air. If, therefore, a piece of soft iron be inserted into the magnetic circuit of a solenoid, the number of lines of force will be greatly increased, and the iron will become highly magnetized.

31. A magnet produced by inserting a magnetic substance into the magnetic circuit of a solenoid is an **electromagnet,** and the magnetic substance around which the current circulates is called the **core.** (See Fig. 27.) The solenoid is generally termed the **magnetizing coil.**

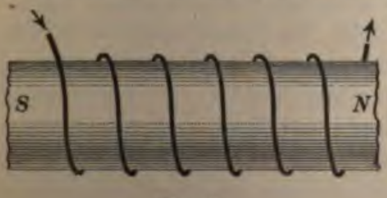


FIG. 27.

In the ordinary form of electromagnet, the magnetizing coil consists of a large number of turns of *insulated* wire, that is, wire covered with a layer or coating of some non-conducting or insulating material, usually silk or cotton; otherwise the current would take a shorter and easier circuit from one coil to the adjacent one or from the first to the last coil through the iron core without circulating around the magnet.

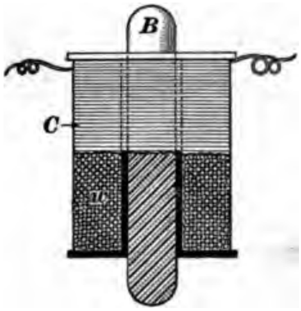


FIG. 28.

The simplest form of an electromagnet is the bar magnet. As usually constructed, it consists of a straight bar of iron or steel B, fitted into a *spool*, or *bobbin*, made of hard vulcanized rubber or some other inflexible insulating material. The magnetizing coil of fine insulated copper wire w is wound in layers in the bobbin, as shown by the cross-section in Fig. 28.

The rule for determining the polarity of a solenoid is the same for an electromagnet. It makes no difference whether the wire is wound in one layer or in any number of layers, or whether it is wound towards one end and then wound back again over the previous layer towards the other end; so long as the current circulates continually in the same direction around the core, the polarity of the magnet will remain unchanged.

32. The most convenient form of electromagnet for a great variety of uses is the *horseshoe*, or **U**-*shaped*, electromagnet, Fig. 30. It consists of a bar of iron bent into the shape of a horseshoe with straight ends and provided with two magnetizing coils, one on each end of the magnet. The two ends which are surrounded by the coils are the *cores* of the magnet, and the arc-shaped piece of iron joining them together is known as the **yoke** of the magnet. The ordinary **U**-shaped electromagnet is made in three parts, namely, two iron cores wound with the magnetizing coils, and a straight bar of iron joining the two cores together for a yoke, as shown in Fig. 29. In looking at the free ends of the two cores, Fig. 30, the current should circulate around one core in an opposite direction to that around the other. If the current circulates around both cores in the same direction, the lines of force produced in the two cores, respectively, oppose one another, forming two like poles at their free ends and a *consequent pole* in the yoke. The total number of lines of force produced by both coils will be greatly diminished, and the magnet will exhibit only a small amount of magnetic attraction.

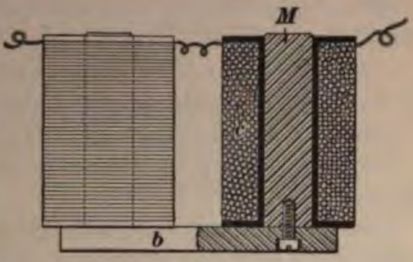


FIG. 29.

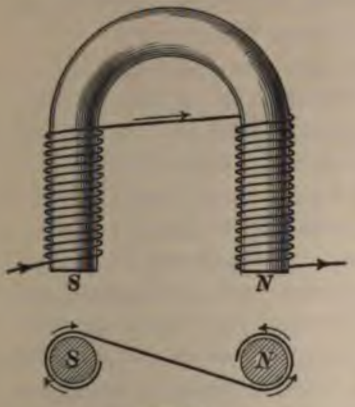


FIG. 30.

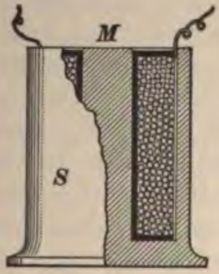


FIG. 31.

Another common form of electromagnet is known as the **iron-clad** electromagnet. In its simplest form, Fig. 31, it contains only one magnetizing coil and one core. The core is fastened to a disk-shaped iron yoke, and the magnetic circuit is completed through an iron shell which rises up from the yoke and completely surrounds and protects the coil.

ELECTRICAL UNITS.

33. The three principal units used in practical measurements of a current of electricity are:

The **ampere,** *or the practical unit denoting the rate of flow of an electric current, or the strength of an electric current.*

The **ohm,** *or the practical unit of resistance.*

The **volt,** *or the practical unit of electrical potential, or pressure.*

Electromotive force, written E. M. F., or simply E., is the *total generated difference of potential* in any electric source or in any circuit. For example, the total difference of potential developed between the plates of a simple voltaic cell would be the *electromotive force* of that cell.

Ordinarily, the term *electromotive force* is used to express any difference of potential between two points.

The relation of these three practical units will be better understood by the analogy of the flow of water through a pipe. The force which causes the water to flow through the pipe is due to the *head*, or *pressure;* that which *resists* the flow is the friction of the water against the inside of the pipe, and the amount will vary with circumstances. The *rate of flow*, or the *current*, may be expressed in *gallons per minute*, and is a ratio between the head, or pressure, and the resistance caused by the friction of the water against the inside of the pipe. For, as the pressure, or head, *increases*, the rate of flow or current *increases* in proportion; as the resistance *increases*, the current *diminishes*.

In the case of electricity flowing through a conductor, the *electromotive force*, or *potential*, corresponds to the pressure, or head, of water, and the resistance which a conductor offers to the flow of electricity to the friction of the water against the pipe. The *strength of an electric current*, or the *rate of flow of electricity*, is also a ratio—the ratio of the electromotive force to the resistance of the conductor through which the current is flowing. This ratio as applied to electricity was first discovered by Dr. G. S. Ohm, and has since been called **Ohm's law.**

34. Ohm's Law.—*The strength of an electric current in any circuit is directly proportional to the electromotive force developed in that circuit and inversely proportional to the resistance of the circuit;* i. e., *it is equal to the electromotive force divided by the resistance.*

Ohm's law is usually expressed algebraically thus:

$$\text{Strength of current} = \frac{\text{electromotive force}}{\text{resistance}}.$$

If the electromotive force (E) is expressed in *volts* and the resistance (R) in *ohms*, the formula will give the strength of current (C) directly in *amperes;* thus, $C = \frac{E}{R}$.

Before giving examples of the application of Ohm's law, the value and significance of each unit will be treated separately.

35. The Ampere, or the Unit Strength of Current.—The strength of an electric current can be described as a *quantity* of electricity flowing continuously every second, or, in other words, it is the rate of flow of electricity, just as the current expressed in *gallons per minute* is the *rate of flow* of liquids. When one unit quantity of electricity is flowing continuously every second, then the rate of flow, or the strength of current, is *one ampere;* if two unit quantities are flowing continuously every second, then the strength of current is *two amperes*, and so on. It makes no difference in the number of amperes whether the current flows for a long period or for only a fraction of a second; if the quantity of electricity that would flow in one second is the same in both cases, then the strength of the current *in amperes* is the same.

The *international ampere* is defined as the strength of an unvarying current, which, when passed through a solution of nitrate of silver and water, deposits silver at the rate of .01725 grain per second.

Electricity possesses neither *weight* nor *extension*, and therefore an electric current cannot be measured by the usual methods adopted for measuring liquids and gases.

In liquids, the strength of the current is determined by measuring or weighing the actual quantity of the liquid which has passed between two points in a certain time and dividing the result by that time. The strength of an electric current, on the contrary, is determined indirectly by the effect it produces, and the actual quantity of electricity which has passed between two points in a certain time is afterwards calculated by multiplying the strength of the current by the time.

36. The principal effects produced by an electric current are given in Art. **3**; of these, the one most generally used for measuring is the action of the current upon a magnetic needle, as shown in Art. **25.** The instrument commonly used in laboratory practice for measuring and detecting small currents of electricity is called the **galvanometer.**

The action of the galvanometer is based upon the principle given in Art. **25,** where a magnetic needle, freely suspended in the center of a looped or coiled conductor, is deflected by a current of electricity passing around the coil, or loop. In ordinary practice, the needle is suspended either upon a pivot projecting into an agate cup fixed in the needle, or by a fiber suspension, as shown by *F* in Fig. 32. In the simpler forms of galvanometers, the magnetic needle itself swings over a dial graduated in degrees; in other forms, a light index needle is rigidly attached to the magnetic needle and swings over a similar dial, as indicated by *I* in Fig. 32; and in the more sensitive galvanometers, Fig. 33, a small reflecting mirror is attached to the fiber suspension and reflects a beam of light upon a horizontal scale situated several inches from the galvanometer.

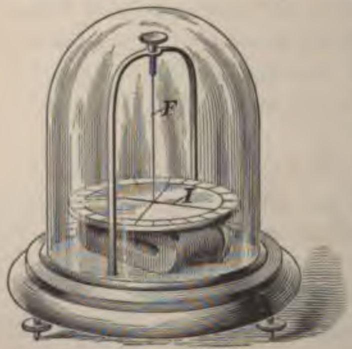


FIG. 32.

In any of these galvanometers, when no current is flowing in the coils, the needle should point in a direction parallel to the length of the coil, Fig. 34. The measuring of currents by most galvanometers depends upon the magnetic needle being held in this position by the magnetic attraction of the earth's magnetism or the attraction of some adjacent magnet. When a current of electricity passes around the coil, its tendency is to deflect the magnetic needle at right angles to its original position, as explained in Art. **25,** while the tendency of the earth's magnetism is to oppose the movement. The couple thereby produced will cause the needle to be deflected a certain number of degrees from its original position, depending upon the relative strengths of the two magnetic fields. The stronger the current in the coil, the greater the deflection. With a

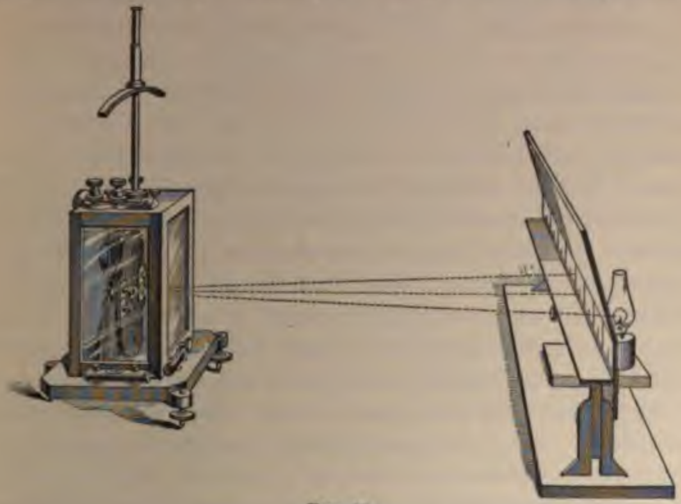


FIG. 33.

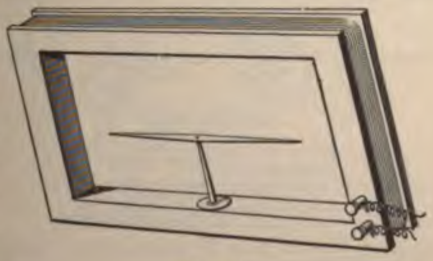


FIG. 34.

galvanometer of standard dimensions and a magnetic field of known strength, such as the earth's magnetism at a convenient place on its surface, a strength of current can be conventionally adopted as a unit which will produce a certain deflection; all other galvanometers can be calibrated from this standard, and their dials graduated to read the strength of current directly in the conventional unit adopted.

37. Commercial and portable instruments are devised for measuring the strength of current directly in *amperes*, and are called **ampere meters**, or simply *ammeters*. The action of the current flowing through the coils in these instruments causes small magnetic needles or other coils of wire to act either against the tension of springs or against gravitational forces. The majority of ammeters are provided with an index needle which travels over a scale or dial graduated in divisions, each division representing one ampere or fractions or multiples of one ampere.

Fig. 35 shows the general form of a standard **Weston ammeter** used for commercial testing purposes. The strength of the current flowing in a circuit can be measured directly in amperes by opening the circuit at any convenient place and connecting the two ends thus formed to the binding post *p* and *p'*. The direction of the current in the circuit should be determined beforehand, so that it passes *into* the instrument by the binding post marked with the positive (+) sign; otherwise the index needle will be deflected off the scale in the wrong direction, which is liable to damage the instrument and cause error in reading when the current passes through in the proper direction.

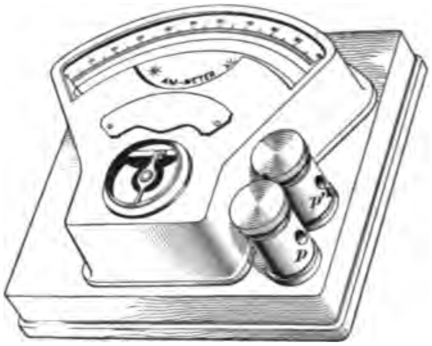


FIG. 35.

38. The Ohm, or the Unit of Resistance.—In Art. 8 it was stated that the resistance varied in different substances; that is, one substance offers a higher resistance to a current of electricity than another. Electrical resistance, therefore, can be defined as a property of matter, varying with different substances, and in virtue of which such matter opposes or resists the passage of electricity.

The resistance which all substances offer to the passage of an electric current is one of the most important quantities in electrical measurements. In the first place, it is that which determines the strength of an electric current in any circuit in which a difference of potential is constantly maintained, as shown by Ohm's law; and in the second place, the unit of resistance, the *ohm*, is the only unit in electrical measurements for which a material standard can be adopted, other quantities being measured by the effect they produce. The basis of any system of physical measurements is generally some material standard conventionally adopted as a unit, physical measurements in each system being made by comparison with the unit of that system.

The unit of electrical resistance now universally adopted is called the **international ohm.** One international ohm is the resistance offered by a column of pure mercury 106.3 centimeters in length and 1 square millimeter in sectional area at 32° F., or the temperature of melting ice. The dimensions of the column expressed in inches are as follows: length, 41.85 inches; sectional area, .00155 square inch. Hereafter the word "international" will be omitted and simply the word "ohm" used; the *international ohm*, however, as defined above, will always be implied unless otherwise stated.

39. If a given conductor offers a resistance of 2 ohms to a current of 1 ampere, it offers the same amount, no more nor less, to a current of 10 amperes. Hence, *the resistance of a given conductor at equal temperatures is always constant, irrespective of the strength of current flowing through it or the electromotive force of the current.*

40. If the length of a conductor be doubled, its resistance will be doubled; that is, the resistance of a given conductor increases as the length of the conductor increases, the resistance being directly proportional to the length of the conductor.

When it is required to find the resistance of a conductor of which the length is varied, and other conditions remain unchanged, the following formula may be used:

$$r_2 = \frac{r_1 l_2}{l_1}. \qquad (1.)$$

In this formula

r_1 = the original resistance;
r_2 = the required or changed resistance;
l_1 = the original length;
l_2 = the changed length.

As in all examples of proportion, the two lengths must be reduced to the same unit.

By this formula, we see that *the resistance of a conductor after its length is changed is equal to the original resistance multiplied by the changed length, and the product divided by the original length.*

EXAMPLE.—Find the resistance of 1 mile of copper wire, if the resistance of 10 feet of the same wire be .013 ohm.

SOLUTION.— $r_1 = .013$ ohm; $l_1 = 10$ feet; $l_2 = 1$ mile $= 5{,}280$ feet. Then, by formula **1,** the required resistance

$$r_2 = \frac{.013 \times 5{,}280}{10} = 6.864 \text{ ohms. Ans.}$$

41. If the sectional area of a conductor is doubled and other conditions remain unchanged, the resistance will be halved. We may, then, obtain the value of the resistance of a conductor for any change in sectional area by the following formula:

$$r_2 = \frac{r_1 a_1}{a_2}, \qquad (2.)$$

in which r_1 = the original resistance of the conductor;
r_2 = the changed resistance;
a_1 = the original sectional area;
a_2 = the changed sectional area.

From the relations here expressed, it will be seen that the resistance varies inversely as the sectional area; that is, *the resistance of a given conductor diminishes as its sectional area increases.*

The resistance of a conductor is independent of the *shape* of its cross-section. For example, this shape may be circular, square, rectangular, or irregular; if the sectional area be the same in all cases, the resistances will be the same, other conditions being similar.

EXAMPLE.—The resistance of a conductor whose sectional area is .025 sq. in. is .32 ohm; what would be the resistance of the conductor if its sectional area were increased to .125 sq. in. and other conditions remain unchanged?

SOLUTION.— $r_1 = .32$ ohm; $a_1 = .025$ sq. in.; and $a_2 = .125$ sq. in. Then, by formula **2**, the required resistance

$$r_2 = \frac{r_1 a_1}{a_2} = \frac{.32 \times .025}{.125} = .064 \text{ ohm. Ans.}$$

EXAMPLE.—The sectional area of a certain conductor is .01 sq. in. and its resistance is 1 ohm; if its sectional area be decreased to .001 sq. in. and other conditions remain unchanged, what will be the resistance?

SOLUTION.— $r_1 = 1$ ohm; $a_1 = .01$ sq. in., and $a_2 = .001$ sq. in. By formula **2**, the resistance

$$r_2 = \frac{1 \times .01}{.001} = 10 \text{ ohms. Ans.}$$

42. When comparing resistances of round copper wires the following formula is used:

$$r_2 = \frac{r_1 D^2}{d^2}, \qquad (3.)$$

in which r_1 = the original or known resistance;
r_2 = the required resistance;
D = the original diameter;
d = the changed diameter.

This formula is based on the rule that, since the sectional area of a round conductor is proportional to the square of its diameter (sectional area = diameter^2 × .7854), *the resistance of a round conductor is inversely proportional to the square of its diameter.*

Example.—The resistance of a round copper wire .2 in. in diameter is 45 ohms; from this calculate the resistance of a round copper wire .3 in. in diameter, other conditions remaining the same in both cases.

Solution.—In this example, $r_1 = 45$ ohms; $D = .2$ inch; and $d = .3$ inch. Hence, by formula **3**, the required resistance

$$r_2 = \frac{45 \times .2^2}{.3^2} = \frac{45 \times .04}{.09} = 20 \text{ ohms. Ans.}$$

Example.—If the resistance of a round German-silver wire $\frac{1}{8}$ in. in diameter is 12.6 ohms, what is the resistance of a round German-silver wire $\frac{1}{16}$ in. in diameter, other conditions being equal in the two cases?

Solution.—In this example, $r_1 = 12.6$ ohms; $D = \frac{1}{8} = .125$ inch; and $d = \frac{1}{16} = .0625$ inch. Hence, by formula **3**,

$$r_2 = \frac{12.6 \times .125^2}{.0625^2} = 50.4 \text{ ohms. Ans.}$$

43. The resistance of two or more conductors connected in *series* (Art. **14**) is equal to the sum of their separate resistances. For example, if four conductors having separate resistances of 8, 12, 22, and 34 ohms, respectively, are connected in series, their total or joint resistance would be 8 + 12 + 22 + 34 = 76 ohms.

44. The **microhm** is a unit of resistance devised to facilitate calculations and measurements of exceedingly small resistances, and is equal to *one millionth* $\left(\frac{1}{1,000,000}\right)$ of an *ohm.* Hence, to express the resistance in *microhms*, multiply the resistance in *ohms* by 1,000,000; and, conversely, to express the resistance in *ohms*, divide the resistance in *microhms* by 1,000,000. For example, .75 ohm = .75 × 1,000,000 = 750,000 microhms; or, 750,000 microhms = 750,000 ÷ 1,000,000 = .75 ohm.

45. The **megohm** is a unit of resistance devised to facilitate calculations and measurements of exceedingly large resistances, and is equal to 1,000,000 ohms. Therefore, to express the resistance in *megohms*, divide the resistance in *ohms* by 1,000,000; and, conversely, to express the resistance in *ohms*, multiply the resistance in *megohms* by 1,000,000. For example, 850,000 ohms $= \frac{850,000}{1,000,000} = .85$ megohm; or, .85 megohm $= .85 \times 1,000,000 = 850,000$ ohms. The megohm is used chiefly to measure the resistance of bad conductors and insulators.

46. In order to compare the resistances of different substances, the dimensions of the pieces to be measured must be equal; for, by changing its dimensions, a good conductor may be made to offer the same resistance as an inferior one. Under like conditions, annealed silver offers the least resistance of all known substances. Soft, annealed copper comes next on the list, and then follow all other metals and conductors.

The resistance of a given conductor, however, is not always constant; it changes with the temperature of the conductor. In all metals, the resistance *increases* as the temperature rises; in liquids and carbons, the resistance *decreases* as the temperature rises. The amount of variations in the resistance caused by a change in temperature of one degree is called the **temperature coefficient.** The temperature coefficients for the common metals are given in Table I for degrees Fahrenheit. These coefficients, however, only hold true for a limited change of temperature, and should not be used with extreme changes. The rules given below, making use of these coefficients, are not absolutely accurate, but enough so for practical purposes. To be strictly correct, the quantity r_1 in formula **4** below should, in each case, represent the resistance at the freezing point.

To find the resistance of a conductor after its temperature has risen, knowing its original resistance and the number of degrees rise, other conditions remaining unchanged:

Let r_1 = the original resistance;
r_2 = the resistance after a change in temperature;
k = the temperature coefficient;
t = rise or fall in temperature, degrees Fahrenheit.

Then, for a *rise* in temperature,

$$r_2 = r_1 (1 + t k). \qquad \textbf{(4.)}$$

That is, *the resistance of a conductor after its temperature has risen may be obtained by multiplying the original resistance by 1 plus the product of the number of degrees rise and the temperature coefficient.*

EXAMPLE.—The resistance of a piece of copper wire at 32° F. is 40 ohms; determine its resistance when its temperature is 52° F.

SOLUTION.— $R = 40$ ohms;
$k = .002155$ (from Table I);
$t = 52 - 32 = 20$ degrees.

By formula **4**, the required resistance

$r_2 = r_1 (1 + t k) = 40 (1 + 20 \times .002155) = 40 \times 1.0431 = 41.724$ ohms. Ans.

47. To find the resistance of a conductor after its temperature has fallen, knowing its original resistance and the number of degrees fall, other conditions remaining unchanged:

For a *fall* in temperature, $r_2 = \frac{r_1}{1 + t k}$. **(5.)**

That is, *the resistance of a conductor after its temperature has fallen may be obtained by dividing the original resistance by one plus the product of the number of degrees fall and the temperature coefficient.*

EXAMPLE.—The original resistance of a piece of German-silver wire is 16 ohms; find its resistance after its temperature has fallen 22° F.

SOLUTION.— $R = 16$ ohms;
$k = .000244$ (from Table I);
$t = 22°$ F.

By formula **5**, the required resistance

$$r_2 = \frac{r_1}{1 + t k} = \frac{16}{1 + 22 \times .000244} = \frac{16}{1.005368} = 15.9145 \text{ ohms. Ans.}$$

48. Specific resistance is the term given to the resistance of substances of unit length and unit sectional area at some standard temperature. In what follows, the specific resistance of a substance is the resistance of a piece of that substance 1 inch in length and 1 square inch in sectional area at 32° F., that is, at the temperature of melting ice; this may also be expressed as the resistance of a cube of that substance taken between two opposing faces.

A list of the common metals is given in Table I, in the order of their relative resistances, beginning with silver, which offers the least resistance. The first column of figures gives the *specific resistance*, in microhms, of 1 cubic inch of the corresponding metal at 32° F. By applying formula **1**, the resistance of any conductor of known dimensions which is made of one of the metals in the table can be determined. The second column of figures gives the relative resistance of the different metals compared with silver. For example,

TABLE I.

Name of Metal.	Resistance, Microhms per Cu. In.	Relative Resistance.	Temperature Coefficient.
Silver, annealed.......	.5921	1.000	.002094
Copper, annealed......	.6292	1.063	.002155
Silver, hard-drawn....	.6433	1.086	.002094
Copper, hard-drawn...	.6433	1.086	.002155
Gold, annealed........	.8102	1.369	.002028
Gold, hard-drawn......	.8247	1.393	.002028
Aluminum, annealed..	1.1470	1.935	
Zinc, pressed..........	2.2150	3.741	.002028
Platinum, annealed....	3.5650	6.022	
Iron, annealed........	3.8250	6.460	
Nickel, annealed......	4.9070	8.285	
Tin, pressed..........	5.2020	8.784	.002028
Lead, pressed.........	7.7280	13.050	.002150
German Silver..	8.2400	13.920	.000244
Antimony, pressed....	13.9800	23.600	.002161
Mercury..............	37.1500	62.730	.000400
Bismuth, pressed......	51.6500	87.230	.001967

the resistance of mercury is 62.73 times the resistance of silver, or the resistance of iron is 6.46 times the resistance of silver, and so on.

Example.—Find the resistance in ohms of a round column of mercury 70' high and .05' in diameter. Ans. 1.3244 ohms.

Example.—Find the resistance in ohms of 1 mile of square iron wire (annealed) .1' on a side. Ans. 24.2352 ohms.

49. In a simple voltaic cell, the *internal* resistance—that is, the resistance of the two plates and the electrolyte—is of great importance, for it determines the maximum strength of current that can possibly be obtained from the cell. In the common forms of cells, the internal resistance may be excessively large, owing to the resistance of the electrolyte, the specific resistance of ordinary liquids used as electrolytes being from 1 to 20 million times that of the common metals. In liquids, as in all conductors, the resistance increases as the length of the circuit increases, and diminishes as its sectional area increases. Hence, the internal resistance of a simple voltaic cell is reduced by decreasing the distance between the plates or elements and by increasing their active surfaces. The internal resistance of the ordinary forms of cells varies from about .2 to 20 ohms.

50. For practical and commercial testing, the standard column of mercury, representing the resistance of 1 ohm, has been replaced by a coil of wire, usually a platinum-silver alloy. The coil is carefully calibrated to offer a resistance of exactly 1 ohm at some convenient temperature, and is enclosed in a metallic case, the connections to the two ends of the coils being made by two heavy terminals of copper wire passing up through the hard-rubber cover. Such coils are known as *standard*

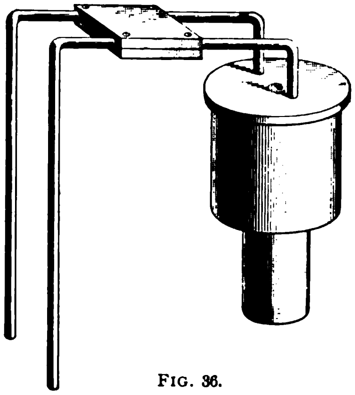


FIG. 36.

ohm coils. The commercial form of standard ohm coils is shown in Fig. 36.

51. A device called a **resistance box** or **rheostat** is largely used for reducing or controlling the strength of currents in various circuits. Such rheostats are connected directly in *series* or *shunt* with the circuit, and are termed *dead resistances.* The resistance in these rheostats is usually made adjustable; that is, the amount of resistance which they offer may be varied at the will of the operator by the use of a sliding contact or by removable plugs. Rheostats in which the amount of resistance is varied by sliding contacts are used mostly where accuracy is of less importance and where the currents are comparatively large.

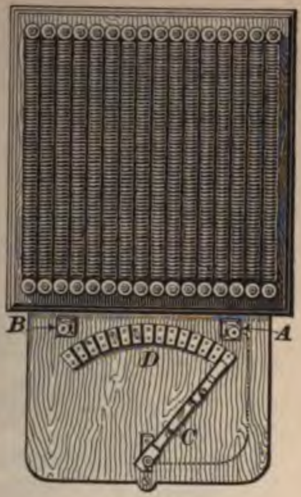


FIG. 37.

Fig. 37 shows a typical form of sliding-contact rheostat. In this particular rheostat, the coils of resistance wire are connected to a row of contact pieces D, as shown in the diagram, Fig. 38. The current enters the rheostat through the terminal A, passes through the movable arm C, and then through all the resistance coils between the contact piece on which the arm rests and the terminal B. When the arm rests upon the first contact piece, as shown by the full lines in this diagram, all the resistance is said to be *in circuit;* that is, the current passes through all the

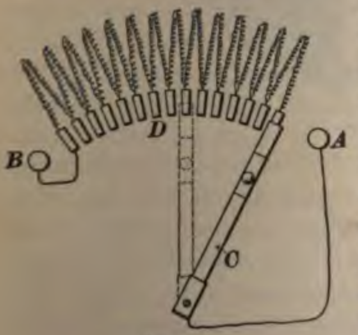


FIG. 38.

coils. By moving the arm to the left, towards the terminal *B*, as shown by the dotted lines, the coils connected to the contact pieces which have been passed over by the arm are said to be *cut out* of circuit, and the current passes through the remaining coils only.

52. Rheostats in which the resistance is adjusted by means of removable plugs are employed in laboratory practice, where small currents are used and where great accuracy is required. The resistance coils in these rheostats are enclosed in a wooden box, and the actual resistance of each coil is carefully determined. A resistance box offering 10,000 ohms resistance is shown in Fig. 39, the separate coils offering resistances from 1 ohm up to 5,000 ohms. The operation of adjusting the resistance by means of the removable plugs can be seen from the diagram in Fig. 40. The contact pieces *a*, *b*, *c*, etc. are arranged side by side on the top of the case and are separated from one another by a small air space. The ends of each contact piece are provided with a tapered recess in such a manner as to allow a metallic plug to be inserted between them and thereby connect the two together electrically. The current passes into the rheostat by the terminal *A*, and when all the plugs are

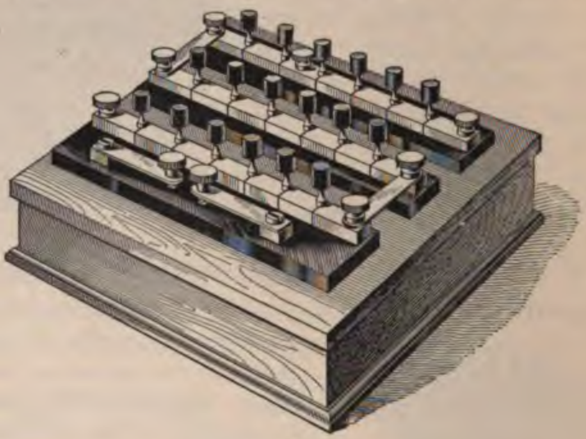


FIG. 39.

removed flows consecutively through all the coils *1*, *2*, *3*, *4*, *5*, and *6* to the terminal *B*. The total resistance of the rheostat can be lowered by inserting the plug *P* between the contact pieces; this operation *short-circuits*, or *cuts out*, the particular coil connected to the two contact pieces, or, in other words, the current, instead of flowing through the coils, passes directly from one contact piece to the other through the metallic plug.

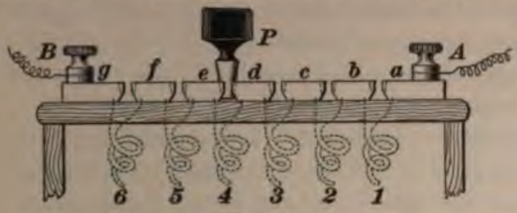


FIG. 40.

53. Electrical resistance may be measured by an apparatus called a **Wheatstone bridge.** A bridge, when completed, ready for taking measurements, consists of three main parts: (1) an adjustable resistance box containing a number of coils, the exact resistance of each coil being known; (2) a galvanometer for detecting small currents; and (3) a battery of several cells. The coils of the resistance box are divided into three groups, two of which are called **proportional** or **balance arms,** and the third is known as the **adjustable arm.** Each proportional arm is composed of three and sometimes four coils of 1, 10, 100, and 1,000 ohms resistance, respectively. The adjustable arm contains a large number of coils ranging from .1 ohm up to 10,000 ohms.

The operation of the bridge depends upon the principle of the relative difference of potential between two points in a divided circuit of two branches. The electrical connections of the bridge are shown in the diagram, Fig. 41. *M* represents the resistance of one of the balance arms, which will be termed for convenience the *upper* balance arm; *N* represents the resistance of the other balance arm, which will be termed the *lower* balance arm; *P* represents the resistance of the adjustable arm, and X represents an unknown resistance, the value of which is to be determined. One terminal of the detecting galvanometer G is connected at c, the junction of the upper balance arm and the unknown resistance; the other terminal is connected at d, the junction of the lower balance arm and the adjustable arm. One pole of the battery is connected at a, the junction of the two balance arms; the other pole at b, the junction of the adjustable resistance and the unknown resistance. The current from the battery divides at a, part of it flowing through resistances M and X, and the rest through N and P. When the resistances M, N, P, and X fulfil the proportion $\frac{M}{N} = \frac{X}{P}$, then the two points c and d will have the same potential, and no current will flow

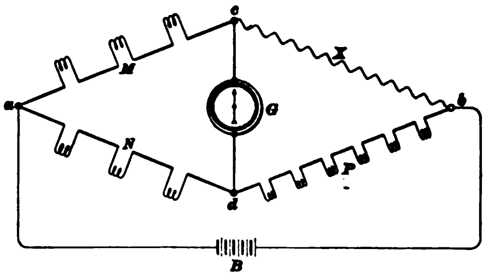


FIG. 41.

through the galvanometer G. Since the resistances of M, N, and P are known, the resistance of X will be given by the fundamental equation $X = \frac{M}{N} \times P$, when the arms are so adjusted as to cause no deflection of the galvanometer. For example, suppose that the two ends of a copper wire are connected to the terminals b and c, and after adjusting the resistance in the arm so that the galvanometer shows no deflection, the resistances of the different arms read as follows: $M = 1$ ohm, $N = 100$ ohms, and $P = 112$ ohms. Then, substituting these values in the fundamental equation gives

$$X = \frac{M}{N} \times P = \frac{1}{100} \times 112 = 1.12 \text{ ohms.}$$

54. The actual various forms of resistance boxes used with the bridges differ widely from the diagram, but all are based upon this same principle and fundamental equation. A common pattern of resistance box for this purpose is constructed similar to the adjustable rheostat, as previously described, where the adjustments are made with removable plugs. Ordinarily, the contact pieces are arranged in the shape of a letter S, and the galvanometer and battery circuits are connected as shown in Fig. 42. The position of the two balance arms and the adjustable arm can be readily seen by comparing the connections of the battery and galvanometer circuits with those in the original diagram. K and K' represent keys for opening the circuits when the plugs are withdrawn or inserted in varying the resistance or when the bridge is not in use. In this particular case, the 1,000-ohm plug in the upper balance arm is supposed to be drawn, and therefore $M = 1{,}000$ ohms. In the lower balance arm the 10-ohm plug is supposed to be drawn, and therefore $N = 10$ ohms. In the adjustable arm the following plugs are supposed to be drawn: 1, 2, 5, 10, 20, 100, 200, 500, 2,000, and 3,000 ohms; therefore, the resistance P is the sum of these resistances, or 5,838 ohms. If, under these conditions, there is no deflection of the galvanometer when the two keys K and K' are pressed and both circuits are closed, the resistance of X will be 583,800 ohms; for, substituting the values of M, N, and P in the fundamental equation gives $X = \frac{M}{N} \times P = \frac{1{,}000}{10} \times 5{,}838 = 583{,}800$ ohms.

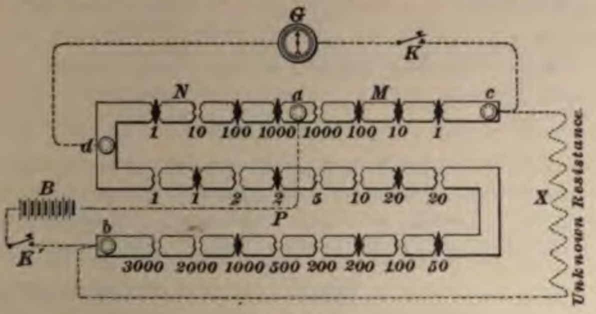


FIG. 42.

Fig. 43 shows a special pattern of resistance box for a Wheatstone bridge, in which the coils of the adjustable arm are arranged in the form of four dials. This pattern is known as the *dial* pattern, and is widely used in making resistance measurements.

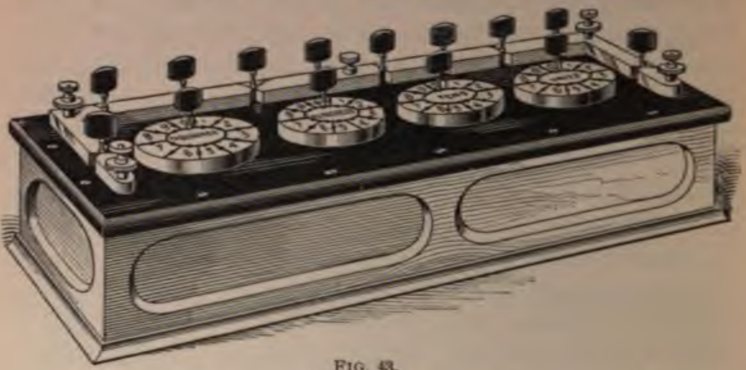


FIG. 43.

EXAMPLE.—The diagram in Fig. 44 represents a particular type of Wheatstone's bridge to which a battery and galvanometer are properly connected for measuring unknown resistances. An unknown resistance x is connected to the terminals A and H; when the plugs a, e, f, g, i, k, m, q, and t are drawn, and when both the contact keys K and K' are pressed, the galvanometer shows no deflection. Determine the resistance of x.

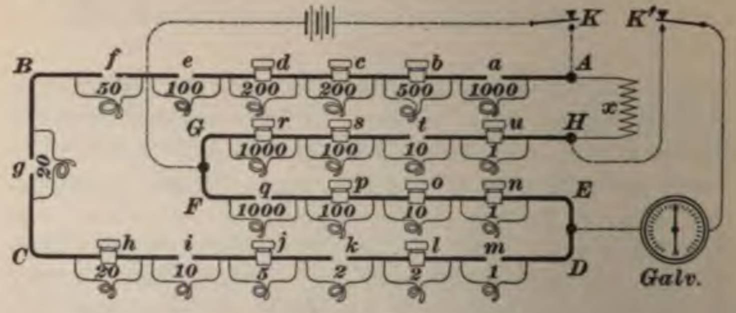


FIG. 44.

SOLUTION.—From the connections of the galvanometer and battery circuits, it will be seen that the resistance coils in line $G H$ represent the upper balance arm M of the bridge; that the coils in the line $E F$

represent the lower balance arm N; and that the coils in the lines $A\,B$ and $C\,D$ represent the adjustable arm P. From the fundamental equation of the Wheatstone bridge, X (the unknown resistance) $= \frac{M}{N} \times P$. In this particular case, the plug t in the upper arm is drawn; hence, $M = 10$ ohms; in the lower arm q is drawn; hence, $N = 1{,}000$ ohms; and in the adjustable arm, the plugs a, e, f, g, i, k, and m are drawn; hence, $P = 1{,}000 + 100 + 50 + 20 + 10 + 2 + 1 = 1{,}183$ ohms. Substituting these values in the fundamental equation gives $X = \frac{M}{N} \times P = \frac{10}{1{,}000} \times 1{,}183 = 11.83$ ohms. Ans.

55. The Volt, or the Practical Unit of Electromotive Force.—In mechanics, pressures of all kinds are measured by the *effects* they produce; similarly, in electrotechnics, *potential* is measured by the effect it produces.

It has been shown that electrical potential will cause an electric current to flow against the resistance of a conductor, and also how the units of resistance and current are obtained. It follows that a *unit potential* would be that electromotive force which would maintain a current of unit strength in a circuit whose resistance is unity. By definition, therefore, the *volt*, or *the practical unit of potential*, is that electromotive force which will maintain a current of *one ampere* in a circuit whose resistance is *one ohm*. With a known resistance in ohms and a known strength of current in amperes, the electromotive force in volts is determined by Ohm's law, Art. **34**, for, by transposing, $E = C\,R$.

This method of determining the potential of a circuit can be readily shown by the following illustration: Suppose, for example, it is desired to determine the electromotive force in volts required to drive a current of 2 amperes through a certain copper wire. In the first place, the resistance of the copper wire is found by Wheatstone's bridge as previously described. For convenience, it is assumed that its resistance is found to be 1.2 ohms. Then the electromotive force E required to drive 2 amperes through the wire will be 2.4 volts, for, by substituting, $E = C\,R = 2 \times 1.2 = 2.4$ volts.

The maximum difference of potential developed by any single voltaic couple placed in any electrolyte is about 2.25 volts; in the common forms of cells, the difference of potential developed averages from .75 to 1.75 volts.

56. When several cells are connected in *series*, the total electromotive force developed will be equal to the sum of the electromotive forces developed by the separate cells; or, if the cells are composed of the same voltaic elements, the total electromotive force developed will be equal to the electromotive force of one cell multiplied by the number of cells in series. For example, a battery is composed of 12 cells connected in series, and the electromotive force in each cell is 1.5 volts; the total electromotive force of the battery is, therefore, 1.5 × 12 = 18 volts.

Connecting cells in *parallel*, or *multiple-arc*, does not increase the electromotive force of a battery; the electromotive force will always be equal to the electromotive force of one cell, no matter how many cells are connected to the main conductors, provided, of course, that all cells develop equal electromotive forces.

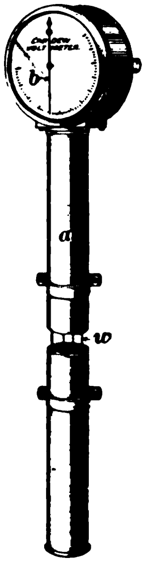


FIG. 45.

57. Measuring instruments called **voltmeters** have been devised for indicating electromotive forces and differences of potential directly in volts.

The **Cardew** voltmeter, Fig. 45, depends for its operation upon the linear expansion of a metallic wire when heated by an electric current. The expansion wire w is enclosed in a long cylindrical case a, and is attached in such a way that its expansion causes a small grooved wheel on the axis of the index needle to revolve in one direction when the wire expands, or lengthens, and in the opposite direction when the wire contracts, or shortens. The movements of this wheel cause the index b to move over the scale. Since the resistance is nearly constant, the current that will flow is proportional to the E. M. F.; the greater the E. M. F. the more the wire will be expanded, and the greater will be the consequent deflection. The resistance of the wire, however, is so large as to permit only a weak current to pass through it when the needle is deflected over the entire scale. A Cardew voltmeter which indicates up to 100 volts has a resistance of about 500 ohms. The circular scale is divided into small divisions, each representing one volt or fractions or multiples of one volt.

58. The **Weston voltmeter,** Fig. 46, is based upon the same principles as the Weston ammeter, and in appearance is quite similar to it. Its internal resistance, as in all voltmeters, is exceedingly large; the resistance of a Weston voltmeter for indicating up to 150 volts is about 19,000 ohms, while the resistance of a Weston ammeter, measuring strengths of currents up to 15 amperes, is only .0022 ohm. It will be seen that, owing to the great resistance, the current passing through a voltmeter is exceedingly small. For example, in the instrument described above, when indicating 150 volts, the current, by Ohm's law, is only 150 ÷ 19,000 = .0079 ampere. All voltmeters are provided with at least two terminals, or binding posts, such as *p* and *p*′, Fig. 46. Connections are made by two separate conductors, called *voltmeter leads*, from these binding posts to two points between which the difference of potential, or the electromotive force, is to be measured.

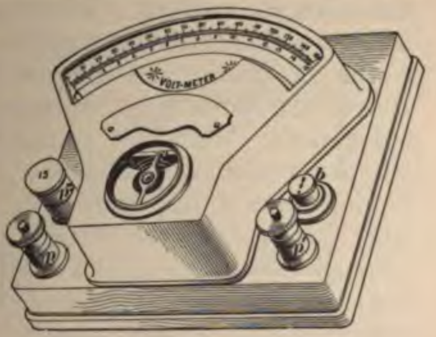


FIG. 46.

The Weston voltmeters usually have a third binding post *p*″, which when used with *p*′ corresponds with a second graduated scale situated directly under the main scale, one division of the upper scale having the value of two lower divisions. The majority of voltmeters are also provided with a contact button *b*, which when pressed closes the circuit and allows the index needle to be deflected by the current. When the pressure upon the button is relaxed, the circuit is opened, and the index needle returns to the zero mark.

59. The methods of connecting voltmeters and ammeters for measuring electromotive forces and currents of various circuits should be thoroughly understood. Suppose, for example, that the terminals of a battery composed of four cells connected in series are connected to an unknown resistance, and it is desired to know the strength of current flowing through the circuit, and also the difference of potential required to drive that current through the unknown resistance when the only instruments available are an ammeter and a voltmeter. In Fig. 47 let B represent the battery and R the unknown resistance; C, C', and C'' are three large conductors for making necessary connections. With the connections as shown, there is practically a continuous current flowing through the closed circuit, that is, from the battery through the conductors and the unknown resistance. The first step is to determine the strength of this current by the use of an ammeter. Assuming that the battery is constant, that is, that the electromotive force developed in it does not vary, then, so long as the resistance of the circuit is not altered, the strength of the current will remain unchanged and *will be the same in all parts of the circuit*. Hence, if an ammeter be inserted in any part of the

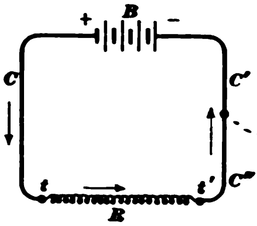


FIG. 47.

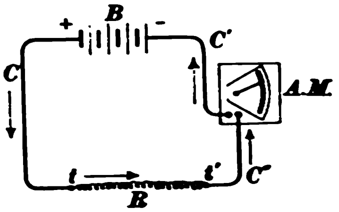


FIG. 48.

circuit, as between C' and C'', Fig. 48, it will measure the total strength of current flowing through the entire circuit. As has been stated, the internal resistance of the ammeter is so small that its insertion makes no appreciable change in the total resistance of the circuit, and therefore does not to any extent affect the current flowing. For convenience, assume that the strength of the current flowing in the circuit is found to be 1.2 amperes. The next operation is to find the electromotive force required to drive a current of 1.2 amperes through the resistance R; or, in other words, to find the difference of potential between the terminals t and t' when a current of 1.2 amperes is flowing in the circuit. This is accomplished by connecting the two terminals t and t', Fig. 49, of the unknown resistance R, to the two binding posts p and p' of the voltmeter $V M$ by two voltmeter leads l and l'. Any small wires of reasonable length can be used for voltmeter leads, as the current they transmit is exceedingly weak, owing to the extremely high resistance of the voltmeter. After pressing the contact button, assume the needle indicates a potential of 6 volts; this, then, is the electromotive force required to force a current of 1.2 amperes through the unknown resistance R; or, in other words, the difference of potential between the terminals t and t' is 6 volts. From these readings of the current and voltage, and by the application of Ohm's law, the resistance R of the circuit between t and t' can be determined. By algebra, Ohm's law can be transposed from the equation $C = \frac{E}{R}$ to $R = \frac{E}{C}$ and be equally true; this signifies that the resistance R of any conductor, or circuit, is equal to the electromotive force, or the difference of potential E in volts, divided by the strength of current C in amperes flowing through that

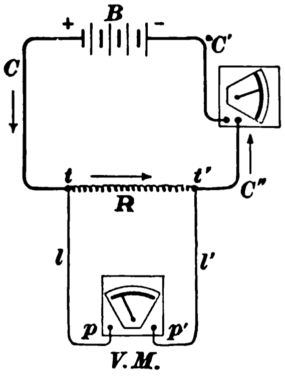


FIG. 49.

circuit or conductor. In the previous case, it has been found that it requires an electromotive force of 6 volts to drive a current of 1.2 amperes through the resistance R; hence, from Ohm's law, $R = \frac{E}{C} = \frac{6}{1.2} = 5$ ohms.

APPLICATIONS OF OHM'S LAW.

TO CLOSED CIRCUITS.

60. The following facts are to be carefully noted regarding the application of Ohm's law to closed circuits:

The strength of current (C) *is the same in all parts of a closed circuit, except in the cases of derived circuits, where the sum of the currents in the separate branches is always equal to the current in the main or undivided circuit.*

The resistance (R) *is the resistance of the internal circuit plus the resistance of the external circuit.*

The electromotive force (E) *in a closed circuit is the total generated difference of potential in that circuit.*

61. The following formula may be used to determine the strength of current in amperes flowing in a closed circuit when the electromotive force and the total resistance are known:

$$C = \frac{E}{R}, \qquad (6.)$$

where C = current in amperes;
E = electromotive force in volts;
R = resistance in ohms.

That is to say, *the strength of current in amperes is found by dividing the electromotive force in volts by the total resistance in ohms.*

EXAMPLE.—The two electrodes of a simple voltaic cell are connected by a conductor whose resistance is 1.6 ohms. If the internal resistance of the cell is 5 ohms and the total electromotive force developed is 1.75 volts, what is the strength of current flowing in the circuit?

Solution.—Let r_i = the internal resistance and r_e = the resistance of the copper wire. Then, $R = r_i + r_e = 1.6 + 5 = 6.6$ ohms, the total resistance of the circuit. Then, by formula **6**, the current

$$C = \frac{E}{R} = \frac{1.75}{6.6} = .265 \text{ ampere. Ans.}$$

62. The following formula may be used to find the total resistance in ohms of a closed circuit when the electromotive force and the strength of current are known:

$$R = \frac{E}{C}, \qquad (7.)$$

the letters having the same significance as in formula **6**. By formula **7** it will be seen that *the resistance in ohms of a closed circuit is found by dividing the electromotive force in volts by the current in amperes.*

Example.—The total electromotive force developed in a closed circuit is 1.8 volts and the strength of the current flowing is .6 ampere; find the resistance in ohms.

Solution.—By formula **7**, the resistance

$$R = \frac{E}{C} = \frac{1.8}{.6} = 3 \text{ ohms. Ans.}$$

63. The following formula may be used to find the total electromotive force in volts developed in a closed circuit when the strength of current and the total resistance are known:

$$E = C R. \qquad (8.)$$

The letters have the same meaning as in formulas **6** and **7**. We find here that *the electromotive force in volts developed in a closed circuit is obtained by multiplying together the current in amperes and the resistance in ohms.*

Example.—The internal resistance of a closed circuit is 2 ohms and the external resistance is 3 ohms; if the current flowing is .4 ampere, what is the electromotive force developed?

Solution.—Let r_i = the internal resistance and r_e = the external resistance. Then, $R = r_i + r_e = 2 + 3 = 5$ ohms. By formula **8**, the electromotive force

$$E = C R = .4 \times 5 = 2.0 \text{ volts. Ans.}$$

TO DROP, OR LOSS, OF POTENTIAL.

64. Referring again to water flowing in a pipe, it is evident that although the *quantity* of water which passes is the same at any cross-section of the pipe, the *pressure per square inch* is not the same. Even in the case of a horizontal pipe of the same diameter throughout the water when flowing suffers a *loss* of head, or pressure. It is this difference of pressure that causes the water to flow between two points against the friction of the pipe.

This is precisely similar to a current of electricity flowing through a conductor. Though the *quantity of electricity* that flows is equal at all cross-sections, the electromotive force is by no means the same at all points along the conductor. It suffers a loss, or drop, of electrical potential in the direction in which the current is flowing, and it is this difference of electrical potential that causes the electricity to flow against the resistance of the conductor. *Ohm's law* not only gives the strength of the current in a closed circuit, but also the *difference of potential* in volts along that circuit. The difference of potential (E') in volts between any two points along a circuit is equal to the product of the strength of the current (C') in amperes and the resistance (R') in ohms of that part of the circuit between those two points, or $E' = C'R'$, which is an example of the use of formula **8.** E' also represents the *loss*, or *drop*, of potential in volts between the two points. If any two of these quantities are known, the third can be readily found; for, by transposing, $C' = \frac{E'}{R'}$, and $R' = \frac{E'}{C'}$, as already given in formulas **6** and **7.**

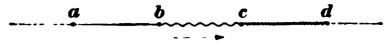


FIG. 50.

EXAMPLE.—Fig. 50 represents part of a circuit in which a current of 3 amperes is flowing. The resistance from *a* to *b* is 1.5 ohms, from *b* to *c* is 2.3 ohms, and from *c* to *d* is 3.6 ohms. Find the difference of potential between *a* and *b*, *b* and *c*, *c* and *d*, and *a* and *d*.

SOLUTION.—Since, by formula 8, $E' = C'R'$, then,

the difference of potential between a and $b = 3 \times 1.5 = 4.5$ volts.
" " " " " b and $c = 3 \times 2.3 = 6.9$ "
" " " " " c and $d = 3 \times 3.6 = 10.8$ "
" " " " " a and $d = 4.5 + 6.9 + 10.8 =$
22.2 volts; or, in other words, the *loss*, or *drop*, of potential caused by a current of 3 amperes flowing between a and d is 22.2 volts.

65. In a great many cases it is desirable to have the current flow from the source a long distance to some electrical receptive device and return without causing an excessive drop, or loss, of potential in the conductors leading to and from the two places. In such circuits, the greater part of the total generated electromotive force is expended in the receptive device itself, and only a small fraction of it is lost in the rest of the circuit. Under these conditions, it is customary to decide upon a certain *drop, or loss, of potential* beforehand, and from that and the current calculate the resistance of the two conductors.

EXAMPLE.—It is desired to transmit a current of 5 amperes to an electrical device situated 500 feet from the source; the total generated E. M. F. is 120 volts, and only $\frac{1}{10}$ of this potential is to be lost in the conductors leading to and from the receptive device. (*a*) Find the resistance of the two conductors, and (*b*) find the resistance per foot of the conductors, assuming each to be 500 feet long.

SOLUTION.—(*a*) $\frac{1}{10}$ of 120 volts $= 12$ volts, which represents the *drop*, or *loss*, of potential on the two conductors. Let $E' = 12$ volts; $C = 5$ amperes; and $R' =$ the total resistance of the two conductors. Then, by formula **7**,

$$R' = \frac{E'}{C} = \frac{12}{5} = 2.4 \text{ ohms. Ans.}$$

(*b*) The resistance per foot of the conductor is found by formula **1**. In this case, $r_1 = 2.4$ ohms; $l_1 = 1,000$ feet; $l_2 = 1$ foot. Then the resistance per foot

$$r_2 = \frac{2.4 \times 1}{1,000} = .0024 \text{ ohm. Ans.}$$

TO VOLTAIC CELLS.

66. The difference of potential between the two electrodes of a simple voltaic cell when no current is flowing—that is, when the circuit is open—is always equal to the total electromotive force developed within the cell; but when a current is flowing—that is, when the circuit is *closed*—a certain amount of potential is expended in forcing the current through the internal resistance of the cell itself. Hence, the difference of potential between the two electrodes when the circuit is closed is always smaller than when the circuit is open. This difference of potential between the two electrodes when the circuit is closed is sometimes called the *available* or *external* eloctromotive force, to distinguish it from the *internal* or *total generated* electromotive force.

67. To find the available electromotive force of a cell,

let E = the total generated E. M. F.;
E' = *available* E. M. F. when the circuit is closed;
C = the current flowing when the circuit is closed;
r_i = the internal resistance of the cell.

Then, the drop, or loss, of potential in the cell = $C r_i$, and the available electromotive force

$$E' = E - C r_i. \qquad \textbf{(9.)}$$

The available electromotive force of a cell is equal to the difference between the total generated electromotive force and the potential expended in forcing the current through the internal resistance of the cell when the circuit is closed. From Ohm's law, this loss, or drop, of potential in the cell itself is equal to the product of the internal resistance in ohms and the strength of the current in amperes flowing through the circuit.

EXAMPLE.—In a voltaic cell, the total generated E. M. F. is 2.2 volts and the internal resistance is .8 ohm. If a current of 1.2 amperes flows through the cell when the circuit is closed, what is the available E. M. F., or, in other words, the difference of potential between the two electrodes?

SOLUTION.—Let E' = the available E. M. F.; E = the total generated electromotive force; C = the current in amperes; and r_i = the internal resistance.

Then, by formula **9**,

$$E' = E - C r_i = 2.2 - (1.2 \times .8) = 1.24 \text{ volts. } \textbf{Ans.}$$

TO DERIVED CIRCUITS.

68. In treating upon derived circuits, only that part of the circuit will be considered which is divided into branches and each branch transmitting part of the total current; the rest of the circuit is assumed to be closed through some electric source, as, for instance, a voltaic battery.

Before applying Ohm's law to derived circuits, the word *conductivity* should be thoroughly understood. Conductivity can be defined as the facility with which a body transmits electricity, and is the opposite of resistance. For example, copper is of low resistance and high conductivity; mercury is of high resistance and low conductivity. In other words, conductivity is the inverse or reciprocal of resistance. There is no established unit of conductivity; it is used merely as a convenience in calculations. For example, if the resistance of a circuit is 2 ohms, its conductivity is represented by one-half; if the resistance is increased to 4 ohms, the conductivity would only be one-half as much as in the former case and would be represented by one-quarter.

The **conductivity** of any conductor is, therefore, unity divided by the *resistance* of that conductor; and, conversely, the resistance of any conductor is unity divided by its conductivity.

69. Fig. 51 represents a derived circuit of two branches.

Let r_1 and r_2 = the separate resistances of the two branches; c_1 and c_2 = the separate currents in each branch, respectively; and C = the sum of the currents in the two branches; that is, the current in the main or undivided branch. Then, $c_1 + c_2 = C$, and $C - c_2 = c_1$.

When the current flows from a to b, if the resistances r_1 and r_2 are equal, the current will divide equally between the two branches; thus, if a current of 2 amperes is flowing in the main circuit, 1 ampere will flow through each branch.

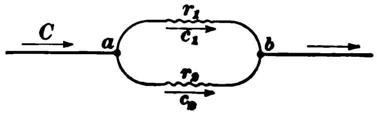


FIG. 51.

When the resistances of the two branches are unequal, the current will divide between them in inverse proportion to their respective resistances. In Fig. 51 the resistances of the two branches are r_1 and r_2. Therefore, $c_1 : c_2 :: r_2 : r_1$.

By algebra, this proportion gives the two following formulas:

For the first branch, $c_1 = \frac{C r_2}{r_1 + r_2}$. **(10.)**

That is, *of two branches in parallel, dividing from a main circuit, the current in the first branch is equal to the current in the main multiplied by the resistance of the second branch, and the product divided by the sum of the resistances of the two branches.*

For the second branch, $c_2 = \frac{C r_1}{r_1 + r_2}$. **(11.)**

Of two branches in parallel, dividing from a main circuit, the current in the second branch is equal to the current in the main multiplied by the resistance of the first branch, and the product divided by the sum of the resistances of the two branches.

EXAMPLE.—Suppose the resistance r_1 of the first branch is 2 ohms, and the resistance r_2 of the second branch is 3 ohms, find the separate currents c_1 and c_2 in the two branches, respectively, when the current C in the main or undivided branch is 60 amperes.

SOLUTION.— $r_1 = 2$ ohms, $r_2 = 3$ ohms, and $C = 60$ amperes. To find the current c_1 in the first branch, substitute these values in formula **10**, which will give

$$c_1 = \frac{C r_2}{r_1 + r_2} = \frac{60 \times 3}{2 + 3} = \frac{180}{5} = 36 \text{ amperes. Ans.}$$

To find the current c_2 in the second branch, substitute these values in formula **11**, which will give

$$c_2 = \frac{C r_1}{r_1 + r_2} = \frac{60 \times 2}{2 + 3} = \frac{120}{5} = 24 \text{ amperes. Ans,}$$

70. It is clear that two conductors in parallel will conduct an electric current more readily than one alone; that is, their *joint conductivity* is greater than either of their separate conductivities taken alone. This being the case, their resistances must follow the inverse law—viz., the joint resistance of two conductors in parallel must be *less* than either of their separate resistances taken alone.

Rule.—*If the separate resistances of two conductors are equal, their joint resistance when connected in parallel is one-half of the resistance of either conductor.*

For example, take two conductors, the separate resistance of each being 2 ohms, and connect them in parallel; their joint resistance will then be one-half their separate resistance, or 1 ohm.

71. When the separate resistances of two conductors in parallel are unequal, the determination of their joint resistance when connected in parallel involves some calculation.

In Fig. 51, the conductivities of the branches are $\frac{1}{r_1}$ and $\frac{1}{r_2}$. Hence, their joint conductivity when connected in parallel is $\frac{1}{r_1} + \frac{1}{r_2} = \frac{r_2 + r_1}{r_1 r_2}$; now, since the resistance of any conductor is the reciprocal of its conductivity, then the *joint resistance* of the two branches in parallel is the reciprocal of their joint conductivity; or, $1 \div \frac{r_2 + r_1}{r_1 r_2} = \frac{r_1 r_2}{r_2 + r_1}$. Hence, joint resistance

$$R' = \frac{r_1 r_2}{r_1 + r_2}. \qquad (12.)$$

That is, *the joint resistance of two conductors connected in parallel is equal to the product of their separate resistances divided by the sum of their separate resistances.*

Example.—In Fig. 51, given $r_1 = 2$ ohms and $r_2 = 3$ ohms; find their joint resistance in parallel.

Solution.—From formula **12**, their joint resistance

$$R' = \frac{r_1 r_2}{r_1 + r_2} = \frac{2 \times 3}{2 + 3} = \frac{6}{5} = 1\tfrac{1}{5} \text{ ohms. Ans.}$$

72. Fig. 52 represents a divided circuit of three branches. Let r_1, r_2, and r_3 be the separate resistances of those branches, respectively. Then, $\frac{1}{r_1}$, $\frac{1}{r_2}$, and $\frac{1}{r_3}$ represent the separate conductivities of the three branches, respectively. Their joint conductivity $= \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3} = \frac{r_2 r_3 + r_1 r_3 + r_1 r_2}{r_1 r_2 r_3}$. Since the joint resistance is the reciprocal of their joint conductivity, then it is equal to

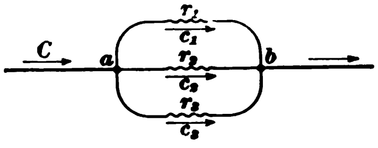


FIG. 52.

$$1 \div \frac{r_2 r_3 + r_1 r_3 + r_1 r_2}{r_1 r_2 r_3} = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}.$$

Hence, the joint resistance of three branches in parallel

$$R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}. \qquad \textbf{(13.)}$$

That is, *the joint resistance of three or more conductors connected in parallel is equal to the reciprocal of their joint conductivity.*

EXAMPLE.—In Fig. 52, given $r_1 = 5$ ohms; $r_2 = 10$ ohms; and $r_3 = 20$ ohms; find their joint resistance from *a* to *b*.

SOLUTION.—By formula **13**, their joint resistance

$$R''' = \frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} = \frac{5 \times 10 \times 20}{10 \times 20 + 5 \times 20 + 5 \times 10} = \frac{1{,}000}{350} = \frac{20}{7}$$

$= 2\frac{6}{7}$ ohms. Ans.

73. In a derived circuit of any number of branches, the difference of potential between where the branches divide and where they unite is equal to the product of the sum of the currents in the separate branches and their joint resistance in parallel, as will be apparent from consideration of Ohm's law, Art. **34.**

For example, if the currents in the three branches, Fig. 52, are 16, 8, and 4 amperes, respectively, and the joint resistance from *a* to *b* is $2\frac{6}{7}$ ohms, then the difference of potential between *a* and *b* $= (16 + 8 + 4) \times 2\frac{6}{7} = 28 \times \frac{20}{7} = 80$ volts.

74. The separate currents in the branches of a derived circuit can be determined by finding the difference of potential between where the branches divide and where they unite, and dividing the result by the separate resistance of each branch.

For example, in Fig. 52, assume that the separate resistances of the three branches are 5, 10, and 20 ohms, respectively, and that the difference of potential between *a* and *b* is 80 volts. Then, the current in the first branch is $\frac{80}{5} = 16$ amperes; in the second, $\frac{80}{10} = 8$ amperes; and in the third, $\frac{80}{20} = 4$ amperes.

75. The separate resistances of the branches of a derived circuit can be determined by finding the difference of potential between where the branches divide and where they unite, and dividing the result by the separate currents in each branch.

For example, in Fig. 52, assume the difference of potential between *a* and *b* to be 80 volts and the currents in the separate branches to be 16, 8, and 4 amperes, respectively; then, the resistance of the first branch is $\frac{80}{16} = 5$ ohms; of the second, $\frac{80}{8} = 10$ ohms; and of the third, $\frac{80}{4} = 20$ ohms.

EXAMPLE.—Fig. 53 represents a closed circuit, part of which, from *a* to *b*, forms a derived, or shunt, circuit of three separate branches *A*, *B*, and *C* in parallel; r_1, r_2, and r_3 represent the separate resistance of the branches, respectively, from *a* to *b*; and R' represents the resistance of the rest of the closed circuit from *b* to *a* in the direction in which the current is supposed to be flowing, including the internal resistance of the battery *K*. Let $r_1 = 2$ ohms; $r_2 = 3.2$ ohms; $r_3 = 4.4$ ohms; and $R' = .8$ ohm. If a current of 2 amperes is flowing in the main, or undivided, circuit, find the total electromotive force developed in the battery *K*.

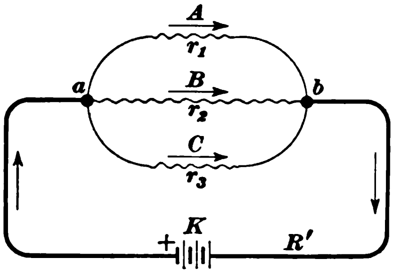


FIG. 53.

SOLUTION.—From the application of Ohm's law to closed circuits, formula **8**, $E = C R$, where *E* is the total electromotive force developed within the electric source; *C* the strength of current flowing; and

R the total resistance of the circuit through which the current passes. In this particular problem, the total resistance of the closed circuit will be the *joint* resistance of the three branches in parallel, plus the resistance R of the rest of the circuit. Hence, first find the joint resistance of the three branches A, B, and C in parallel from a to b. By formula **13**, the joint resistance of three conductors in parallel is $\frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2}$, where r_1, r_2, and r_3 represent the separate resistances of the three conductors. Substituting gives

$$\frac{2 \times 3.2 \times 4.4}{3.2 \times 4.4 + 2 \times 4.4 + 2 \times 3.2} = \frac{28.16}{14.08 + 8.8 + 6.4} = \frac{28.16}{29.28} = .9617 \text{ ohm},$$

the joint resistance of the three branches A, B, and C in parallel from a to b. The total resistance of the closed circuit is, therefore, $.9617 + .8 = 1.7617$ ohms, and $E = C \times R = 2 \times 1.7617 = 3.5234$ volts. Ans.

ELECTRICAL QUANTITY.

76. The rate of flow of liquids is expressed in units of quantity per second or minute, and similarly the strength of an electric current can be defined as a quantity of electricity flowing per second. The practical unit of *electrical quantity* is called the **coulomb.**

The coulomb is such a quantity of electricity as will pass in one second through a circuit in which the strength of current is one ampere.

As stated previously, the *quantity* of electricity is calculated from the strength of current; it cannot be actually measured. For example, suppose the strength of current in a closed circuit to be 10 amperes, as measured by an ammeter; if such a current flows for only one second, the quantity of electricity which has passed around the circuit is 10 coulombs; but if the current flows for two seconds, the quantity of electricity would be 20 coulombs.

Hence, to calculate the quantity of electricity which has passed in a circuit in a certain time when the strength of the current in amperes is known:

Let Q = the quantity of electricity in coulombs, C the strength of current in amperes, and t the time in seconds.

Then, $$Q = C t. \qquad \textbf{(14.)}$$

If any two of these quantities are known, the third can be readily found. By transposition, $C = \frac{Q}{t}$ and $t = \frac{Q}{C}$.

Therefore, to obtain the quantity of current which has passed through a circuit in a given time, *multiply the strength of current in amperes by the time in seconds.*

EXAMPLE.—Find the quantity of electricity in coulombs that flows around in a closed circuit in 1½ hours when the strength of current is 12 amperes.

SOLUTION.—Reducing the time to seconds gives $1.5 \times 60 \times 60 = 5{,}400$ seconds; hence, $t = 5{,}400$ seconds and $C = 12$ amperes. Then from formula **14**,

$$Q = C\,t = 12 \times 5{,}400 = 64{,}800 \text{ coulombs. Ans.}$$

ELECTRICAL WORK.

77. When an electric current flows from a higher to a lower potential, *electrical energy* is expended and *work* is done by the current. The principle of the *conservation of energy* teaches that energy can never be destroyed; it follows, therefore, that if energy has to be expended in forcing a quantity of electricity against a certain amount of resistance, the equivalent of that energy must be transformed into some other form. This other form is usually *heat;* that is, when a quantity of electricity flows against the resistance of a conductor, a certain amount of *electrical* energy is transformed into *heat* energy.

The actual amount of heat developed is an exact equivalent of the *work done* in overcoming the resistance of the conductor, and varies directly as that resistance. For example, take two wires, the resistance of one being twice that of the other, and send currents of equal strengths through each. The amount of heat developed in the wire of higher resistance will be twice that developed in the wire offering the lower resistance.

The unit used to express the amount of mechanical work done is known as the *foot-pound.* The work done in raising any mass through any height is found by multiplying the weight of the body lifted by the vertical height through which it is raised; similarly, the practical unit of *electrical work* is that amount accomplished when a unit quantity of electricity, *one coulomb*, flows between potentials differing by *one volt*.

The unit of electrical work is, therefore, the *volt-coulomb*, and is called the **joule.**

1 *joule* = .7373 foot-pound.

78. By means of the following formulas, we may find directly the amount of electrical work accomplished in *joules* during a given time in any circuit:

Let J = electrical work in joules;
C = current in amperes;
t = time in seconds during which the current flows;
E = potential, or E. M. F., of circuit;
R = resistance of circuit.

When the current and electromotive force are known,

$$J = C E t. \qquad \textbf{(15.)}$$

When the current and resistance are known,

$$J = C^2 R t. \qquad \textbf{(16.)}$$

When the resistance and electromotive force are known,

$$J = \frac{E^2 t}{R}. \qquad \textbf{(17.)}$$

To determine, therefore, the electrical work done in a given time, *multiply the quantity of electricity in coulombs which has passed in the circuit during that time by the loss, or drop, of potential as measured directly, or as computed from the values of the current and resistance.*

Example.—Find the amount of work done in joules when a current of 15 amperes flows for ½ an hour against a resistance of 2 ohms.

Solution.—Reducing the time to seconds gives $30 \times 60 = 1{,}800$ seconds $= t$. The current $= C = 15$ amperes, and the resistance $=$ 2 ohms $= R$. Then, by formula **16**, the electrical work done

$$J = 15 \times 15 \times 2 \times 1{,}800 = 810{,}000 \text{ joules. Ans.}$$

79. When the work in joules is known, the work in foot-pounds

$$\text{F. P.} = .7373\,J. \qquad \textbf{(18.)}$$

That is, *the equivalent work done in foot-pounds is obtained by multiplying the number of joules by .7373.*

Example.—Express the work done in foot-pounds in a circuit when a current of 8 amperes flows for 2 hours between potentials differing by 10 volts.

Solution.—Reducing the time to seconds gives $2 \times 60 \times 60 = 7{,}200$ seconds $= t$. The current $= 8$ amperes $= C$, and the electromotive force $= 10$ volts $= E$. Then, by formula **15**, the electrical work done $= J = 8 \times 10 \times 7{,}200 = 576{,}000$ joules. Expressed in foot-pounds, this will be, by formula **18**,

$$\text{F. P.} = .7373 \times 576{,}000 = 424{,}684.8 \text{ foot-pounds. Ans.}$$

ELECTRICAL POWER.

80. Power, or *rate of doing work*, is found by dividing the amount of work done by the time required to do it. In mechanics, the unit of power is called the **horsepower;** in electrotechnics, the unit of power is the **watt.** It is found by dividing the amount of electrical work done by the time required to do it.

Let $E =$ the electromotive force in volts; Q, the quantity of electricity in coulombs; C, the current in amperes; and W, the power in watts.

By formula **15**, the amount of electrical work $J = C E t$.

Then, $$W = \frac{C E t}{t} = C E. \qquad \textbf{(19.)}$$

The power in watts is equal to the strength of current in amperes multiplied by the electromotive force in volts.

Example.—What is the power in watts developed in a closed circuit in which a current of 12 amperes is flowing between potentials differing by 25 volts?

Solution.— $E = 25$ volts and $C = 12$ amperes. Hence, by formula **19**,

$$W = C E = 12 \times 25 = 300 \text{ watts. Ans.}$$

By taking into consideration the resistance of the circuit, the equation for determining the power in watts may be expressed in two other ways:

By derivation from formula **16**,

$$W = \frac{C^2 R t}{t} = C^2 R. \qquad \textbf{(20.)}$$

That is, ***the power in watts is equal to the strength of current in amperes squared, multiplied by the resistance in ohms.***

Example.—Find the power in watts in a closed circuit in which a current of 30 amperes is flowing against a resistance of 3 ohms.

Solution.— $C = 30$ **and** $R = 3$. **Hence, by formula 20,**

$$W = C^2 R = 30^2 \times 3 = 2{,}700 \text{ watts. Ans.}$$

By derivation from formula **17**,

$$W = \frac{E^2 t}{R t} = \frac{E^2}{R}. \qquad \textbf{(21.)}$$

That is, *the power in watts is the quotient arising from dividing the electromotive force in volts squared by the resistance in ohms.*

Example.—The drop of potential in a closed circuit when a current is flowing is 20 volts and the resistance is 10 ohms; what is the power in watts expended?

Solution.— $E = 20$ volts and $R = 10$ ohms. Hence, by formula **21**,

$$W = \frac{E^2}{R} = \frac{20^2}{10} = 40 \text{ watts. Ans.}$$

81. One watt equals $\frac{1}{746}$ of a horsepower; or, 1 horsepower equals 746 watts.

If H. P. = horsepower,

$$\text{H. P.} = \frac{W}{746}. \qquad \textbf{(22.)}$$

That is, *to express the rate of doing electrical work in horsepower units, find the number of watts and divide the result by 746.*

The horsepower may also be expressed by three other equations, by expressing the watts in terms of electromotive force, current, and resistance, as obtained from formulas **19, 20, 21,** viz.:

$$\text{H. P.} = \frac{E\,C}{746};\ \text{H. P.} = \frac{C^2 R}{746};\ \text{and H. P.} = \frac{E^2}{746\,R}.$$

Example.—Given, current = 50 amperes and electromotive force = 250 volts; express the power directly in horsepower units.

Solution.— $E = 250$ volts; $C = 50$ amperes; hence,

$$\text{H. P.} = \frac{E\,C}{746} = \frac{250 \times 50}{746} = 16.756 \text{ horsepower. Ans.}$$

Example.—Given, strength of current = 25 amperes and resistance = 14.92 ohms; express the power directly in horsepower units.

Solution.— $C = 25$ amperes; $R = 14.92$ ohms; hence,

$$\text{H. P.} = \frac{C^2 R}{746} = \frac{25^2 \times 14.92}{746} = 12.5 \text{ horsepower. Ans.}$$

Example.—Given, electromotive force = 110 volts and resistance = 4 ohms; express the power directly in horsepower units.

Solution.— $E = 110$ volts; $R = 4$ ohms; hence,

$$\text{H. P.} = \frac{E^2}{746\,R} = \frac{110^2}{746 \times 4} = 4.055 \text{ horsepower. Ans.}$$

82. To express the power in watts when the horsepower is known, use the following formula:

$$W = \text{H. P.} \times 746. \qquad (\mathbf{23.})$$

That is to say, *the power in watts is found by multiplying the horsepower by 746.*

Example.—Express the equivalent of 4.35 horsepower in watts.

Solution.— H. P. = 4.35; by formula **23,** the electrical power $W = 4.35 \times 746 = 3{,}245.1$ watts. Ans.

83. The watt is too small a unit for convenient use in expressing the output of large dynamos, so the **kilowatt** is generally used. One kilowatt is equal to 1,000 watts or about 1⅓ horsepower. For example, if a dynamo were rated at 75 kilowatts, it would have an output of 75,000 watts or roughly about 100 horsepower. The **kilowatt-hour** is a unit of *work* commonly used in electrical work. It is the amount of *work* done when 1 kilowatt is expended for 1 hour, or ½ kilowatt for 2 hours, etc. The kilowatt-hours are, therefore, found by multiplying the average number of kilowatts by the average number of hours during which the kilowatts were expended. Since 1 kilowatt = 1,000 watts, 1 kilowatt-hour = 1,000 watt-hours. Now 1 watt expended for 1 second is equal to 1 joule; hence, 1 kilowatt-hour = 1,000 × 3,600 = 3,600,000 joules, or 3,600,000 × .7373 = 2,654,280 foot-pounds. The kilowatt-hour represents a definite amount of work, whereas the kilowatt expresses the rate at which work is done, and is, therefore, a unit of power.

DYNAMOS AND MOTORS.

ELECTROMAGNETIC INDUCTION.

1. It has been shown that an electric current circulating around a coiled conductor produces lines of force which thread through the coil, entering at one end and leaving at the other. So long as the current in the coil remains at a constant strength, the lines of force have direction and position only; unless influenced by some exterior magnetic substance, they do not increase or diminish in number, or change their position relatively to the coil. Fig. 1 repre-

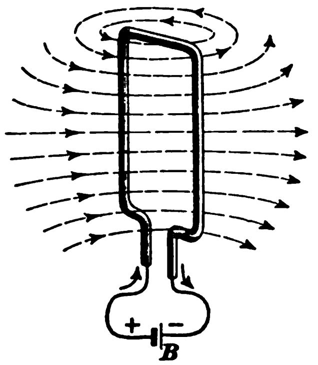


FIG. 1.

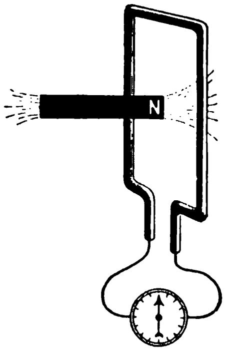


FIG. 2.

NOTE.—This section is the same as that formerly entitled *Dynamos and Motors*, Part 2.

For notice of the copyright, see page immediately following the title page.

sents such a coil around which a current is flowing from the battery B. Suppose the battery is disconnected from the coil and a galvanometer for detecting small currents is inserted in its place. A magnetic pole suddenly thrown into the coil, as represented in Fig. 2, will cause a deflection of the galvanometer needle; the needle, however, will return to its original position as soon as the magnet comes to rest. Withdrawing the magnet from the coil also causes a deflection of the needle, but in the opposite direction. In the first case, a momentary current is induced in the circuit, as shown by the deflection of the galvanometer needle while the magnet is being inserted into the coil; this current immediately subsides when the magnet ceases to move. In the second case, the same effects are produced, with the exception that the current induced in the coil flows in an opposite direction to that in the first case.

These induced currents are caused by a change in the number of lines of force which pass through the coil. In passing into or out of the coil, the lines of force from the magnet set up an E. M. F. in that portion of the conductor in which the number of lines of force is changing, and this E. M. F. tends to send a current through the circuit.

2. In place of a small magnetic pole, imagine the coil to be suddenly inserted into a large uniform magnetic

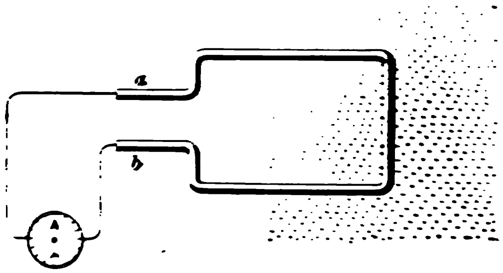


Fig. 2.

field where all the lines of force are parallel to one another. The diagram, Fig. 3, represents a cross-sectional view of such a field. The dots represent the ends of the lines of force; their direction is assumed to be downwards, piercing the paper, or, in other words, the observer is looking along the lines of force toward the face of a south magnetic pole. As the coil enters the magnetic field with its plane at right angles to the lines of force, a current will be induced in the coil and the galvanometer needle will be deflected; this induced current is produced by a change in the number of lines of force which pass through the coil, as in the previous case. Withdrawing the coil from the magnetic field will also induce a current in the circuit, but it will deflect the galvanometer needle in an opposite direction, showing that the current in the circuit is reversed.

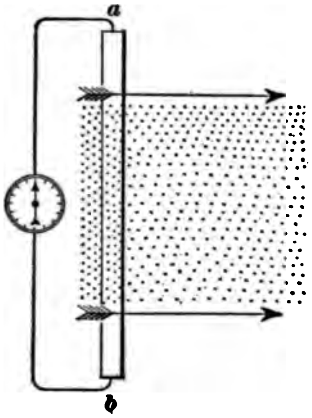


FIG. 4.

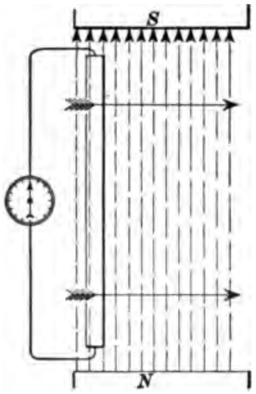


FIG. 5.

If the coiled conductor be straightened out, forming one long conductor, and then moved across the magnetic field at right angles to the lines of force, as represented in Fig. 4, a current will be generated in the circuit. The current, however, immediately subsides when the motion ceases, no matter whether the conductor is in the magnetic field or otherwise. Should the conductor be moved in the magnetic field, with its length parallel to the lines of force, as in Fig. 5, no current will be generated in the circuit. From these two experiments the following principle is deduced: *When a conductor is moved across a magnetic field so that* coiled conductor acts as if possessing *inertia;* any sudden change in the strength of the current produces a corresponding electromotive force which opposes that change and tends to keep the current at a constant strength.

6. In **mutual induction,** two separate coiled conductors, one conveying a current of electricity, are placed near each other, so that the magnetic circuit produced by the one in which the current flows is enclosed by the other, as shown in Fig. 7, where the current circulates around the coil *P* when the circuit is closed at key *b*. The coil *P* is called the **primary,** or **exciting, coil;** the other coil *S* is the **secondary coil.**

Any sudden change in the strength of the current circulating around the primary coil, as, for instance, breaking the circuit at *b*, produces a corresponding change in the number of lines of force in the magnetic circuit which passes through both coils, and hence an electromotive force is induced in the secondary coil. If the primary circuit is completed at *b* and the current tends to rise in that coil, the electromotive force induced in the secondary coil causes a current to circulate around in it in the *opposite* direction to the current in the primary coil. If, on the contrary, the circuit at *b* is suddenly opened and the current in the primary decreases, the induced electromotive force in the secondary causes a current to circulate around in it in the *same* direction as the current in the primary coil.

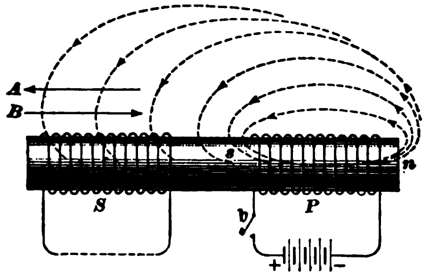


FIG. 7.

To make this clear, suppose, in Fig. 7, the current in the primary coil to be suddenly established by closing the switch at *b*. The lines of force will surround the conductors and spread out in all directions. The lines of force spreading out in the direction of arrow A cut the conductors of the secondary coil. The resulting current in the secondary would have the same direction were the lines of force stationary, as shown, and the coil S moved along the core in the direction of arrow B. Then, according to the thumb-and-finger rule, the current will flow in the secondary coil in a direction opposite to that in the primary. Similar reasoning will show that when the primary circuit is broken and the lines of force collapse, the direction of the current in the secondary coil S will be the *same* as that which existed in the primary.

7. The direction of an induced current in a coil depends upon the direction of the lines of force in the coil and whether their number is increasing or diminishing. If these two facts are known, the direction in which the current circulates around the coil is determined by the following rule:

Rule.—*If the effect of the action is to diminish the number of lines of force that pass through the coil, the current will circulate around the coil in the direction of the movement of the hands of a watch as viewed by a person looking along the magnetic field in the direction of the lines of force; but if the effect is to increase the number of lines of force that pass through the coil, the current will circulate around in the opposite direction.*

For example, in the diagram, Fig. 3, when the coil is inserted into the magnetic field, thereby *increasing* the number of lines of force which pass through the coil, the current circulates from b around the coil to a, and thence through the galvanometer to b again; when the coil is withdrawn and the number of lines *diminishes*, the current circulates in the opposite direction, that is, from a around the coil to b, and thence through the galvanometer to a again. That end of the coiled conductor *from* which the current flows *to* the external circuit, as from a through the galvanometer, in the first case, is the *positive* pole or inal of the coil; in the second case, *b* is the *positive* pole erminal.

ferring to the straight conductor in which a cur- ıs erated by moving it across a magnetic field at ai s to the lines of force, the direction of the current uductor depends upon the relation of the direction ies of force to that of the moving conductor. The tor must necessarily be moved across the magnetic some angle to the lines of force, and the current gen- ated in the conductor will tend to flow at right angles to lines of force and at ri angles to the direction in h the conductor is moving. In Fig. 4, if the conductor ed from left to right across the lines of force, the cur- erated in it will tend flow upwards through the uctor; that is, from *b* to *a* through the conductor, then m *a* to *b* through the galvan neter. If the conductor is ved in the opposite direction, that is, from right to left, nt in the conductor will tend to flow in a reversed , that is, from *a* to *b* through the conductor and from *b* to *a* through the galvanometer. A convenient method for remembering the direction of a current generated in a straight conductor, when the conductor is moved in a magnetic field at right angles to the lines of force, is a follows:

Rule.—*Place thumb, forefinger, and middle finger of the right hand so that each will be perpendicular to the other two; if the forefinger points in the direction of the lines of force and the thumb points in the direction toward which the conductor is moving, then the middle finger will point in the direction toward which the current generated in the conductor tends to flow.*

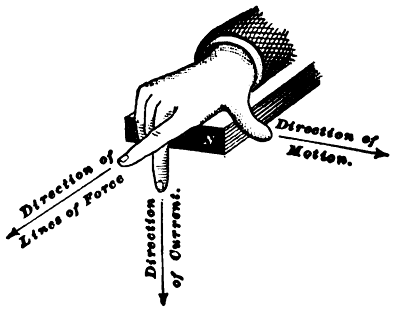

Direction of Lines of Force
Direction of Motion.
Direction of Current.

Fig. 8.

For example, in Fig. 8, if a vertical conductor be moved across the front of the north pole *N* of the magnet in the direction toward which the thumb points, the current generated in the conductor will flow downwards, that is, in the direction toward which the middle finger is pointing.

The summary of these electromagnetic induction experiments can be stated as follows: *Electromotive forces are generated in a conductor moving in a magnetic field at right angles to the direction of the lines of force, or are induced in a coiled conductor when a change occurs in the number of lines of force which pass through the coil.*

PHYSICAL THEORY OF THE DYNAMO.

9. In Fig. 9, a rectangular coil of copper wire is placed in the center of a uniform field with its plane lying perpendicular to the lines of force; in this position, the coil encloses the greatest number of lines of force. A voltmeter *V M* for measuring small E. M. F.'s is connected to the two ends of the coil, as shown in the diagram. The circuit in the voltmeter is kept closed, and any E. M. F. generated in the conductor will be indicated by the deflection of the index needle. So long as the coil remains at rest in the magnetic field no E. M. F. is generated; but imagine the coil to be rotated on an axis in its own plane, such as represented by the broken line *m n*, in the direction indicated by the curved

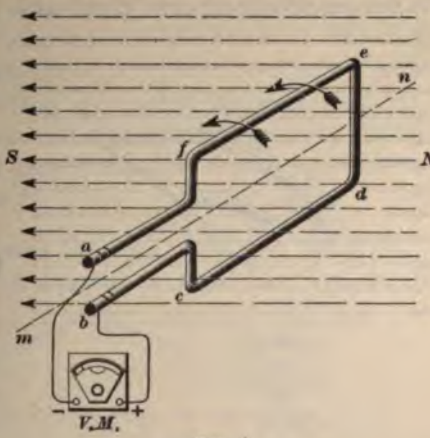


FIG. 9.

arrows. As the coil starts to rotate, its sides *c d* and *e f* begin to cut the lines of force at right angles, thus generating an E. M. F. in each side. From the thumb-and-finger rule, the E. M. F. generated in the upper side tends to cause a current to flow from *f* to *e*; and in the lower side, the current tends to flow from *d* to *c*. Hence, the E. M. F.'s generated in the two sides are added together, and the total E. M. F. generated by the coil is indicated by the *V M* between *a* and *b*, the end *b* forming the *positive* terminal of the coil. If the coil is rotated at a uniform angular velocity, that is, if the speed of rotation is constant throughout each revolution, the deflection of the voltmeter becomes greater as the coil revolves from its vertical position until it passes through one-quarter of a revolution and reaches a position where its plane lies parallel to the lines of force.

10. The diagram, Fig. 10, represents an end view of the coil in two positions: position *1*, as shown by the dotted lines, represents the coil standing vertically at the moment of starting, and position *2*, as shown by the full lines, represents the coil lying horizontally after passing through one-quarter of a revolution. The deflection of the needle, if read at frequent intervals during this quarter of a revolution, gradually increases, beginning at zero in position *1* and reaching a maximum at position *2*. The gradual rise of the E. M. F. in the circuit while the coil is revolving from position *1* to position *2* can be graphically shown by means of cross-section paper, Fig. 11. The horizontal divisions represent equal intervals of time, and the sum of the divisions between *A*

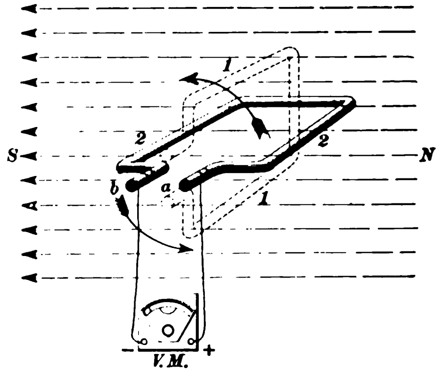


FIG. 10.

and *B* is the total time occupied by the coil in revolving one-quarter of a revolution; the vertical divisions represent E. M. F., and the sum of the divisions between *A* and *Y* is the total E. M. F. that is being generated in the coil when it is passing through position *2*. The vertical distances between the line *A B* and the curved line represent the E. M. F. which is being generated in the coil at every instant during its rotation between positions *1* and *2*. For example, let each vertical division represent 2.5 volts; then, the distance between *A* and *Y* represents 10 volts. When the coil has revolved one-third of the distance between positions *1* and *2*, Fig. 10, it has consumed one-third of the time; hence, at this instant the E. M. F. that is being generated in the coil is represented by the number of divisions between the line *A B* and the curved line, at one-third the distance toward *B*, which equals two divisions; or 2 × 2.5 = 5 volts. When the coil travels two-thirds the distance between positions *1* and *2*, the E. M. F. that is being generated at that instant is represented by the number of divisions between the line *A B* and the curved line at two-thirds the distance toward *B*, which equals about 3.48 divisions, or 3.48 × 2.5 = 8.7 volts.

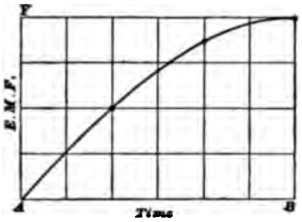


FIG. 11.

11. After the coil passes through position *2*, the E. M. F. that is being generated begins to diminish, and by the time the coil has revolved one-half of a revolution and is once more in a vertical position, the E. M. F. falls to zero

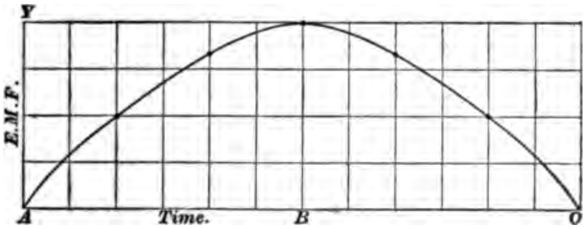


FIG. 12.

again. The E. M. F. that is being generated at every instant during one-half of a revolution can be shown by a continuation of the curve on cross-section paper, Fig. 12. The sum of the divisions between *A* and *C* represents the total time occupied by the coil in rotating one-half of a revolution. It will be seen that the maximum E. M. F. that is being generated at any instant is at position *2*, Fig. 10, which corresponds to *B*, Fig. 12. In this position the plane of the coil lies parallel to the lines of force, and its sides, corresponding to *c d* and *e f*, Fig. 9, are cutting the lines of force at exactly right angles. The sides of the coil at the moment of passing through this position are cutting more lines of force for equal intervals of time than in any other position during the first half of a revolution.

From this fact the following principle is deduced: *The E. M. F. generated in a moving conductor cutting lines of force at right angles is directly proportional to the rate of cutting.* Suppose, for example, that a magnetic field contains 100,000 lines of force, and that a conductor is moved across the field at right angles in such manner as to cut every line of force. If the time occupied by the conductor in passing across the field is one second, then the rate of cutting is 100,000 lines per second; or, if it occupied two seconds, the rate of cutting is 50,000 lines per second, and so on. The E. M. F. generated in the former case is twice as great as that generated in the latter. The method for determining the number of lines of force in a magnetic field will be described later.

12. Fig. 13 shows the coil after being rotated one-half of a revolution. As soon as the coil starts on the last half of the revolution, its sides *c d* and *e f* cut a few lines of force, and, consequently, an E. M. F. is generated in each side. The E. M. F., however, tends to cause a current to flow in the coil in an opposite direction to that which tends to flow during the first half of the revolution. For, by applying the thumb-and-finger rule, the E. M. F. generated in the sides tends to cause a current to flow from *c* to *d* and from *e* to *f*; the end *a* of the coil, which in the first half of the revolution was the negative terminal of the coil, now forms the positive terminal. Hence, in order to allow the current to enter the positive binding post of the voltmeter, the connections must be reversed.

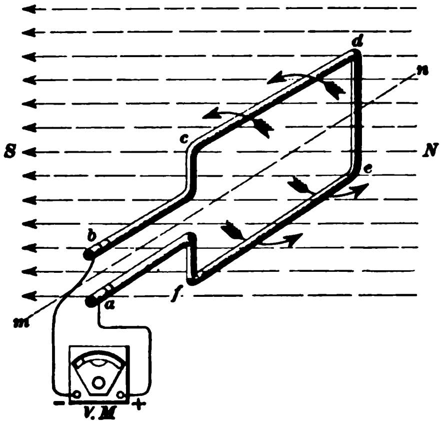


FIG. 13.

The E. M. F. that is generated as the coil is rotated through the last half of the revolution gradually rises as in the first half, reaching a maximum height when the plane of the coil lies parallel to the lines of force, and afterwards falling to zero again as the coil reaches a vertical position. In Fig. 14, the E. M. F. that is generated in the coil at every instant during one complete revolution is graphically shown by the use of the cross-section paper. The sum of the divisions between *A* and *E* represents the time occupied by the coil in making one complete revolution; the divisions between *A* and *Y* represent the E. M. F. which tends to send a current in one direction through the coil, as in the first half of the revolution, and the divisions between *A* and

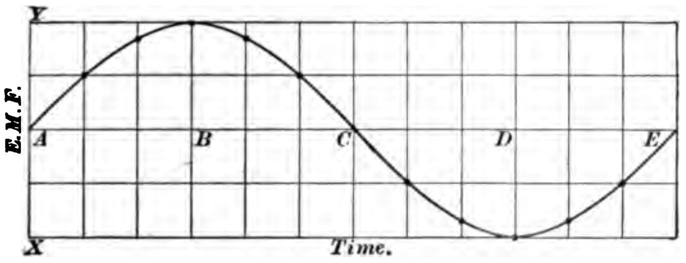


FIG. 14.

X represent the E. M. F. which tends to send a current through the coil in an opposite direction, as in the last half of the revolution. The divisions between the curved line and the line *A E*, or **base line**, give the E. M. F. that is being generated in the coil at any instant during the revolution, and the direction in which the E. M. F. tends to act depends upon whether this E. M. F. falls above or below the base line *A E*. For convenience, let the direction in which the E. M. F. tends to act in the first half of the revolution be called the **positive** (+) **direction**, and in the last half the **negative** (−) **direction**. For example, the E. M. F. that is generated in the coil when it has revolved three-quarters of a revolution is represented by the distance between *D* and the curved line, which, in this case, is two divisions; and since these divisions are below the base line, the direction in which this E. M. F. tends to act is negative.

13. In Fig. 15, instead of connecting the external circuit directly to the ends of the coil, suppose the wires *o* and *p* to be brought to two brushes *r* and *s*, which lie in a horizontal position and bear on the two *collector rings* *x* and *y*, respectively. These collector rings, it will be seen, are connected to the two ends of the coil; *x* to *a* and *y* to *b*.

The resistance of the entire circuit, including the coil, ammeter, collector rings, and brushes, is comparatively small; hence, any E. M. F. generated in the coil causes a corresponding current to flow through the circuit, and its strength is indicated by the ammeter *A. M.* When the coil begins to revolve, a feeble E. M. F. is generated in it, as previously described. This E. M. F. causes a corresponding current to flow through the circuit in a positive direction; as the E. M. F. becomes larger, the strength of current in the circuit becomes greater, and vice versa. After the coil is rotated one-half of a revolution and the direction in which the E. M. F. tends to act becomes negative, the direction of the current in the circuit is also reversed. If there is no self-induction to retard the rise and fall of the current in the circuit, the strength of the current in the circuit at any instant is exactly proportional to the E. M. F. that is being generated in the coil at that moment; for, according to Ohm's law, the strength of current in any circuit is equal to the E. M. F. generated in that circuit, divided by its resistance. The rising and falling and also the reversing of the current in all parts of the circuit for each revolution, therefore, can be represented graphically on cross-section paper in the same manner as previously described for the E. M. F. Fig. 16 represents the rising, falling, and reversing of the current in the circuit for three complete and consecutive revolutions of the coil; the divisions between

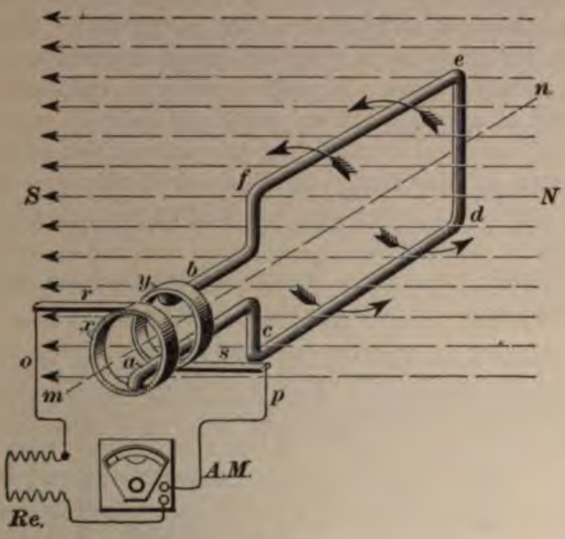


FIG. 15.

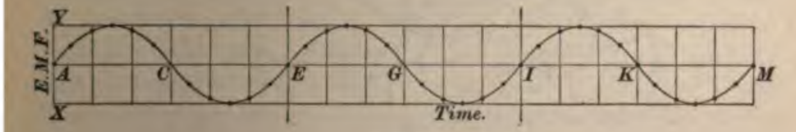


FIG. 16.

A and *E*, *E* and *I*, and *I* and *M* represent the time of each revolution, respectively. The divisions between the base line *A M* and the curved line above the base line represent the strength of current in the circuit when the direction of flow is positive, and those below represent the strength of current when the direction of flow is negative. Revolving the coil, therefore, at a constant speed generates a current in the circuit, which, in every complete revolution, rises gradually to a maximum strength and falls to zero in one direction, then is reversed, and the same effect is produced in the opposite direction. In other words, the current in the circuit *alternates* from one direction to the opposite direction in each revolution.

An electric current of this character flowing through a circuit is termed an **alternating current.**

14. The next step is to demonstrate the principle of changing, or *commuting*, this alternating current into a *continuous*, or *direct*, current; that is, a current which always flows in the same direction through the external circuit. In Fig. 17, the two ends of the coil are fastened to two

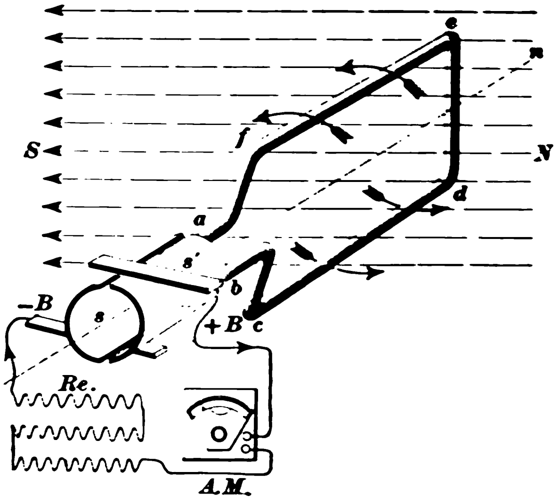


FIG. 17.

halves s and s' of a metallic tube. These halves are called **segments,** and in this case are separated by a small air space, the rigidity of the coil holding them apart. The combination of the two segments, or, in fact, any number of segments held together in this position, is called the **commutator.** Two copper strips $+B$ and $-B$, called **brushes,** press against the segments, and are held in a horizontal position while the coil is rotated. The brushes rub, or *brush*, against the segments and make electrical contact only.

When the coil is in a vertical position, as represented in the figure, both brushes rest against both segments; but as soon as the coil starts on the first half of a revolution in the direction indicated by the arrows, the brush $-B$ leaves segment s' and rubs only against segment s; brush $+B$ leaves segment s and rubs only against segment s'. As previously described, the electromotive force that is generated in the coil during the first half of a revolution causes a current to flow from a through the coil to b, and from b through the external circuit to a again, making b the positive end of the coil. Hence, in this case, $+B$ is the positive brush, and the current in the external circuit flows in the direction indicated by the arrowheads. As the coil starts on the last half of a revolution, the direction of the current in the coil changes, and a becomes the positive end of the coil. But the current in the external circuit continues to flow in the same direction as in the first half of the revolution, and $+B$ remains the positive brush. For, at the beginning of the second half of a revolution, when end a of the coil becomes positive, $-B$ leaves segment s and makes contact with s', and $+B$ leaves s' and makes contact with s. Hence, the current in the external circuit, during a complete revolution, flows from the positive brush $+B$ through the ammeter $A\ M$ and the resistance Re to the negative brush $-B$; that is, the current in the external circuit flows continually in the same direction, while the current in the coil itself flows in two directions during every revolution. But the strength of the current in the external circuit is by no means constant; it rises from zero to a maximum strength and falls again to zero twice in every revolution, but always in the same direction. The effect is graphically shown in Fig. 18 by the use of cross-section paper, where the divisions between A and E, E and I, and I and M represent the time occupied by the coil in rotating each revolution, respectively, and the vertical divisions between the base line $A\,M$ and the curved line represent the strength of the current in the external circuit at every instant during the three revolutions. The effect is produced continually in the external circuit if the coil is rotated at a constant speed. These impulses in the strength of the current give it the name of **pulsating current.**

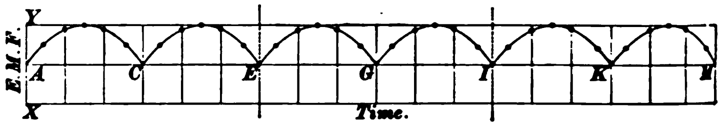


FIG. 18.

A consideration of the preceding paragraphs will show the student that direct-current dynamos require commutators, while alternating-current dynamos employ only collector rings.

15. In Fig. 19, two separate coils are placed in a magnetic field at right angles to each other. Four metallic segments s, s', s'', and s''' are cut from a cylindrical ring to form the commutator, and are separated from one another by small air spaces; the two ends of each coil are connected to two opposite segments in such manner that an imaginary diameter connecting the two segments together would lie at right angles to the plane of their coil, as shown in the figure. Two metallic brushes $+B$ and $-B$ rub against the commutator, touching the two segments diametrically opposite to each other. A line drawn through the center of the commutator, connecting the contact ends of the two brushes, should lie at right angles to the direction of the lines of force in the magnetic field in which the coils are rotated. As the two coils and commutator are rotated in the direction indicated by the arrows, the two brushes rub against the segments consecutively and always make contact with the two opposite ones. The brushes are connected to an exernal circuit consisting of the ammeter *A M* and the resistance *Re*. At the position of the coils in the figure, the brushes are rubbing against the segments s and s'', which are connected to the ends of the horizontal coil. From previous experiments, it will be seen that at this position the horizontal coil is generating a maximum E. M. F., which tends to send a current from a through the coil to b; hence, the current is flowing in the external circuit from $+B$ to $-B$. After the coils and commutator are rotated one-eighth of a revolution from this position, and the E. M. F. in the coil begins to fall, the brush $+B$ passes from segment s

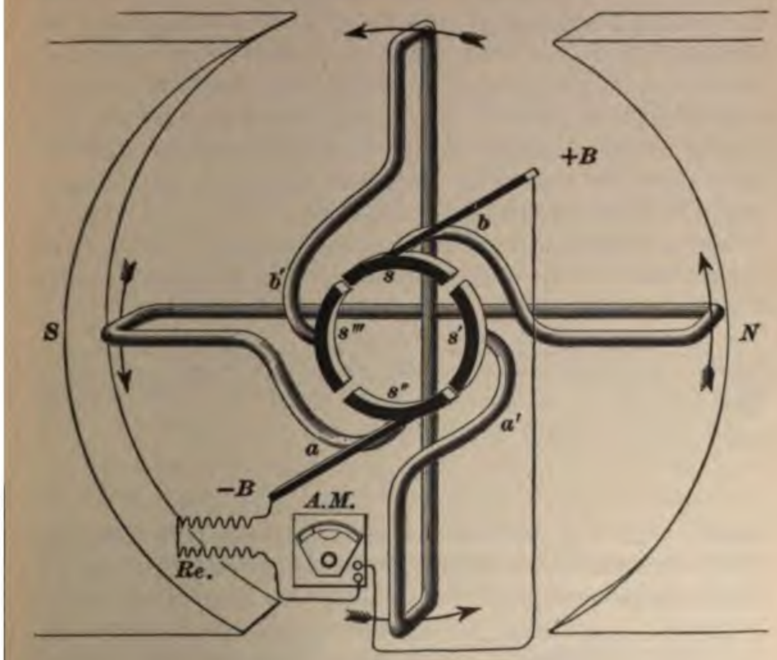


FIG. 19.

to segment s', and brush $-B$ passes from s' to s'''. The E. M. F. that is being generated in the vertical coil when the brushes pass to segments s' and s''' is nearly maximum. Consequently, the strength of the current which has been flowing in the external circuit from the other coil does not decrease to zero; it only diminishes a small amount before the segments of the next coil make contact with the brushes, when it begins to increase again. It will be seen that during one complete revolution of the moving parts, the brushes passed over four segments; also that the direction of the current produced is *from* the coils *to* brush $+B$, and *into* them *from* brush $-B$. These actions produce a direct current in the external circuit which flows continually in the same direction, but whose strength fluctuates, or changes, regularly four times in every revolution.

By resorting again to the cross-section paper, the fluctuations of the current in the exterior circuit can be graphically shown. In Fig. 20, the divisions between the base line $A\,M$ represent the strength of current in the external circuit for three complete revolutions. So long as the speed of rotation is uniform, the current decreases to a little less than three-quarters of its maximum strength, provided, of course, the resistance of the external circuit is not altered; the dotted curved lines indicate how the strength of the current would fall to zero if only one of the coils were used.

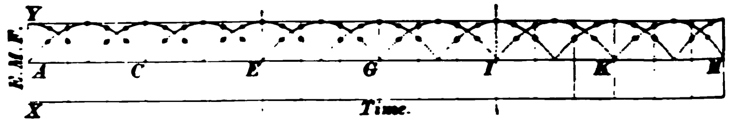


FIG. 20.

The strength of such currents can be made more uniform and the pulsations less noticeable by using several coils connected to the segments of a commutator, the planes of the coils being placed at equal angles from one another. A continuous current of uniform strength is known as a constant current

16. It has already been stated that the *permeability* of iron is much greater than that of air; or, in other words, if a piece of iron were inserted in a magnetic field, the number of lines of force in the field would be greatly increased. Hence, if the coils are wound around a cylindrical drum of iron, as shown in Fig. 21, the number of lines of force passing through the coils is increased, and the E. M. F. that is generated is greater, since the E. M. F. is proportional to the rate of cutting of the lines of force. The coils are entirely insulated from the iron core by some non-conducting material, such as cloth, mica, or paper; otherwise, they would be short-circuited on the core; that is, the current would flow through the iron instead of passing into the external circuit. The other conditions remain unchanged; i. e., the lines of force have the same direction as in the previous cases, and remain in one position while the coils are revolved. The core should not be made of one solid mass of iron; for, if such were the case, the core, when rotated, would act as a large closed conductor, cutting lines of force at right angles. The E. M. F. generated in the core would cause **local,** or **eddy,** currents to flow through the iron itself, heating it and uselessly dissipating a large amount of energy. An idea of how these

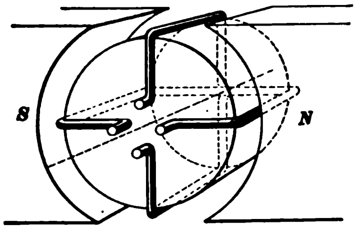


FIG. 21.

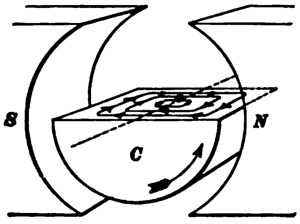


FIG. 22.

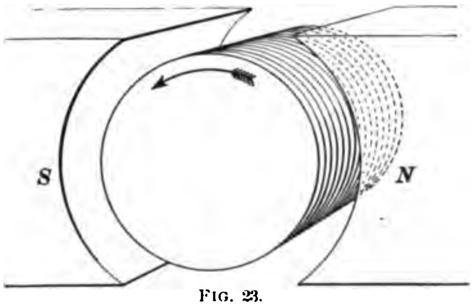


FIG. 23.

eddy currents would circulate in a solid iron core can be formed from Fig. 22. *C* represents the solid iron core, the top half of which is cut away. The curved lines and arrowheads show the direction in which the eddy currents would flow if the core were rotated in the direction indicated by the large arrow. To overcome this difficulty, the core is made of a large number of round, thin iron plates, or disks, each disk being insulated from the adjacent ones by some nonconducting material, such as tissue paper, insulating japan, or simply by the oxide formed on the surface of the disk during the process of its manufacture. The disks should be fastened together in such a manner that, when rotated in a magnetic field, their flat surfaces are parallel to the direction of the lines of force and to the direction of rotation, as shown by Fig. 23. Dividing the core into disks in no way diminishes the magnetic permeability of the iron, and for all practical purposes, it prevents the eddy currents from flowing. A core made in this way is said to be **laminated.**

17. Iron cores are generally made in two styles: **drum** or **ring.**

A drum core may be defined as a laminated cylinder, the length being generally greater than the diameter, such as shown in Fig. 23.

A ring core may be defined as a laminated rim of rectangular cross-section, such as *R* in Fig. 24.

An iron core inserted between the poles of a magnet not only increases the total number of lines of force from the magnet, but attracts nearly all the stray lines of force from the surrounding air; that is, the lines of force prefer to complete their circuit through iron rather than through air or other non-magnetic substances. For example, in Fig. 24, an iron ring *R* is placed between the poles *N* and *S* of a magnet; the lines of force pass out from the north pole *N* and enter the iron ring. When passing across the air gap, they are uniformly distributed, but after entering the ring they crowd together and remain in the iron as long as possible. If the total number of the lines of force is large in comparison with the cross-sectional area of the iron ring on $x\,y$, a few will pass through the air in the inside of the ring, as shown in the cut; but in most cases the number of such stray lines is not large enough to be considered. Consequently, in Fig. 25, if a loop of insulated wire $a\ b\ c\ d$ is wound around the iron ring, and the ring and loop are

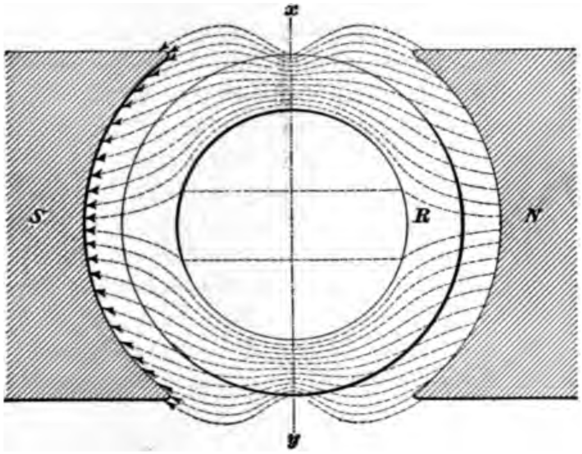


FIG. 24.

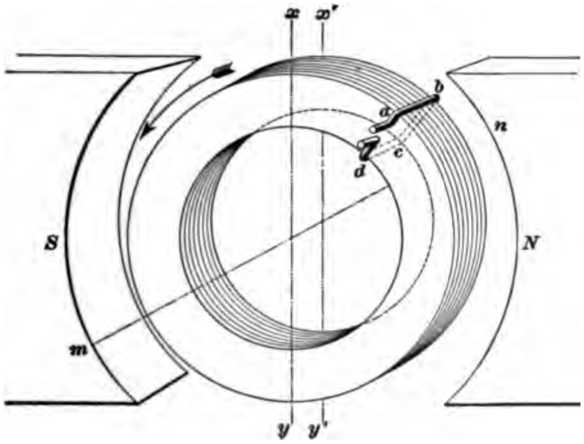
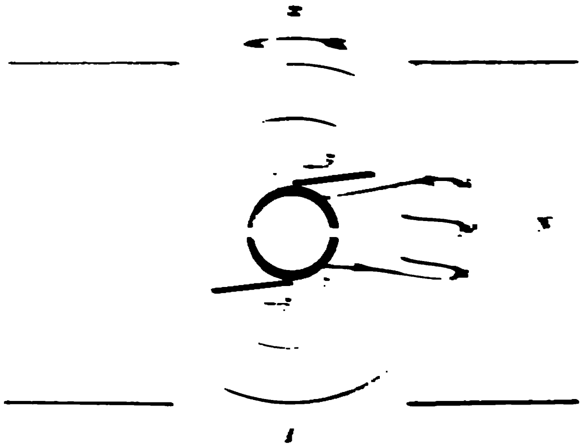


FIG. 25.

[illegible]

[illegible]



[illegible] between [illegible] and [illegible] the same [illegible] and [illegible] tends [illegible] between [illegible] and [illegible] tends [illegible] act away from *c*. Hence, there is no difference of potential between the ends *a* and *d*, and no current will flow through an external circuit.

After replacing the iron ring again, suppose the insulated wire to be wound around it several times, as represented in Fig. 26, and the ends of the coil connected to two metallic segments S' and S''. By applying the rule, it will be seen that the electromotive forces generated in the separate turns at *a*, *b*, and *c* are added together; that is, the difference of potential between the brushes $+B$ and $-B$ is the sum of the electromotive forces generated in the separate turns. The current obtained from such a coil is *pulsating*. For all practical purposes, the total E. M. F. generated by such a coil is directly proportional to the number of turns. For example, if a coil of 1 turn generates 2 volts at a certain position and angular velocity, then a coil of 4 turns will generate 8 volts under the same conditions, and so on. But the turns in each coil must be approximately close together. For, if the coil is wound over a large portion of the ring, some of the turns, at one position of the coil, will be cutting the lines of force as they pass out from the north pole, while other turns will be cutting the lines of force as they enter the south pole, the electromotive forces generated in the

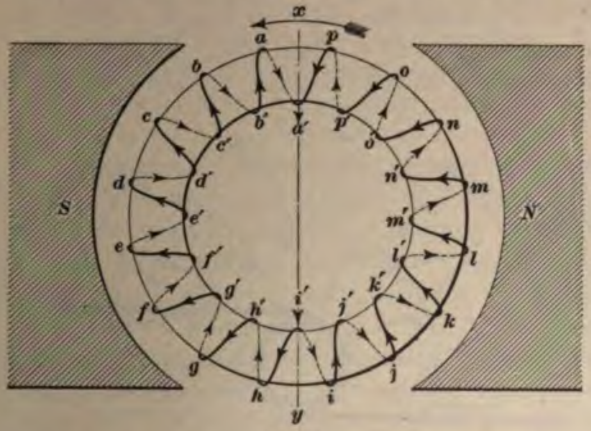


FIG. 27.

two cases being opposed to each other. This action will be readily understood by winding the entire core with one large coil of several turns and connecting the two ends of the coil together, as represented in Fig. 27. This is known as a *ring* winding, or one in which the conductors are wound in the form of a helix on a ring core. At the instant the ring and coils reach the position shown in the figure, the E. M. F. generated in the separate turns tends to act in the direction indicated by the arrowheads upon the winding. No current can flow around the coil, because the electromotive forces generated in the two halves act toward each other at *a'* and away from each other at *i'*.

19. It is possible, however, to obtain a continuous current from the coil by the addition of a commutator with several segments, as will presently be seen. If the ends of a voltmeter are touched to *a'* and *i'* during the instant the coil occupies the position in Fig. 27, a difference of potential between the two points will be indicated, *a'* being the positive point and *i'* the negative. Hence, if these two

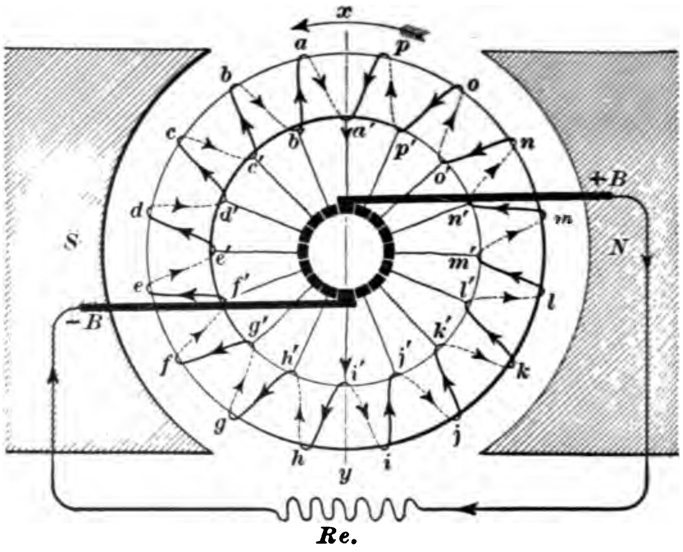


FIG. 28.

points are connected to an external circuit, a current will flow through it from a' to i', while the coil is at the position shown in the figure. As soon, however, as the coil is rotated about one-sixteenth of a revolution, the difference of potential between a' and i' will begin to fall, and the greatest difference will now be found between p' and h'. About another sixteenth of a revolution will bring the greatest difference of potential between o' and g', and so on. In short, as the coil is rotated, the greatest difference of potential will always be found between any two turns situated diametrically opposite each other when they pass through the vertical diameter $x y$. The next operation is to provide some means to utilize this difference of potential between each pair of turns as they arrive in a vertical position. This is accomplished by connecting each turn to a separate segment of a commutator by a small conductor, and allowing two brushes to rub against the commutator at two points diametrically opposite each other on the vertical diameter $x y$, Fig. 28. From an examination of the figure, it will be seen that the two halves of the coil are connected in parallel or multiple; that is, the current divides at i', one half passing through the turns i, j, k, l, etc. and the other through h, g, f, e, etc. to a', where it again unites. The maximum E. M. F. that is obtained from the coil is equal, therefore, to the E. M. F. generated in one-half of the coil. This statement will be better understood by comparing the coil to a battery of voltaic cells connected in multiple-series. For example, in Fig. 29, the separate cells from a to h, inclusive, correspond to the separate turns on one half of the coil, and the cells from i to p correspond to the turns on the other half. The total E. M. F. of the above battery is equal to the E. M. F. of either of the two sets which are connected in parallel; and the total E. M. F. of either of the two sets is

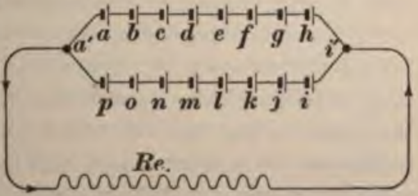


FIG. 29.

the product of the E. M. F. of 1 cell and the number of cells which are connected in series, as from *a* to *h*, inclusive.

If a comparatively large number of turns and segments is used, the current flowing from $+B$, Fig. 28, through the external circuit to $-B$ will be practically continuous, that is, non-pulsating; the fluctuations caused by the brushes when passing from one segment to another are extremely minute, and produce no appreciable change in the strength of the current in the external circuit.

20. A conductor wound upon a core in the manner shown in Figs. 27 and 28 is termed a **closed-coil winding**, since all the turns are connected together in one *continuous*, or *closed*, coil, and the current is obtained from it by tapping into each turn or set of turns. In the case where the turns or sets of turns are separate and distinct from one another and their ends are connected to opposite segments of a commutator, as in Figs. 19 and 26, the winding is termed an **open-coil** winding.

21. A *closed-coil winding* can be applied to a cylindrical drum core, as previously described, and a continuous non-pulsating current obtained from the brushes, as in the case of the ring core. The method of winding is somewhat similar to that of the ring, and each turn or set of turns is tapped into and connected to the segment of a commutator by a separate lead, as will be seen from the diagram, Fig. 30. This is known as a drum winding, or one in which the conductors are wound longitudinally upon the surface of a drum core. A drum winding may also be applied to a ring core, as will be seen. The conductor is started at any convenient place on the core, as, for example, at *a*, and wound across the face of the drum to the rear end; then, wound nearly diametrically across the end, and from there along the face of the core to the front end at *a'*. From *a'*, the conductor is wound across the front end to a point somewhat in advance or behind the original starting point *a*, as, for example, to *b*; from *b* it makes another complete turn in like manner, which is followed by a third, and so on, until the last turn is connected to the first by joining the two ends of the coil together at *a*. A separate lead *L* is tapped into the conductor at every complete turn where it is wound across the front end of the core and connected to the separate segments of a commutator. From an examination of the diagram, it will be seen that only a part of the wires on the face of the drum are cutting the lines of force as they enter and pass out of the core at any one instant during a revolution. At the position represented, the wires *e'*, *a*, *f'* and *b'*, *e*, *a'* are the inactive ones, so far as the lines of force are concerned; but they still perform the important function of completing the circuit for the current. The parts of the core where the wires are not cutting the lines of force as the core is rotated are called the **neutral spaces**; and the two opposite parts of the commutator to which the coils are connected are called the **neutral points** of the commutator. Each individual wire becomes inactive twice during every revolution and passes through two neutral spaces; but this fact does not change the positions of the neutral spaces—they lie on an imaginary diameter approximately

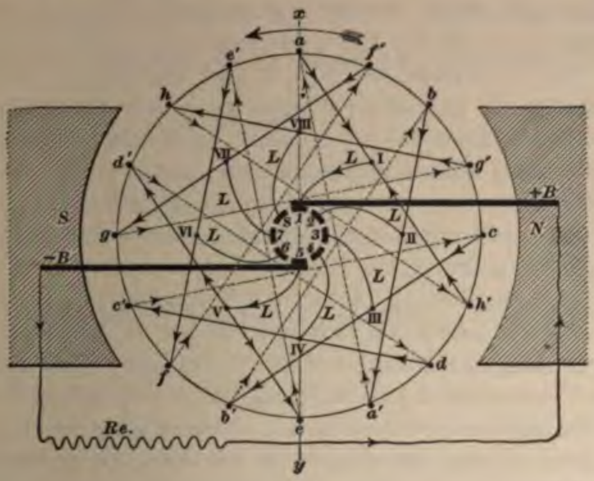


FIG. 30.

perpendicular to the lines of force. This same effect takes place in the commutator, i. e., each segment passes through two neutral points during one complete revolution, but the neutral points remain in a fixed position relative to the neutral spaces of the core. The neutral segments of the commutator, at any instant during a revolution, are those segments which are connected to the wires passing through the two neutral spaces at that instant. The neutral points, however, can be shifted to different points around the commutator by changing the leads from the coil to the segments. For example, in Fig. 30, the two neutral points lie opposite each other on the commutator along the vertical diameter $x\,y$. But if the lead from *I* is connected to segment No. *7*, instead of No. *1*, and the lead from *II* to segment No. *8*, and so on around the commutator, then the two neutral points will lie opposite each other on the commutator along a horizontal diameter, and in order to collect any current from the commutator, the brushes $+B$ and $-B$ must be shifted around a quarter of a revolution to these new neutral points.

The current flowing through the winding divides at one neutral space and flows through the coil in opposite directions, uniting again at the other neutral space, as indicated by the arrowheads. According to the thumb-and-finger rule, the current in all the active wires in front of the north pole flows along the periphery of the core towards the observer; that in the wires in front of the south pole flows away from the observer.

22. The next step is to determine the magnitude of the E. M. F. in volts generated in a closed coil. As previously stated, the E. M. F. generated in a conductor cutting lines of force at right angles is proportional to the *rate of cutting*. Consider the case of a single conductor moving across a magnetic field in which the total number of lines of force is known; the rate of cutting is equal to the total number of lines of force cut by the conductor divided by the time required to cut them. This may be expressed in the form of an equation: thus, *rate of cutting* $= \frac{N}{t}$, where N is the total number of lines cut and t is the time required to cut them. By definition, *one volt* is that E. M. F. generated in a conductor when it is cutting lines of force at the rate of one hundred million (100,000,000) per second. Hence, $E = \frac{N}{10^8 t}$, where E is the E. M. F. in volts and t the time in seconds, since $100{,}000{,}000 = 10^8$.

For example, suppose a magnetic field contains 4,500,000 lines of force, and a conductor cuts the total number in the same direction in 1.5 seconds. The E. M. F. that is being generated in the conductor is equal to .03 volt, since $E = \frac{N}{10^8 t}$ $= \frac{4{,}500{,}000}{100{,}000{,}000 \times 1.5} = .03$ volt.

When two or more conductors are cutting lines of force at equal rates, the E. M. F. obtained by connecting them in series is equal to the E. M. F. developed by one conductor multiplied by the number of conductors. Consequently, if S is the number of conductors in series, then $E = \frac{N S}{10^8 t}$, where E is the total E. M. F. in volts that can be obtained from S conductors cutting N lines in t seconds. For example, if 8 conductors are moved across the magnetic field containing 4,500,000 lines of force in 1.5 seconds, and they are connected in series, then $E = \frac{N S}{10^8 t} = \frac{4{,}500{,}000 \times 8}{100{,}000{,}000 \times 1.5}$ $= .24$ volt.

Next, imagine these 8 conductors to be moved across the magnetic field in the same direction at the rate of 30 times per second for 1.5 seconds; then, the number of lines cut in one second is $4{,}500{,}000 \times 30 = 135{,}000{,}000$, and the total number of lines cut in 1.5 seconds is, therefore,

$$135{,}000{,}000 \times 1.5 = 202{,}500{,}000. \text{ Hence,}$$

$$E = \frac{(N n t)\, S}{10^8\, t} = \frac{202{,}500{,}000 \times 8}{100{,}000{,}000 \times 1.5} = 10.8 \text{ volts.}$$

Here $n =$ the number of times per second that one conductor cuts the lines of force.

But, in general, the E. M. F. that is obtained from several conductors connected in series moving continually across the same magnetic field at a constant number of times per second is independent of the length of time the operation is continued. For, in the above equation, $E = \frac{(N\,n\,t)\,S}{10^8\,t}$, the two t's cancel each other, leaving the equation $E = \frac{N\,S\,n}{10^8}$. In the above example, for instance, so long as the 8 conductors are moved across the magnetic field at the rate of 30 times per second, the E. M. F. generated in them is always 10.8 volts, no matter whether the operation is continued for 1.5 seconds or for 1 hour. The time of 1.5 seconds was used merely to make the demonstration clearer by using a specific value for t.

23. The equation $E = \frac{N\,S\,n}{10^8}$ can now be applied with some modifications to the closed-coil conductor wound upon either the ring or drum core. The ring core, Fig. 28, will first be considered. In the equation, E is the maximum E. M. F. in volts that is obtained from the brushes $+B$ and $-B$ when the core is revolved; N is the total number of lines of force passing from the north pole through the core to the south pole. Each wire, therefore, on the periphery of the core cuts the total number of lines twice during every revolution; or, in other words, each outside wire cuts $2\,N$ lines of force per revolution. S is the number of outside wires on the periphery through which the current flows in *series*, and n is the number of complete revolutions per second of the core. Therefore, the maximum E. M. F. in volts that is obtained from the brushes is found by the formula

$$E = \frac{2\,N\,S\,n}{10^8}. \qquad \textbf{(1.)}$$

That is to say, *the E. M. F. obtained from a number of* *conductors connected in series and moved across a magnetic field is equal to twice the number of lines of force multiplied by the number of conductors in series and by the revolutions per second of the core, divided by 100,000,000.* For example, assume the total number of lines N passing from the north pole through the core to be 3,000,000, or $N = 3,000,000$. In the diagram, Fig. 28, there are 8 outside wires in series, or $S = 8$. If the core is rotated at 2,100 revolutions per minute, $n = \frac{2,100}{60} = 35$ revolutions per second. Substituting the values in the formula gives $E = \frac{2NSn}{10^8}$ $= \frac{2 \times 3,000,000 \times 8 \times 35}{100,000,000} = 16.8$ volts, or the difference of potential between the brushes $-B$ and $-B$ on open circuit. The difference of potential between the brushes when the external circuit is closed is somewhat smaller than when no current is flowing; because, as in the case of the voltaic cell, a part of the total E. M. F. developed is required to overcome the internal resistance of the coil itself.

The formula $E = \frac{2NSn}{10^8}$ holds equally true for the drum core, Fig. 30. In both cases, the number of outside wires through which the current flows in *series* is equal to one-half the total number of outside wires. Hence, by using the same magnetic field and rotating the cores at equal speeds, the E. M. F. generated in both cases will be equal.

24. The foregoing articles demonstrate the elementary principles and physical theory of a *dynamo*. A **dynamo**, therefore, is a machine for converting mechanical energy into electrical energy by electromagnetic induction. It has three essential features, viz.: (1) a magnetic field; (2) a conductor, or several conductors, called an **armature**, in which the electromotive force is generated by som
ment relative to the lines of force in the magnetic
(3) a *commutator*, or a *collector*, from which the
collected by two or more conducting

Here n = the number of times per second that one conductor cuts the lines of force.

But, in general, the E. M. F. that is obtained from several conductors connected in series moving uniformly across the same magnetic field at a constant number of times per second is independent of the length of time the operation is continued. For, in the above equation, $E = \frac{N\,n\,t\,S}{10^8\,t}$, the two t's cancel each other, leaving the equation $E = \frac{N\,S\,n}{10^8}$. In the above example, for instance, so long as the 8 conductors are moved across the magnetic field at the rate of 30 times per second, the E. M. F. generated in them is always 1.8 volts, no matter whether the operation is continued for 1.5 seconds or for 1 hour. The time of 1.5 seconds was used merely to make the demonstration clearer by using a specific value for t.

23. The equation $E = \frac{N\,S\,n}{10^8}$ can now be applied with some modifications to the closed-coil conductor wound upon either the ring or drum core. The ring core, Fig. 25, will first be considered. In the equation, E is the maximum E. M. F. in volts that is obtained from the brushes $+B$ and $-B$ when the core is revolved; N is the total number of lines of force passing from the north pole through the core to the south pole. Each wire, therefore, on the periphery of the core cuts the total number of lines twice during every revolution; or, in other words, each outside wire cuts $2\,N$ lines of force per revolution. S is the number of outside wires on the periphery through which the current flows in *series*, and n is the number of complete revolutions per *second* of the core. Therefore, the maximum E. M. F. in volts that is obtained from the brushes is found by the formula

$$E = \frac{2\,N\,S\,n}{10^8}. \qquad \textbf{(1.)}$$

That is to say, *the E. M. F. obtained from a number of* *conductors connected in series and moved across a magnetic field is equal to twice the number of lines of force multiplied by the number of conductors in series and by the revolutions per second of the core, divided by* 100,000,000. For example, assume the total number of lines N passing from the north pole through the core to be 3,000,000, or $N = 3{,}000{,}000$. In the diagram, Fig. 28, there are 8 outside wires in series, or $S = 8$. If the core is rotated at 2,100 revolutions per minute, $n = \frac{2{,}100}{60} = 35$ revolutions per second. Substituting the values in the formula gives $E = \frac{2\,N\,S\,n}{10^8}$ $= \frac{2 \times 3{,}000{,}000 \times 8 \times 35}{100{,}000{,}000} = 16.8$ volts, or the difference of potential between the brushes $+B$ and $-B$ on open circuit. The difference of potential between the brushes when the external circuit is closed is somewhat smaller than when no current is flowing; because, as in the case of the voltaic cell, a part of the total E. M. F. developed is required to overcome the internal resistance of the coil itself.

The formula $E = \frac{2\,N\,S\,n}{10^8}$ holds equally true for the drum core, Fig. 30. In both cases, the number of outside wires through which the current flows in *series* is equal to one-half the total number of outside wires. Hence, by using the same magnetic field and rotating the cores at equal speeds, the E. M. F. generated in both cases will be equal.

24. The foregoing articles demonstrate the elementary principles and physical theory of a *dynamo*. A **dynamo,** therefore, is a machine for converting mechanical energy into electrical energy by electromagnetic induction. It has three essential features, viz.: (1) a magnetic field; (2) a conductor, or several conductors, called an **armature,** in which the electromotive force is generated by some movement relative to the lines of force in the magnetic field; and (3) a *commutator*, or a *collector*, from which the current is collected by two or more conducting brushes.

In all dynamos, the magnetic field is produced either by a permanent magnet or by an electromagnet, and they are classified accordingly; for present purposes, however, it is sufficient to consider only the uniform magnetic field lying between the poles of some large magnet. In the preceding article, the armature core and commutator were assumed to be fastened rigidly to a shaft and the shaft supported by suitable bearings in such a position that the core would rotate in the magnetic field with its axis of rotation at right angles to the lines of force. The shaft with core and commutator was assumed to be rotated by some exterior mechanical power. The armature conductors were wound directly upon the core and rotated with it. If it were not for mechanical considerations, however, only the armature conductors would need to be rotated; the core could remain stationary.

ARMATURE REACTIONS.

25. When the current is flowing through the armature conductors, it produces several effects upon the magnetic field; and the field, in return, reacts upon the current. These effects will be considered before describing the typical forms of dynamos.

Consider the case of a single conductor in which a current is flowing from a voltaic battery or a continuous-current dynamo, and a magnet. It has been shown that a magnet and a conductor conveying an electric current exert a mutual force upon each other; or, in other words, each tends to produce motion in the other. In the case of a compass placed over or under a conductor conveying a current, if the magnetic needle be held rigidly and the conductor be allowed to swing freely in a horizontal plane, it would tend to place itself at right angles to the length of the needle. In general, *when a conductor conveying an electric current is placed in a magnetic field, the conductor will tend to move in a definite direction and with a certain force, depending* *upon the strength and direction of the current, and upon the direction and density of the lines of force in that field.*

Imagine that a conductor conveying an electric current is placed across a uniform magnetic field, and that it lies in a position at right angles to the lines of force. For example, the diagram in Fig. 31 represents a cross-sectional view of a uniform magnetic field, the dots representing the ends of the lines of force and the heavy line a conductor conveying a current. The direction of the lines of force is assumed to be downwards, that is, piercing the paper; or, in other words, the observer is looking along the lines of force toward the face of a south magnetic pole. The lines of force along the conductor from the top to the bottom of the magnetic field act upon the current in the conductor with equal intensities, and all tend to move the conductor in the same direction. This action, if the magnetic field is uniform, is similar to that of a uniformly distributed load upon a beam tending to move or bend it.

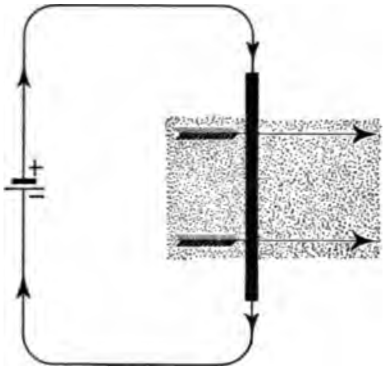


FIG. 31.

The motion imparted to the conductor is perpendicular to the lines of force, and also perpendicular to the flow of current in the conductor. To fulfil these conditions, therefore, the conductor in Fig. 31 must tend to move bodily either to the right or left across the field; in which of these two directions it moves depends upon the relative direction of the lines of force with the direction of the current in the conductor. In this case, if the direction of the lines is downwards, piercing the paper, and the current flows *from* the top *to* to the bottom of the diagram, as indicated by the small arrowheads, the conductor will tend to move from the left to the right in the direction in which the two large arrows are pointing. If the direction of the lines of force only is changed, the conductor will tend to move in the opposite direction, i. e., from the right to the left; or, if the direction of the current in the conductor only is reversed, the conductor will tend to move also from right to left across the field. But should both the direction of the lines of force and the direction of the current in the conductor be changed, the conductor would still tend to move from left to right.

26. There is a convenient thumb-and-finger rule for remembering the direction of motion imparted to a conductor conveying an electric current when placed in a magnetic field; it is similar to the rule for generated currents, with the exception that the *left hand* is used instead of the *right.*

Rule.—*Place thumb, forefinger, and middle finger of the left hand each at right angles to the other two; if the forefinger points in the direction of the lines of force and the middle finger points in the direction toward which the current flows, then the thumb will point in the direction of movement imparted to the conductor.*

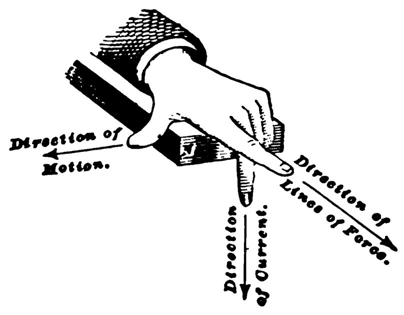


FIG. 32.

For example, in Fig. 32, if a vertical conductor in which a current is flowing downwards is placed in front of the north pole N of a magnet, it will tend to move in the direction as indicated by the thumb.

27. Comparing the rule in Art. **8** with that given above, it will be seen that the two appear to oppose each other; or, in other words, the current which flows in the former case, according to the latter rule tends to oppose the motion of the conductor and move it in the opposite direction. This is exactly what takes place. When a conductor is moved across lines of force, an electromotive force is generated which tends to send a current in a definite direction; if the circuit is open and no current flows, it requires no force to move the conductor across the field; but if the circuit is closed and a current flows through the conductor, then the action of the lines of force on the current opposes the original motion and tends to stop or retard the conductor. The opposing force is proportional to the strength of current flowing in the conductor; that is, if a current of 10 amperes acts with a certain force, a current of 20 amperes will act with twice that force, and so forth. Hence, the stronger the current in the conductor, the greater will be the force necessary to keep the conductor moving in the original direction. The above explanation will be made clearer by the graphical illustration in Fig. 33. The diagram represents a cross-sectional view of a magnetic field, the direction of the lines of force being downwards, piercing the paper. If the conductor *c c'* be moved across the field by some exterior motive power in the direction indicated by the arrows *a*, *a*, a current will flow through the circuit in the direction indicated by the small arrowheads. The length of the arrows *a*, *a* may also serve to represent the magnitude of the force that moves the conductor. As the current flows through the conductor, the lines of force immediately react upon it, producing a *counter* force which tends to stop the conductor and move it in the direction indicated by the arrows *b*, *b*. The counter force would never actually move the wire in the direction of the arrows *b*, *b*, but it exerts a dragging effect

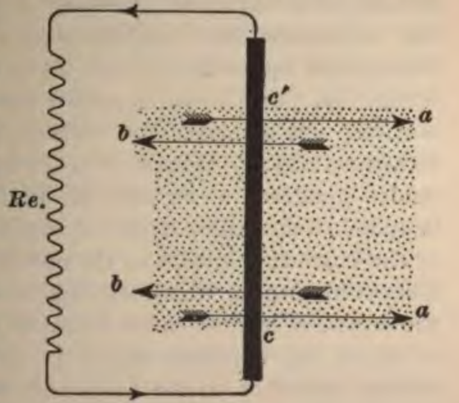


FIG. 33.

upon the conductor, which would reduce its speed and almost stop its motion if the exterior motive force were not increased. So long as the conductor is moved, the applied motive force is always larger than the counter force, as graphically represented by the relative lengths of the arrows.

28. The above principle explains the action of converting the mechanical energy into electrical energy in the dynamo. For example, suppose that an armature is rotated at a constant speed in a magnetic field by some exterior motive force, as, for instance, by a belt from an engine. If the armature is properly wound and connected to a commutator, an electromotive force is generated in the outside conductors on the core, causing a difference of potential between the brushes. If the brushes are not connected to an external circuit and no current is flowing through the armature, it requires no energy to rotate the armature, excepting a small amount to overcome the friction of the shaft in the bearings and the loss in the armature iron by eddy currents. By connecting the brushes to an external circuit, however, and allowing a current to flow through the armature, the conditions are altered. The lines of force react upon the current in the conductors, tending to rotate the core in an opposite direction and to retard its motion; the stronger the current, the greater will be the retarding effect. Hence, in order to keep the speed constant and to generate a constant E. M. F., more energy must be supplied to the pulley from the engine. This retarding effect of the current is known as the **counter torque** of a dynamo. The word torque, which will appear later in connection with the action of motors, means simply *turning* force.

It can be mathematically proved that the mechanical energy delivered to the armature from any exterior source is exactly equal to the electrical energy obtained from the armature plus the energy lost in mechanical friction, eddy currents in the iron, and other small losses, which will be described subsequently.

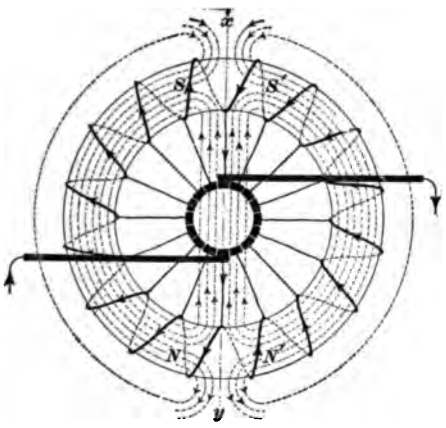


FIG. 34.

29. Besides producing a counter torque in the armature, the current tends to distort or crowd the lines of force from their original position in the magnetic field. This effect is termed **armature reaction,** and will be understood by investigating the magnetic effects of the current in the armature when the armature is removed from between the poles of the field magnets. In the diagram, Fig. 34, the current is flowing through the armature coil in the same direction as represented in Fig. 28. The current circulating around the armature coil in two directions acts as a magnetizing force upon the core and produces two electromagnets. According to the rule for magnetic polarity, the two magnets thus formed oppose each other at the two neutral spaces of the armature; that is, their like poles N, N' and S, S' tend to act in opposite directions at the neutral spaces. As previously explained, lines of force can never intersect each other, and will always produce consequent poles when acting in opposite directions at one place. Therefore, in this case, two consequent poles are formed in the core, one at each neutral space, as shown in the diagram. The polarity of the consequent poles, of course, depends upon the direction in which the coil is wound upon the core and the direction in which the current is generated. The same action occurs when the armature is rotated between the poles of a magnet and a current flows through the coil, although the conditions are somewhat altered. The lines of force from the magnet tend to pass through the core nearly at right angles to those produced by the current. The lines can never intersect, however, and they crowd and distort one another in order coincide in direction. The lines that pass out from t north pole of the magnet tend to enter the core at the sou consequent pole and to pass out from the core at the no consequent pole. At the same time, the south conseque pole is shifted toward the north pole of the magnet, a the north consequent pole toward the south pole of t magnet. The diagram in Fig. 35 represents the manner which the magnetic field is distorted by the reaction of t armature current. In the case where the armature w

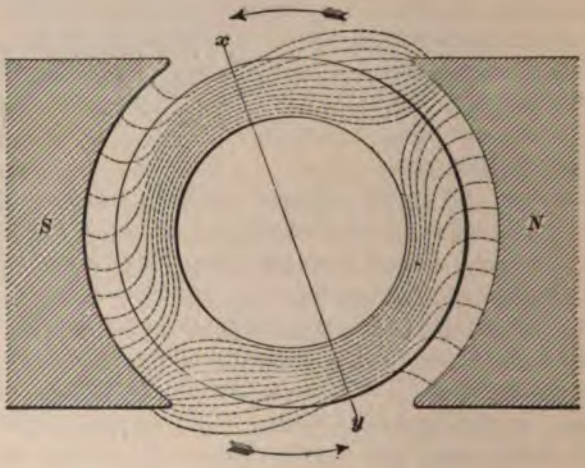


FIG. 35.

removed from the magnetic field, the consequent poles coincided with the neutral space; but when the armature is replaced, as in the diagram, the consequent poles are shifted backwards against the direction of rotation, and the neutral spaces are moved forwards in the opposite direction, as indicated by the imaginary diameter $x\,y$. As the positions of the neutral points on the commutator depend upon the positions of the neutral spaces on the core, they are also shifted forwards in the direction of rotation when the current flows through the armature; hence, the brushes must be moved forwards in order to obtain the full E. M. F. generated in the coil. The stronger the current, the farther forwards the brushes should be shifted.

30. From the fact that in all dynamos of this character the relation of the lines of force, direction of rotation, and direction of current are constant, *the neutral spaces are always shifted forwards in the direction of rotation when the current becomes stronger*, no matter how the coil is wound upon the armature or in which direction the lines of force pass through the core.

These armature reactions are not confined entirely to the ring core, but are produced with the same effects in a drum-core armature, such as represented in Fig. 30. If the direction of the current is traced by the arrowheads upon the conductors, it will be seen that the current is flowing upwards along the face of the core in front of the north pole, as represented by the open circles, Fig. 36, and downwards in front of the south pole, as represented by the solid circles. The lines of force surrounding each conductor in which the current is flowing coincide with those around the adjacent conductors, forming a large number of long lines which pass through the core and produce consequent poles at the neutral spaces, as shown in Fig. 36. The direction of the lines of force around the conductors in which the current is flowing downwards corresponds with the movements of the

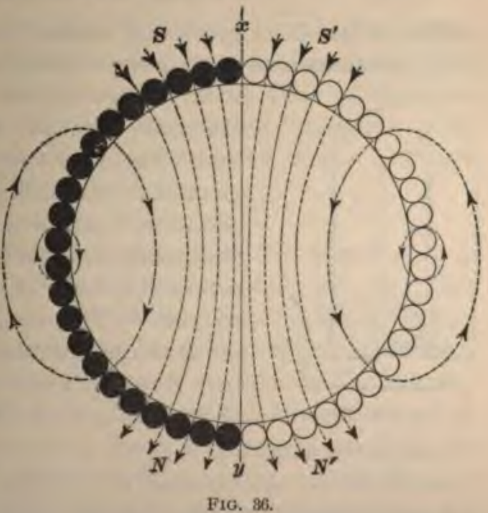


FIG. 36.

hands of a watch, while the direction of the lines around the other conductors is opposite. The lines from all conductors, however, coincide in direction in passing through the center of the core. When the armature is rotated between the poles of a magnet, the field is distorted and the neutral spaces shifted forwards in a manner similar to that described for the ring core.

31. Armature reactions not only distort the magnetic field, but also have a tendency to reduce the total number of lines of force from the magnet, and thereby diminish the E. M. F. generated in the armature. This effect, however, can be almost entirely eliminated by increasing the strength of the field, or, in other words, by increasing the number of lines of force passing through the core. This fact leads to the consideration of *field magnets.*

FIELD MAGNETS.

32. It has been previously stated that the magnetic field in all dynamos is produced from either a permanent magnet or an electromagnet. A dynamo of the first class is called a **magneto-machine.** Such machines are necessarily small on account of the difficulty of making large permanent magnets; in fact, the field in most magneto-machines is produced by several permanent magnets placed side by side. The magnets are usually of the U-shaped pattern, of hard steel, and with a recess bored out between the ends of the poles to admit the armature, as shown in the diagram, Fig. 37.

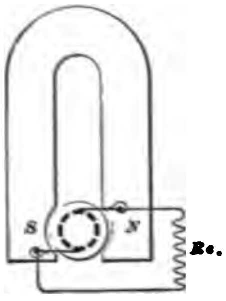


FIG. 37.

As the majority of magneto-machines are made for testing and signaling purposes where alternating currents can be used to advantage, the armature is wound with one large coil of wire, and the two ends of the coil are connected to two separate collector rings, as shown in Fig. 38. The alternating current is obtained from two brushes, one rubbing against each collector ring. The brushes can bear upon the collector ring at any position relative to the coil and the field magnets, since all parts of one collector ring are at the same potential in any instant. By comparing this coil with that in Fig. 13, it will be seen that the current obtained from the two brushes flows in two directions during every revolution.

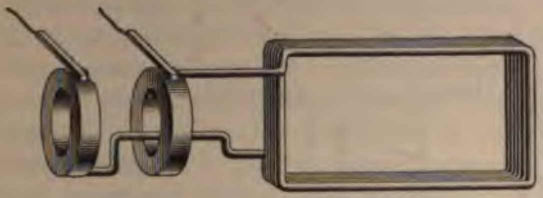


FIG. 38.

33. In nearly all dynamos furnishing current for lamps, power, and other commercial purposes, the magnetic field is produced by an electromognet. This class of dynamos is divided into various types, depending upon the manner in which the current is obtained to excite the field magnets.

34. The first class of machines to be considered is termed a **separately excited dynamo,** from the fact that its field magnets are excited or magnetized by a current from some external source, as, for instance, a voltaic battery, or another continuous-current dynamo. The connections of a separately excited dynamo are represented in Fig. 39. The magnetizing coils are wound around the cores of a magnet and connected to the terminals of a voltaic battery *B*. The exciting current flows from the battery around the cores of the field magnet in such a direction as to produce a closed magnetic circuit through the armature, and has no connection whatever with the current obtained from the brushes by rotating the armature. If the strength of the exciting current is not changed, the difference of potential between the brushes of the dynamo when the armature is rotated at a uniform speed remains constant so long as the external circuit is open; but when the external circuit is closed, the difference of potential gradually diminishes as the strength of current increases, owing to the internal resistance of the armature conductors and the reactions of the armature current on the field.

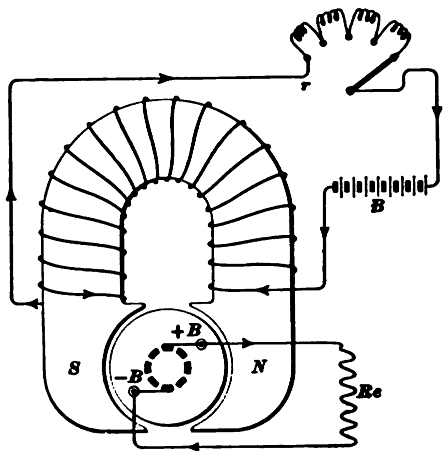
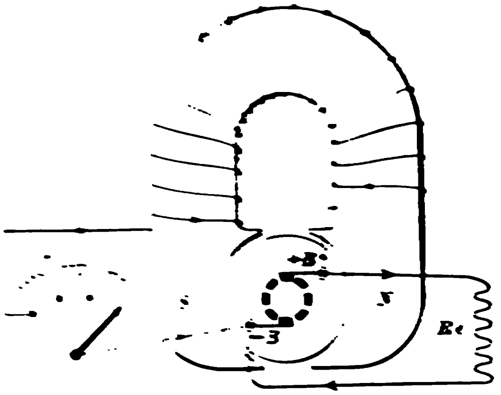


FIG. 39.

35. The **magnetizing force** is that which produces the lines of force in the magnet. Its strength is proportional to the strength of current flowing and to the number of coils or complete turns around which the current circulates. The total number of turns multiplied by the strength of the current in amperes will give the magnetizing force in **ampere-turns.** It has been proved that 10 amperes circulating around 20 turns exert precisely the same magnetizing force as 1 ampere circulating around 200 turns, or as 200 amperes circulating around 1 turn. In each of these cases, the magnetizing force is 200 *ampere-turns.* But the number of lines of force produced in an electromagnet is not directly proportional to the magnetizing force in *ampere-turns.* The strength of the magnet in lines of force depends upon the permeability of the magnetic substances used in the core. The permeability varies greatly in different magnetic substances, depending upon both the physical condition and the chemical composition of the substance. In general, *wrought iron*, *soft sheet iron*, and *steel* have greater permeability than cast iron, and, whenever available, should be used in field magnets in preference. The permeability, however, of all magnetic substances changes with every stage of magnetization. In all kinds of magnetic substances, the permeability decreases when the magnetism is increased beyond a certain limit. This tendency of the substance to become less permeable is called **magnetic saturation**; that is, the substance becomes *saturated* with lines of force and cannot hold any more. A limit is never reached where actual saturation takes place, but there is a limit beyond which it becomes impracticable to magnetize the substance. The practical saturation in wrought iron, soft sheet iron, and cast steel is when there are between 120,000 and 130,000 lines of force per sq. in. of sectional area of the iron, measured on a plane at right angles to the lines of force in the magnet. In gray cast iron, the practical saturation limit is from 60,000 to 70,000 lines of force per sq. in. Hence, when these limits are exceeded, it requires an enormous increase in the *ampere-turns* to produce a slight change in the number of lines of force in the magnet. In general, however, the field magnets of dynamos are designed with the density of the lines of force below the saturation limits, and it is safe to assume that any change in the strength of the current circulating around the magnetizing coils produces a corresponding change in the number of lines of force passing through the magnetic circuit. Consequently, if the strength of the current in the field coils of a *separately excited* dynamo is increased as the current in the armature becomes stronger, the E. M. F. obtained from the brushes will remain practically constant. This is usually accomplished by inserting an adjustable resistance box, or field rheostat r, in series with the battery and field coils, and decreasing the resistance as the difference of potential between the brushes tends to drop.

[illegible] [illegible] electromagnet [illegible] **self-exciting shunt dynamos** [illegible] simply a **shunt dynamo.** [illegible] current for [illegible] the dynamo itself, the field [illegible] with the external circuit from [illegible] terminals of the magnetizing [illegible] positive brush, and the other to a [illegible] negative brush is [illegible] If the resistance [illegible] it will be seen that [illegible] exists between the terminals



[illegible]

[illegible] the dynamo is generating its [illegible] magnetizing coils of a shunt [illegible] large number of turns of [illegible] the resistance large in comparison [illegible] potential between the field terminals. [illegible] the resistance of the shunt [illegible] large enough to allow not more than about [illegible] current of the dynamo to pass through the field coils [illegible] Ohm's law, the strength of current in amperes circulating around the field coils is equal to the difference of potential in volts between the brushes, divided by the resistance in ohms in the field coil, neglecting the resistance of the rheostat. For example, suppose that the difference of potential between the brushes of a shunt dynamo is 500 volts when a current of 10 amperes is flowing from the armature. If 5% of this current is required to excite the field magnets, the strength of current circulating around the field coils is $10 \times .05 = .5$ ampere; and if E_e is the E. M. F. at the brushes, C_s is the current in the shunt field, and R_s is the resistance of the shunt field, then, according to Ohm's law, $R_s = \frac{E_e}{C_s} = \frac{500}{.5} = 1{,}000$ ohms.

37. When a shunt dynamo is rotated at a constant speed, an appreciable length of time elapses before the armature generates a maximum E. M. F. after the field circuit is closed, and in some cases a self-exciting dynamo will generate no E. M. F. until after it has been once separately excited. The starting of a dynamo to generate an E. M. F. is termed **picking-up,** or **building-up.** If the field circuit of a dynamo is open so that no current flows through the magnetizing coil, the armature will generate no E. M. F. when rotated, provided the field magnets are not permanent magnets; consequently, when the field circuit is closed on a shunt dynamo, no current will flow through the magnetizing coils, because there is no difference of potential between their terminals. But nearly all magnetic substances become permanent magnets in a slight degree after once being magnetized.

This permanent magnetism is called **residual magnetism,** since it *resides* in the metal after the magnetizing force has been removed. In general, soft iron and annealed steel retain only a small amount of magnetism, and in some cases the residual magnetism is imperceptible. Chilled iron and hardened steel retain residual magnetism in large quantities. Artificial or permanent magnets are made by placing a piece of hardened steel in a dense magnetic field or in contact with another magnet. Lodestone is the result of a natural residual magnetism. Iron and its alloys will also become slightly magnetized in the process of refining and working.

From these facts it will be seen that the cases where field magnets do not exhibit some residual magnetism are exceedingly rare. The armature conductors when cutting the lines of force of the residual magnetism generate a small E. M. F., and this E. M. F., in turn, causes a feeble current to circulate around the magnetizing coils when the field circuit is closed. The residual magnetism is, therefore, reenforced by the magnetizing effect of the current, which is followed by an increase in the E. M. F. generated, and that, in turn, by a stronger current in the field. These actions and reactions continue until a limit is reached where the fields become saturated with magnetism, and the number of lines do not increase at such a rapid rate; finally, both the E. M. F. and the current in the field become constant.

38. The difference of potential between the brushes of shunt dynamos gradually decreases as the current from the armature becomes stronger, on account of the internal resistance of the armature conductors and the reactions of the current on the field. The effect is even more marked than in separately excited dynamos, because a decrease in the difference of potential between the brushes causes a corresponding decrease on the field terminals, thereby weakening the current in the magnetizing coils. In order to compensate for the decrease in the E. M. F., a field rheostat *r* of comparatively high resistance is connected in the field circuit, and so adjusted that when no current is flowing in the external circuit only enough current flows through the field to produce the normal difference of potential between the brushes; this normal difference of potential between the brushes is kept constant as the load increases by gradually cutting out, or short-circuiting, the resistance coils of the rheostat.

NOTE.—The word *load* as used above is a common expression for *current* in dynamos generating a constant potential, and the student should become familiar with its use.

39. The third class of machines whose field magnets are excited by an electric current are termed **self-exciting series dynamos,** or simply **series dynamos.** The magnetizing coils of a series dynamo are connected directly in *series* with the external circuit; that is, all the current from the armature circulates around the magnetizing coils and flows through the external circuit. The connections of a *series* dynamo are shown in Fig. 41. The current starts from the positive brush $+B$, circulates around the external circuit Re, from thence through the magnetizing coils back to the negative brush $-B$. The action of a series dynamo differs widely from that of a shunt dynamo. In the first place, no E. M. F. is generated in the armature unless the external circuit is closed and a current flows from the brushes, that is, neglecting the small E. M. F. generated by the residual magnetism. In the second place, the difference of potential between the brushes depends upon the strength of current flowing from the armature. The E. M. F., however, is not directly proportional to the strength of the current unless the internal resistance and reactions of the armature are negligible. Compared with the coils on a shunt dynamo, the magnetizing coils of a series dynamo are made of a few turns of a large conductor. This is necessary, because the coils usually are required to carry the total current from the armature; the conductor is made large to carry the current without heating, and only a few turns are used to secure the proper degree of magnetization, since that is proportional to the *ampere-turns.*

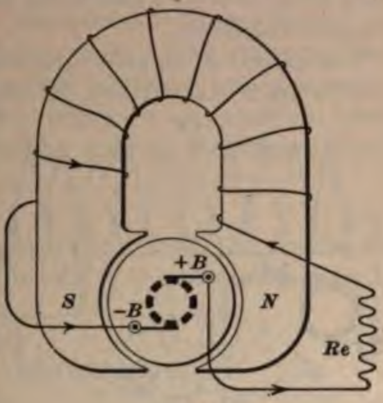
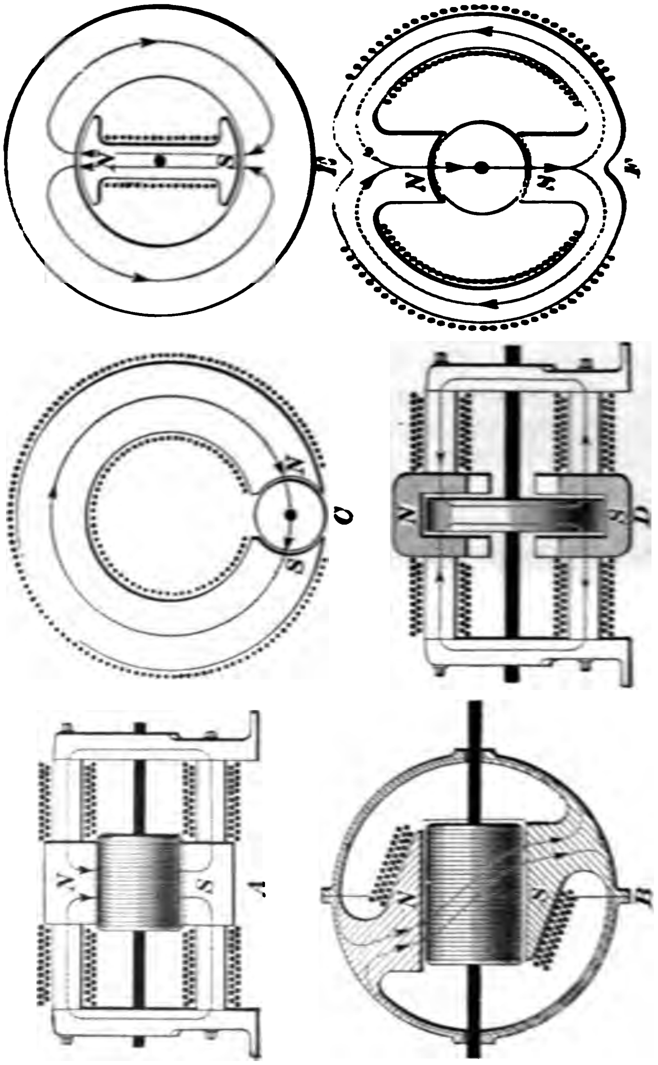


FIG. 41.

40. The E. M. F. of a series dynamo may be regulated in three different ways, viz.: (1) By controlling the strength the shunt current being reenforced by the series current. When the dynamo is not loaded, that is, when no current is



flowing in the external circuit, and the armature is rotated at normal speed, the normal E. M. F. is generated in the armature due to the magnetic field produced by the shunt coils alone. Upon closing the external circuit, however, the difference of potential between the brushes *tends* to

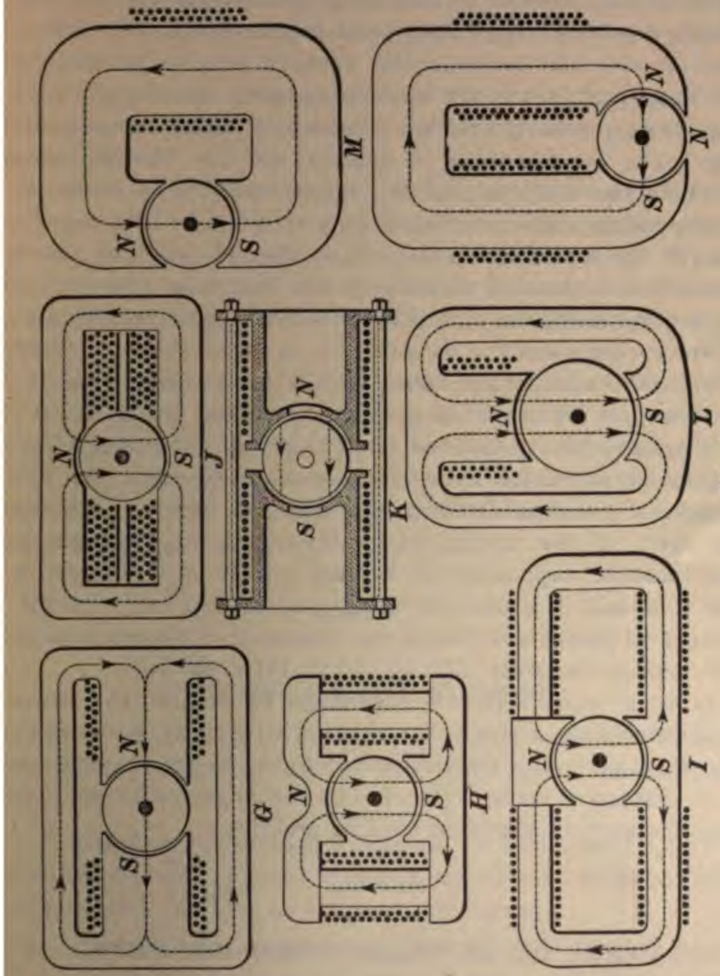


FIG. 45.

decrease, and would continue to decrease, as previously described in a simple shunt machine, if the series coils were neglected. The current circulating through these, however, reenforces the magnetizing force of the shunt coils, and immediately increases the number of lines of force in the field, which, in turn, raise the difference of potential between the brushes to normal. These actions are produced simultaneously, and, to all appearances, the difference of potential between the brushes remains normal for all changes of load in the external circuit. This method of regulating the E. M. F. of a dynamo is called **compounding.** The **terminals** of a dynamo are the binding posts to which the external circuit is connected; in a series, or compound, dynamo one terminal is attached to the outside end of the series coils, as $-T$ in Fig. 44, and the other terminal is connected directly to the brush, as represented by $-T$ in the figure. It is desirable in a great many cases to **over-compound** a dynamo, or, in other words, to wind a sufficient number of turns on the series coils so as to increase the difference of potential between the terminals of a dynamo above normal when the load increases. The expression **per cent. over-compound** means that the difference of potential between the terminals increases a given per cent. of the normal when the load is at a maximum. For example, supposing the normal voltage of a dynamo is 500 volts and it is 10% over-compound at full load; the difference of potential between the terminals of the machine at full load is, therefore, $500 + (500 \times .10) = 550$ volts.

In some cases it is an advantage to connect the shunt field outside the series coils; that is, in Fig. 44, to connect the negative end of the shunt coil to the negative terminal $-T$, instead of being connected to the negative brush $-B$. This connection is seldom used in practice.

TYPES OF BIPOLAR FIELD MAGNETS.

43. The various types of field magnets for dynamos in which the armature revolves between only one pair of poles are shown in Fig. 45. It is customary to speak of such machines as **bipolar dynamos**, from the fact that only one pair of poles is presented to the armature. The broken lines and arrowheads in each of the separate cuts represent the paths of the lines of force which must pass lengthwise through the coils from the north pole to the south pole. The black dots indicate a cross-section through the wires which form the coils.

Field poles are distinguished as follows with respect to the coils producing them: (*a*) **salient poles**; (*b*) **consequent poles.**

In all cases where a single coil is used, or where, if two coils are used, they are wound so as to produce unlike poles at their free ends, the poles are called salient poles. When two coils are used and wound so as to make their adjacent poles similar, the resultant poles are called consequent poles.

Referring to Fig. 45, salient poles exist in fields *B*, *C*, *G*, *J*, *K*, *L*, *M*, *N*, and consequent poles in *A*, *D*, *E*, *F*, *H*, *I*. The adjacent coils in *A*, Fig. 45, have their adjacent poles at *N* and *S* similar. Were these poles opposite, the magnetic flux would circulate around the magnets without passing through the armature.

TYPES OF DYNAMOS.

44. Dynamos are divided into three general types, depending on the character of their currents. These three types are:

1. **Constant-potential dynamos,** in which the E. M. F. remains constant and the strength of current (continuous) changes with the load or external resistance.

2. **Constant-current dynamos,** in which the strength of current (continuous and pulsating) remains constant and the E. M. F. changes with the load.

3. **Alternating-current dynamos,** the current from which alternates or reverses direction with great rapidity

and whose E. M. F. is constant. In ordinary alternating-current dynamos, the reversals average generally either 7,200 or 16,000 per minute.

NOTE.—A dynamo which generates current for power purposes has been conventionally termed a *generator*, to distinguish it from a machine for lighting.

CONSTANT-POTENTIAL DYNAMOS AND GENERATORS.

45. The foregoing articles have demonstrated the principle and regulation of constant-potential dynamos, but only one form has been considered, namely, a dynamo in which a ring or drum armature is rotated between only one pair of poles from a U-shaped magnet. Theoretically, however, constant-potential dynamos can be built with one armature revolving between any number of pairs of poles, although in practice eight pairs of poles are seldom exceeded, except in machines for large output and slow speed. Machines having more than one pair of poles are called **multipolar dynamos.**

In multipolar dynamos the pole pieces and field cores are fastened into one magnetic yoke, more or less circular in shape, as shown in Fig. 46, which represents the magnetic circuits of a four-pole dynamo. A magnetizing coil is wound upon each field core, and the four coils are connected in series in such a manner that when a current circulates around the coil, it produces first a north pole and then a south pole. The lines of force from each field core divide into two magnetic circuits in

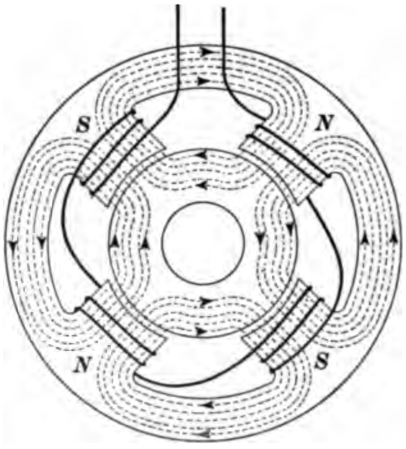


FIG. 46.

the yoke and armature, as represented in the diagram. Their density is practically uniform, however, where they pass from the north pole into the armature core, or from the armature core into the south pole. In nearly all multipolar dynamos this same principle of polarity is applied, that is, every *other* pole is of like polarity, and lines of force from each core divide into two magnetic circuits, in the armature and in the field yoke.

46. The process of generating an E. M. F. is similar to that in bipolar machines, but there are some points which should be understood. Consider first the case of a ring core with a closed-coil winding as shown in the diagram, Fig. 47. If the armature is rotated in the direction of the large arrow, the E. M. F. generated in the conductors in front of the south poles will tend to act downwards along the face of the pole, while that generated in front of the north pole will

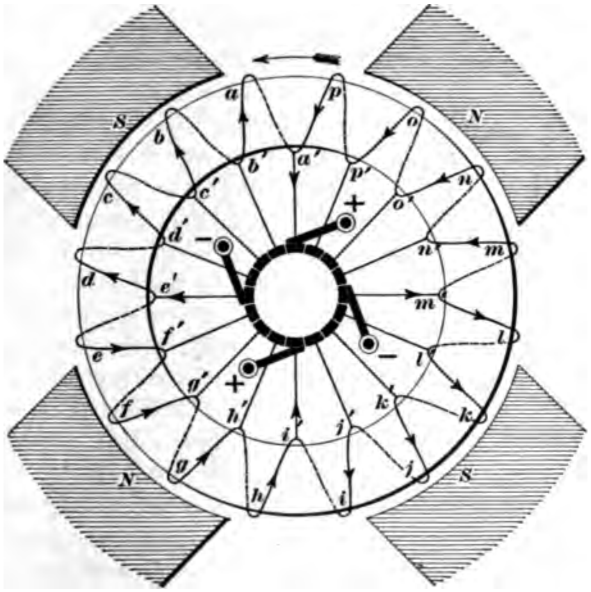


FIG. 47.

nd to act upwards. By tracing out, by aid of the small rrowheads on the conductors, the direction in which the E. M. F. acts, it will be seen that there are four points where the E. M. F. acts in opposite directions. The action of the electromotive forces is to meet at a' and i' and to divide at e' and m'. The segments connected to a' and i' have the same potential and form two *positive* neutral points of the commutator; the segments connected to e' and m' have the same potential and form two neutral points of the commutator. Hence, four brushes are necessary—two positive and two negative. The current is obtained from the armature by connecting the two positive brushes in parallel to one terminal of the external circuit and the two negative brushes to the other terminal, as shown in Fig. 48. The currents from the positive brushes unite to form the current in the external circuit and divide again between the negative brushes. The current in the armature is divided into four circuits in parallel instead of two, as in bipolar dynamos, and the maximum E. M. F. that is obtainable from the brushes is equal to that generated by the active conductors in one of the circuits only. For example, the difference of potential between the positive and negative brushes in Fig. 47, when no current is flowing, is equal to the E. M. F. generated in one-quarter of the outside wires on the core; or, in other words, the total E. M. F. of the armature is proportional to the number of outside wires connected in series between brushes of opposite polarity.

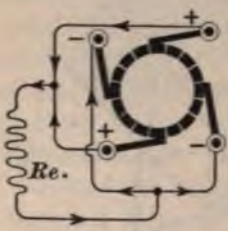


FIG. 48.

The current in a ring armature wound and connected in this manner, if placed in a field magnet of six poles, would divide into six circuits in parallel; if the armature is placed in a field magnet of eight poles, the current would divide into eight circuits in parallel, and so on. An armature winding of this character is called a **parallel,** or **multiple, inding,** since the current divides into as many circuits in allel as there are poles in the field magnet.

47. It is possible, however, to connect and group the conductors in an armature for a multipolar dynamo so that

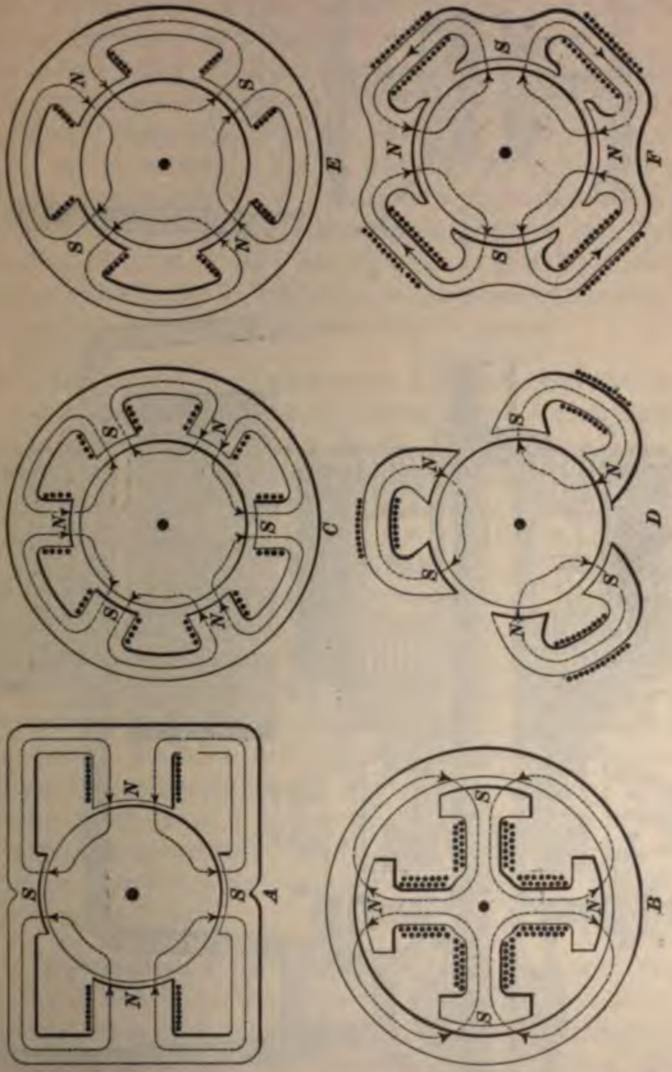


FIG. 49.

the current divides into two circuits only, making the number of active conductors in series equal to one-half the total

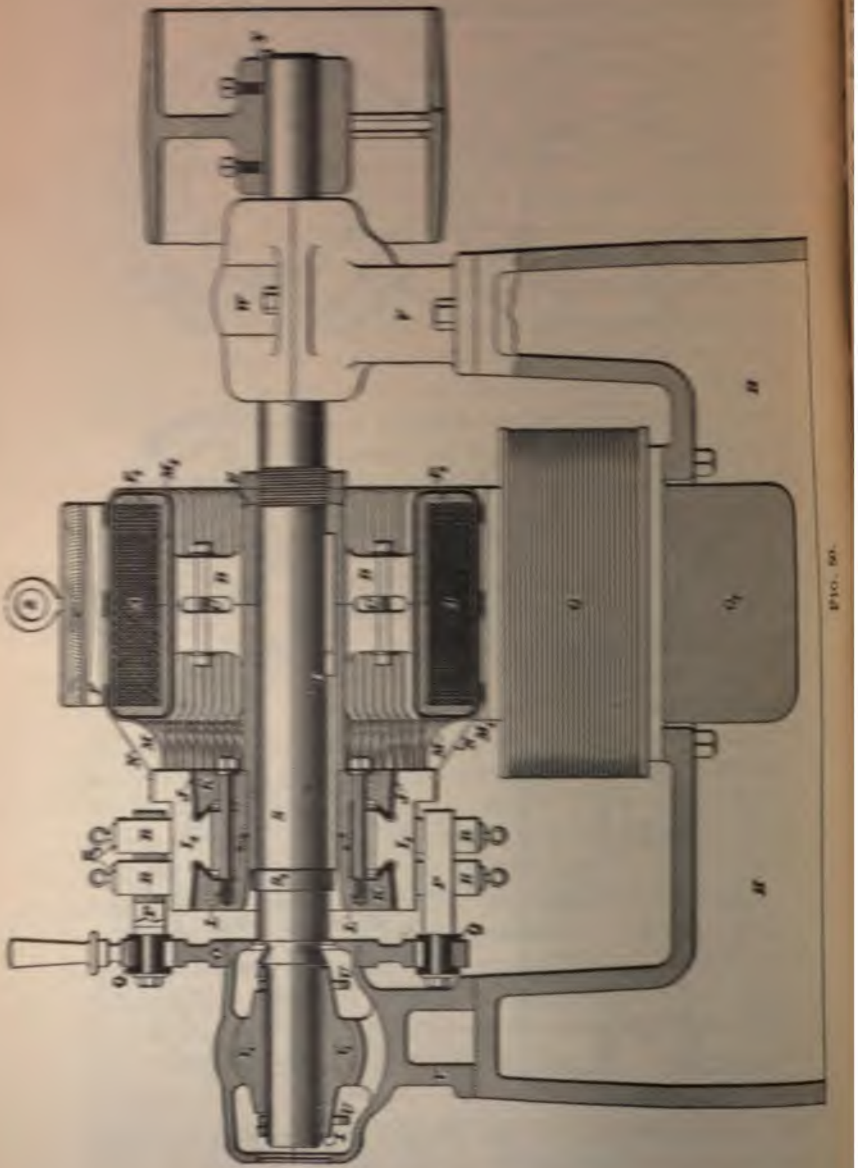


FIG. 30.

number of outside wires on the core. This armature winding is termed a *series winding*, since one-half of the total outside wires is the largest possible number that can be connected in series and produce a continuous current.

There are many different methods of connecting and winding armatures for generating a continuous current, the method used depending upon the character of the current and E. M. F. desired. Drum windings as well as ring windings are connected in a variety of ways for multipolar and bipolar dynamos, but the principle of commutation and generation of E. M. F. does not differ from that previously described; the E. M. F. is always proportional to the number of outside or active wires connected in series.

48. The regulation of multipolar dynamos for constant potential is accomplished by the changing of the strength of the magnetizing force, as in the bipolar machines. In a compound dynamo, the series coils are wound on each field core and all connected together in parallel or series, whichever is more expedient.

49. Types of Multipolar Field Magnets.—The various types of multipolar field magnets are shown in Fig. 49. Consequent and salient poles are used as in bipolar field magnets, but the type generally employed has salient poles alone, as in *C* and *E*. *A* embodies both consequent and salient poles. In *B* the field magnet is surrounded by the armature and is known as an *internal-pole* dynamo. The armature in all cases is that part of a dynamo in which the current is generated. Each type of field magnet in the above figure has its own special advantages, but all represent good design.

50. Mechanical Construction.—Heretofore, only the principles of a dynamo have been considered; its mechanical construction in detail depends upon the requirements of the machine and upon the originality and taste of the designer. A few general remarks, however, on the construction of the principal parts of the machine are necessary to give the student a clear conception of a complete dynamo ready for operating.

The mechanical construction of a typical bipolar dynamo is shown in Fig. 30, which is a vertical section taken along the center of the armature shaft. The parts of the machine shown in the figure are lettered, and the names of the parts corresponding to the letters are as follows:

A = Armature core, which may be either punchings from sheet iron or built up of fine annealed iron wire.
B = Armature spider for connecting core to shaft.
C = Armature spider bolts.
D = Armature key for fastening spider to shaft.
E = Armature locknut.
F = Pole piece.
G_1 = Magnetic yoke.
G = Magnetizing or field coil.
H = Frame.
I_1 = Commutator bars or segments.
J = Commutator insulation.
K = Commutator shell or body, and rings for holding commutator segments in place.
L = Bolt for clamping commutator frame.
M = Armature leads, connecting armature winding to commutator.
N = Armature dressing or covering.
O = Rocker-arm or brush-holder yoke.
P = Brush holder.
Q = Insulating bushings.
R = Carbon brushes.
R_1 = Carbon-brush hammers.
S = Shaft.
T = Bearing or brass.
U = Oil rings.
V = Standard.
W = Cap for standard.
Complete outfit called pillow-block.
X = Pulley.
Y = Key for pulley.
Z = Eyebolt.

51. Frame.—The frame is made up of two castings; the upper one forms the magnetic yoke *G*, and pole pieces *F*, and is bolted to the lower one *H*, which forms the base and is extended on either side to support the standards *V*, *V*. The pole pieces are bored out to admit the armature core when wound; the standards are bolted to the base casting and are so adjusted as to allow the armature core to revolve centrally between the pole pieces. The magnetizing or field coils *G*, only one of which is shown in this cut, are wound on separate bobbins or spools, and one is slipped over each pole piece.

52. Armature.—As generally used, the word **armature** includes the wound core and commutator mounted on the shaft ready for operating. In Fig. 50, the armature spider *B* is made in halves; each half is provided with flanges F_1 at the ends to hold the disks or sheets of iron *A* in place. The disks are punched in circular rings from thin sheet iron annealed, and a large number are slipped over each half of the spider, which is then bolted together by long spider bolts *C* as shown. The spider usually has three or four arms joining the flanges to the hub, the armature conductors on the inside of the ring, in case of ring winding, being wound between the arms. The hub of the spider is bored out to slip over a portion of the shaft *S*; it rests against a turned shoulder S_1, and is held in this position by the armature nut *E*. The spider and core are made to revolve with the shaft by the aid of a key or feather *D*, fitted into the spider hub and into the shaft. The core and spider are insulated by mica, cloth, paper, etc., M_1, and the armature conductors are wound on them in the manner previously described, with armature leads properly connected to the winding at suitable places. After the core has been wound and the leads connected to the commutator, the winding is sometimes covered or dressed with cloth of suitable texture to prevent flying particles and dust injuring or short-circuiting the coils. The armature leads should be made of a flexible conductor or cable, insulated from one another with cotton or rubber tape; an electrical contact of two leads will short-circuit and burn out the intervening coil. It is sometimes the practice to use the armature conductors themselves for leads by looping the conductor and connecting the end of the loop to the commutator. This is bad practice, however, and, except for small dynamos, ought not to be followed. A large, solid copper wire is liable to become crystallized by the repeated vibration of the machine and will eventually give way.

53. Commutator.—Every maker of dynamos has a special design of commutator, but all embody the same general construction. Fig. 51 shows two enlarged views of a commutator such as is shown in place in Fig. 50. It will be noticed that the segments are broader on the outside of the commutator than near the center, thus providing for an equal thickness of insulation between all parts of adjacent bars. A portion a of each segment projects above the general level of the commutator surface, and is provided with a slot into which the armature leads are securely fastened by screws s, s, as shown at l. Sometimes the leads are soldered to the segments. The method of clamping and securely holding the segments is shown in the lower view. The commutator shell, it will be seen, consists of two rings $c\,c$ and $c_1\,c_1$, clamped together by bolts b, b. The notches n in the segments fit over corresponding projections on the rings, and as the bolts are tightened the segments are drawn firmly against the insulation which separates them. The commutator shell is usually made of brass, sometimes of cast iron. This shell is, of course, thoroughly insulated from the commutator segments. A key is fitted into the commutator shell and shaft to cause the commutator to turn with the shaft. The armature leads from the winding are soldered or screwed to ears or clips extending from each commutator bar, as shown by the cross-sectional view.

54. Brushes and Brush Holders.—In the cut of the machine in Fig. 50, the brushes shown are made of carbon and rub against the segments of the commutator radially, the pressure being regulated by a spring which is attached to a hammer pressing on top of the carbons. The carbons slide in slots in the brush holders, fitting snugly, with but little play or lost motion sideways. Both brush holders are provided with studs which pass through holes in the rocker-arm, each stud being insulated from the arm by insulating bushings, as shown in the cross-sectional view. The rocker-arm is fitted over the journal-box, and can be rocked or rotated to change the position of the brushes on the commutator as the position of the neutral points changes when

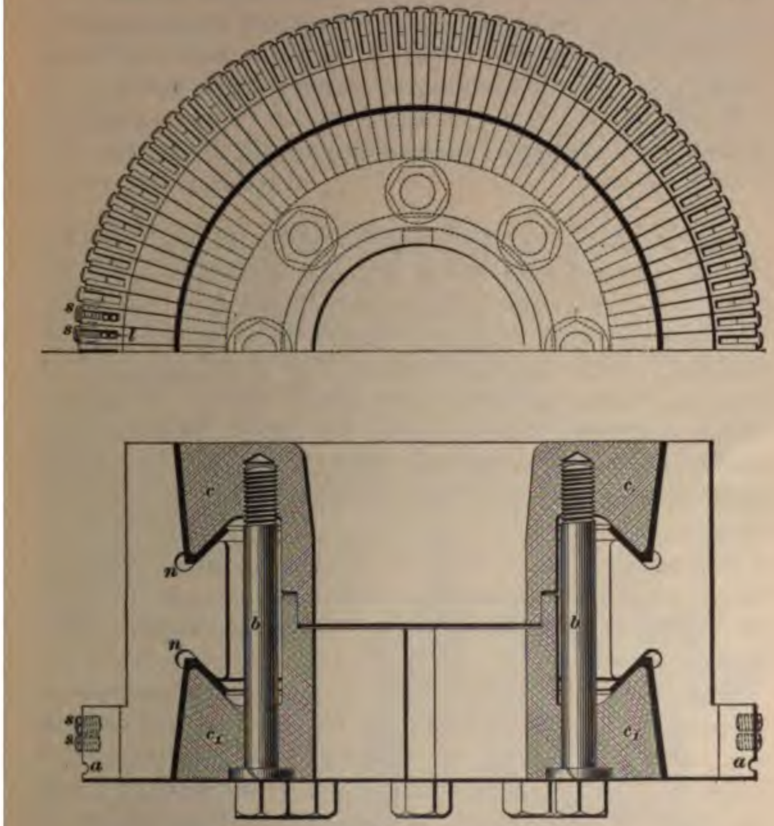


FIG. 51.

the load is varied. This action is usually accomplished by a handle attached to the rocker-arm; and a thumbscrew or setscrew is provided to hold the rocker-arm in position when properly adjusted. The current is taken from the brushes by a cable or flexible conductor connected to the brush holder, generally by the use of a small cable clip surrounding the stud. On some kinds of dynamos it is customary to use copper brushes; that is, brushes made either of copper leaves, strips, wires, or gauze. Such brushes are built in a great variety of ways, and on constant-potential machines are generally used where the E. M. F. does not exceed 125 volts.

55. Journals or Bearings.—The armatures of most dynamos are generally driven at a high speed compared with the average rotating machinery, and hence it is important that the journals or bearings should be of the best design possible. In the dynamo shown in Fig. 50 the bearings are called **self-alining** boxes; that is, the linings are allowed to find their own alinement with the shaft. This is accomplished by turning a spherical surface I'_1 around the center of the lining and turning the cap and standard to match, as shown in the cross-sectional view. The linings I in such a bearing are usually made of some composition metal, as bronze or gun-metal, for small machines; on large machines the linings are made of cast iron covered on the inside with babbitt metal.

The best practice in lubricating high-speed journals in dynamos is to make the bearings *self-oiling* or *self-lubricating*; that is, to design the bearings with a reservoir of oil below the journal, using some device to carry the oil from the reservoirs to the top of the journal, from whence it flows around the journals and drops back into the reservoirs again. This method produces a constant circulation of oil around the journals and allows the oil to be used over and over again.

A good method of automatically oiling or lubricating bearings on journals is shown in the cross-sectional view in

Fig. 50. Two slots are cut across the top of each lining, permitting two circular *oil rings U* to rest upon the journals of the shaft; the diameters of the rings are made large in comparison with the diameter of the shaft, and their lower parts dip into the reservoirs of oil. When the shaft is rotated, the friction between it and the inside of the oil rings causes the latter to revolve, thus carrying the oil which adheres to the bottom part of the rings to the top of the journal, where it finds its way between the linings and the shaft.

In general, any freely lubricated journals can be used in dynamos or generators.

56. Driving Mechanism.—The armatures of nearly all dynamos are driven in one of the following ways: (1) By using a flat belt passing over a pulley on the armature shaft. (2) By using several ropes, side by side, running in a grooved pulley. (3) By connecting the armature directly to the crank-shaft or shaft of the driving machine, which, in most cases, consists of a steam engine, steam turbine, or water-wheel. In any of the above methods, the driving mechanism should be amply capable of transmitting the total output of the dynamo with a suitable factor of safety.

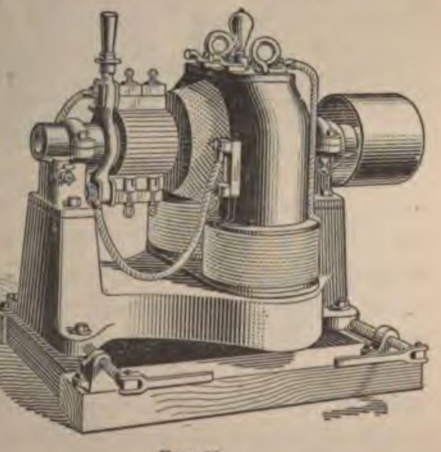


FIG. 52.

57. A perspective view of the bipolar dynamo just described is shown in Fig. 52. In the figure the machine is represented as ready for operating, and is mounted upon sliding rails which are attached to the wooden *bedplate*. Two adjusting screws, one on each side of the machine, are used to move the dynamo along the rails, thereby loosening or tightening the belt as the circumstances may require. The current passes from the brush holders through flexible copper cables to two terminals fastened to, but insulated from, the pole pieces; from the terminals the current passes through the series winding on the field or magnetizing coils, and thence to a small connection board on the top of the pole pieces. An incandescent lamp is connected between the main terminals of the connection board, and is used to indicate when the machine is generating its normal [illegible]. A lamp used for this purpose [illegible] called a **pilot lamp.**

[illegible] bipolar dynamo for developing a constant potential and ready for operating is shown in Fig. 53. In

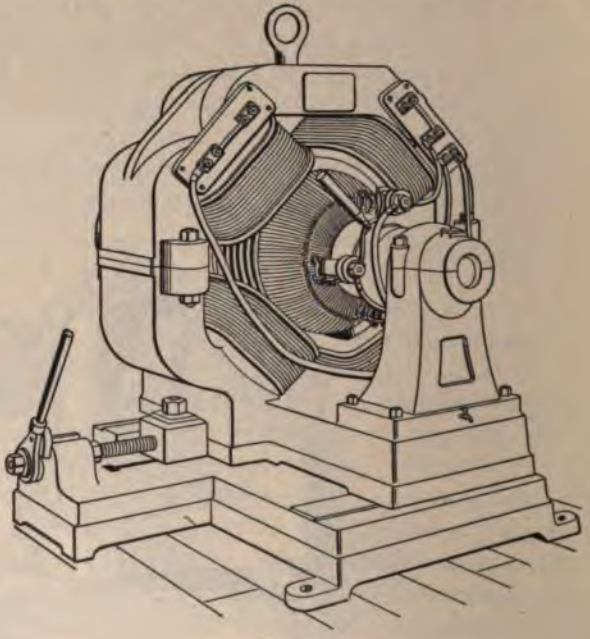


FIG. 53.

this machine the frame is made of two main castings; one consisting of the upper magnetic yoke and two pole pieces, and the other consisting of the lower magnetic yoke and two pole pieces, from which project two extensions for supporting the pillow-blocks. The dynamo slides upon a cast-iron bedplate, and adjustment is made by a screw, as in the case of the bipolar dynamo.

The two dynamos previously described are illustrations taken from actual practice and embody some special features which are not found in other machines of the same character; they were selected, however, on account of their simplicity, to convey to the student a general idea of how electrical prin[illegible] with mechanical construction.

EFFICIENCY OF CONSTANT-POTENTIAL DYNAMOS.

59. As previously stated, a dynamo is a machine for converting or transforming mechanical into electrical energy. In any transformation of energy, the total amount of energy is constant; when energy which is manifested in one form disappears, the same quantity will always appear again in another form or in several other different forms. This action is exactly that which takes place in a dynamo. A certain amount of mechanical energy is delivered to the armature shaft of the dynamo by a belt or some other transmitting device; a large portion of the energy is converted into electrical energy in the armature conductors and is transmitted to the external circuit, while the rest of the energy, usually the smaller portion, is converted directly or indirectly into heat energy in the different parts of the dynamo itself. The amount of energy delivered to the armature shaft is always equal to the energy appearing in the external circuit from the brushes, plus the energy converted into heat in the dynamo itself.

In a dynamo the mechanical energy delivered to the armature shaft is usually called the **input**; the electrical energy appearing in the external circuit from the brushes is called the **output**; and the energy converted into heat directly or indirectly in the dynamo itself is termed **energy losses**, or simply **losses**. This last term is not a strictly true one; for the energy converted into heat in the dynamo is lost only in relation to its utility—it cannot be utilized to an advantage, and if too intense, endangers the life of the machine.

From what has been stated, it will be seen that the *input* of a dynamo is always equal to the *output* at the brushes, plus the *losses* in the machine itself; or, in other words, the losses in the dynamo are equal to the difference between the *input* and the *output*. It is assumed in the above statement that the *input*, *output*, and *losses* are reduced to the same units. For example, suppose that 20 horsepower is delivered to the armature shaft of a dynamo where the *output* from the brushes to the external circuit is 13,428 watts. Reducing the 20 horsepower to watts gives 20 × 746 = 14,920 watts; hence, the losses in the dynamo are equal to the difference between the input of 14,920 watts and the output of 13,428 watts, or 14,920 − 13,428 = 1,492 watts.

60. It is more convenient, however, to express the relation of the *input*, *output*, and *losses* of a dynamo in percentage; that is, the output as well as the losses may be expressed as a certain per cent. of the input. The relation of the input to the output of a dynamo, expressed in percentage, is termed the **efficiency** of the machine.

Let I = the input of a dynamo;
O = the output;
E = the per cent. efficiency.

Then, the per cent. efficiency of a dynamo may be found by the formula

$$\mathsf{E} = \frac{100 \times O}{I}. \qquad (2.)$$

That is, *to find the per cent. efficiency of a dynamo, divide the output in watts by the input in watts and multiply by 100.*

For instance, in the above example, the efficiency, by formula **2**,

$$\mathsf{E} = \frac{100 \times 13,428}{14,920} = 90 \text{ per cent.}$$

61. The relation of the input to the heat losses in a dynamo, expressed in percentage, is termed the **per cent. loss.**

Let L = per cent. loss. Then, the per cent. loss in a dynamo may be found by the formula

$$L = \frac{100(I - O)}{I}. \qquad \textbf{(3.)}$$

That is, *to find the total per cent. loss in a dynamo, divide the difference between the input and the output in watts by the input in watts and multiply by 100.*

EXAMPLE.—(*a*) What is the per cent. efficiency of a dynamo if 10 horsepower is delivered to the armature shaft and the output from the brushes is equivalent to 6,341 watts? (*b*) What is the total per cent. loss in the dynamo when running under these conditions?

SOLUTION.—Reducing the input of 10 H. P. gives 10 × 746 = 7,460 watts input. (*a*) By formula **2**, the efficiency

$$E = \frac{100 \times 6{,}341}{7{,}460} = 85 \text{ per cent. Ans.}$$

(*b*) By formula **3**, the total loss

$$L = \frac{100\ (7{,}460 - 6{,}341)}{7{,}460} = 15 \text{ per cent. Ans.}$$

The efficiency of a dynamo depends upon its character, construction, condition when tested, its capacity (or output), losses, and various other conditions; in fact, two dynamos of the same construction and capacity seldom show exactly the same efficiency. The following list, however, will give the student a general idea of the approximate per cent. efficiencies which should be obtained from constant-potential machines of different capacities, or outputs, under ordinary conditions met with in practice:

From 750 to 1,500 watts output inclusive, about 75% efficiency.

From 3,000 to 5,000 watts output inclusive, about 80% efficiency.

From 7,500 to 10,000 watts output inclusive, about 85% efficiency.

From 15,000 to 100,000 watts output inclusive, about 90% efficiency.

From 150,000 watts output and upwards, from 91 to 93% efficiency.

The method of actually testing a dynamo to find its efficiency and losses is beyond the scope of this section; the above, however, will serve as a guide to the student when computing the necessary power required to drive dynamos of different capacities or outputs.

62. When the output of a dynamo and its corresponding efficiency are given, the input necessary may be found by the formula

$$I = \frac{100 \times O}{E}. \qquad (4.)$$

That is, *the input necessary to drive a dynamo, when its output and efficiency at that output are given, is obtained by dividing the output by the per cent. efficiency and multiplying the quotient by 100.*

Example 1.—The efficiency of a constant-potential dynamo is found to be 85% when giving an output of 6,341 watts; find the input in horsepower necessary to drive its armature shaft under these conditions.

Solution.—By formula 4, the input necessary $I = \frac{100 \times 6,341}{85}$ $= 7,460$ watts. The equivalent of 7,460 watts in horsepower is $\frac{7,460}{746}$ $= 10$ horsepower, which is the power required to drive the armature shaft of the dynamo under the stated conditions. Ans.

When the input of a dynamo and its corresponding efficiency are given, the output may be found by the formula

$$O = \frac{I\,E}{100}. \qquad (5.)$$

That is, *the output of a dynamo, of which the input and the efficiency at that input are given, is obtained by multiplying the input by the per cent. efficiency and dividing by 100.*

Example 2.—An input of 35 horsepower is delivered to the shaft of a dynamo; if its efficiency at that input is 89.5%, find its output in watts.

SOLUTION.—The equivalent of 35 horsepower is 35 × 746 = 26,110 watts. By formula **5**, the output of the dynamo under these conditions,

$$O = \frac{26,110 \times 89.5}{100} = 23,368.45 \text{ watts. Ans.}$$

63. The total loss of power in a dynamo can be separated into smaller losses, depending upon the manner in which the loss is produced and the part of the dynamo in which it occurs. In ordinary cases, all the losses will come under one or more of the following heads:

1. Mechanical-friction loss.
2. Core loss.
3. Field loss.
4. Armature loss.

64. Friction Losses.—The larger part of the loss due to mechanical friction takes place between the bearings and journals. The brushes rubbing on the commutator produce some friction and consequent loss, but the amount is small, unless carbon brushes, under heavy pressure, are used. The per cent. of power lost in mechanical friction necessarily depends upon the construction and condition of the bearings and journals, upon the size of the machine, and, to some extent, on the method of driving the armature shaft. Under ordinary conditions, the loss in mechanical friction should not exceed 5% of the input of dynamos from 1,500 up to about 10,000 watts output, and 3% of the input of dynamos from 15,000 to 100,000 watts output. For example, suppose that a dynamo has an efficiency of 88% at its rated output of 22,000 watts, and a test shows that 2.5% of the input is lost in mechanical friction. The total loss in the machine is 100 − 88 = 12%, of which 2.5% is lost in friction; the remaining 9.5% loss is due to other causes. The total input to the machine, from formula **4**, is $\frac{22000}{88} \times 100 = 25,000$ watts; hence, the power lost in friction is $\frac{25,000 \times 2.5}{100} = 625$ watts.

65. Core Losses.—The **core loss** is the energy converted into heat in the iron disks of the armature core when they are rotated in the magnetic field. A small portion of this loss is due to eddy currents generated in the revolving core disks, as explained in Art. **16**; the larger portion of the loss is due to a *magnetic friction* which occurs whenever the direction of the lines of force is rapidly changed in a magnetic substance. When the magnetism of an electromagnet is rapidly reversed—that is, when the direction of the lines of force is suddenly changed several times in rapid succession by reversing the direction of the magnetizing current—the iron or steel in the core becomes heated, which necessitates a certain amount of energy being expended. This effect is due to a kind of internal *magnetic friction* by reason of which the rapid changes of magnetism cause the iron to grow hot. This effect is called **hysteresis.**

The energy expended by hysteresis is furnished by the force which causes the change in the magnetism, and in the case of an electromagnet where the magnetism is reversed by the magnetizing current being reversed, the energy is supplied by the magnetizing current.

The same effect is produced when the iron of the armature core is rapidly rotated in the constant magnetic field of the dynamo; this case differs from the electromagnet only in the fact that the magnetic lines of force remain at rest and the iron core is made to rotate. Since the core is rotated from the armature shaft, the energy lost in hysteresis is furnished by the force which drives the shaft.

The loss of energy due to hysteresis depends (1) upon the hardness and quality of the magnetic substance in which the magnetic change takes place, (2) upon the amount of metal in which the reversal takes place, (3) upon the number of complete reversals of magnetism per second, and (4) upon the maximum density of the lines of force in the metal. Building the core of iron disks does not affect the hysteretic loss; it only reduces the eddy currents. Hysteretic loss is greatly reduced by using soft annealed iron, which exhibits only slight traces of residual magnetism; for where the residual magnetism is large, the loss due to hysteresis is large in proportion. The hysteretic loss increases in a certain ratio with the magnetic density and the number of reversals per second; hence, these quantities are kept within reasonable limits. In well-designed dynamos the magnetic density in the armature rarely exceeds 85,000 lines of force per square inch, and the maximum number of complete reversals of magnetism in the armature core is about 133 per second. In bipolar dynamos the number of complete reversals of magnetism in the armature is equal to the number of revolutions per second at which the armature shaft is driven; in multipolar machines the number of reversals is equal to the number of revolutions of the armature shaft multiplied by the number of *pairs of poles*. For example, if the armature of a four-pole dynamo is driven at 600 revolutions per minute, or 10 revolutions per second, the number of complete reversals of magnetism in the armature core is 10 × 2 = 20 per second.

In a well-designed dynamo, the core loss, including eddy currents and hysteresis, should not exceed 2% of its input when delivering its rated output from the brushes.

66. Field Losses.—In self-exciting dynamos, a portion of the electrical energy generated in the armature is required to excite the field magnets. This energy is considered as one of the losses of the dynamo, since it does not appear in the external circuit and it is entirely dissipated in the form of heat.

In a series-connected dynamo, where the total current from the armature passes through the magnetizing coils, the power in watts is equal to the square of the current, multiplied by the resistance of the series turns. If, then, C is the total current from the armature, r is the total resistance of the series coils, and W is the watts lost in the series coils, then, $W = C^2 r$. For example, suppose that a series dynamo generates 200 volts between its terminals when a current of 100 amperes is flowing from its brushes through its series coils and through the external circuit. The total output of the dynamo is, then, 100 × 200 = 20,000 watts. If the total resistance of the series coils is .1 ohm, then the number of watts (W) required to excite the field magnets $= C^2 r = 100^2 \times .1 = 100 \times 100 \times .1 = 1{,}000$ watts.

67. In a shunt dynamo which generates a nearly constant potential for limited strengths of current in the armature, the field coils usually consist of a large number of turns of fine wire, offering a high resistance compared with the field coils of a series dynamo. The two terminals or ends of the shunt field coils are connected to the positive and negative brushes, respectively, of the dynamo in parallel with the external circuit, thereby allowing the full potential of the dynamo to act against the resistance of the coils. Then, from Ohm's law, the current in the shunt coil is equal to the electromotive force of the brushes divided by the resistance of the coils. Let E_e represent the difference of potential between the brushes of the dynamo when running at normal speed and fully excited, let r_s represent the resistance of the shunt coils, and C_s represent the current in the shunt coils. Then, from Ohm's law, the current in the shunt coils is given by the formula $C_s = \frac{E_e}{r_s}$. For example, suppose that a shunt dynamo, when running at a constant speed, generates a constant difference of potential of 110 volts, and the resistance of the magnetizing coils from the positive connection to the negative connection is 55 ohms; or $E_e = 110$ volts and $r_s = 55$ ohms. Then, the current in the shunt coils would be given by substituting these values in the above formula, or $C_s = \frac{E_e}{r_s} = \frac{110}{55} = 2$ amperes.

This gives the strength of current in the shunt coils, but does not indicate the amount of power required to constantly excite the field magnets. The power in watts is $W = CE$; that is, it is equal to the current in amperes flowing through the shunt coils multiplied by the difference of potential in volts between the terminals of the shunt coils. We have found in this case that the current $C = 2$ amperes and the E. M. F. $E = 110$ volts; then, $W = 2 \times 110 = 220$ watts, which represents the power required to excite the field magnets.

All other conditions being similar, the same number of watts will be dissipated in a shunt field coil as in a series coil, provided an equal amount of magnetizing force is produced in the two cases.

68. In a compound-wound dynamo, the field loss consists of two losses, one in the series coil and the other in the shunt coil. The loss in the series coil depends upon the strength of current flowing from the dynamo, as in the case of a simple series dynamo, while the loss in the shunt coil is constant, irrespective of the load on the machine; provided, of course, the dynamo generates a constant electromotive force for all loads. This can readily be understood from the following example: A dynamo is compounded to generate 220 volts between its terminals for all loads up to its rated capacity; that is, when the current from the armature becomes stronger and the difference of potential between the terminals tends to fall, the current in passing through the series coil strengthens the field magnets sufficiently to keep a difference of exactly 220 volts between the terminals of the dynamo. Assume the resistance of the shunt coil to be 275 ohms and that of the series coil to be .055 ohm. At a rated output of 4,400 watts, the current flowing through the series coil and into the external circuit is $\frac{4400}{220} = 20$ amperes (assuming the connections are made for a *short shunt*). At all loads the current in the shunt coil is $C_s = \frac{E_e}{r_s} = \frac{220}{275} = .8$ ampere, and the loss of power in the shunt coil is $W_s = E_e \times C_s = 220 \times .8 = 176$ watts; even when the external circuit is open the loss in the shunt coil remains constant, or 176 watts in this particular case. The loss in the series coil, however, varies directly with the square of the current passing through it. In this example, the loss in the series coil is $W = C^2 \times r = 20^2 \times .055 = 22$ watts; at half load, or 10 amperes, the loss is $W = 10^2 \times .055 = 5.5$ watts, etc.; at no load there is no current in the series coil, and, consequently, no loss. The total field loss in a compound dynamo is the sum of the losses in the series and shunt coils. For instance, in this example, the total field loss at full load is 198 watts; at half load, 181.5 watts, and at no load, 176 watts.

69. The amount of power lost or dissipated in the field coils of a dynamo depends (1) upon the capacity of the dynamo, (2) upon its design, and (3) upon the amount of copper used in the coils. In the last condition it is obvious that in order to produce a certain number of *ampere-turns*, the current in amperes required could be made exceedingly small by using a large number of turns of copper wire, thereby reducing the electrical loss. A limit is reached, however, where it is not economical from a commercial standpoint to increase the amount of copper in order to save in electrical loss.

The per cent. loss in the field coils of dynamos varies from about 10% of the input to dynamos having an output of about 1,000 watts to as low as 1.5% to 2% of the input to dynamos having an output of 100,000 watts and upwards. For example, suppose that the input to a dynamo from an engine was 100 horsepower and the loss in the field coils was 2.5%. Under these conditions, how many watts are lost or dissipated in the field coils? Changing the input from horsepower to watts gives 100 × 746 = 74,600 watts, since 1 horsepower is equivalent to 746 watts. Hence, the number of watts lost in the field coils is 74,600 × .025 = 1,865 watts.

70. Armature Losses.—The principal armature loss is that produced by the current in flowing against the internal resistance of the armature, that is, the resistance of the armature *conductors*. The *core losses* previously described could also be classed as part of the armature losses, but it is usual to consider them apart. The armature loss proper is usually termed the **copper**, or **wire, loss**, since it is due to the resistance of the armature conductors, which are composed of copper wire or bars. The internal resistance of an armature is an exceedingly variable quantity, depending upon the form, construction, size, number of conductors, size of conductors, etc. In constant-potential dynamos, generally speaking, the internal resistance of the armature must necessarily be comparatively small, since it determines the maximum strength of current that can be obtained from the dynamo, as will be seen subsequently.

The armature loss depends upon the amount of internal resistance and upon the strength of current flowing through the armature conductors. In a given armature the internal resistance remains constant at equal temperatures, while the strength of current varies with the load upon the dynamo at that particular moment; in other words, this loss only occurs when there is a current flowing through the armature—the stronger the current, the greater is the loss, and vice versa. As previously shown, in all cases where an electric current flows against the resistance of a conductor, the loss of power in watts is equal to the resistance of the conductor in ohms multiplied by the square of the current in amperes; hence, in an armature the number of watts lost in the armature conductors is equal to the square of the current in amperes flowing through the armature multiplied by the internal resistance in ohms of the armature from the positive to the negative brush. If C represents the total current in amperes flowing through the armature and r_i the internal resistance in ohms from the positive to the negative brush, then $W_i = C^2 r_i$, where W_i is the number of watts lost in the armature conductors. From this fact, this armature loss is also designated as the **C^2 r loss.** For example, suppose that the internal resistance of an armature from brush to brush is .125 ohm, and a total current of 40 amperes is flowing through the armature. Determine the number of watts lost in the armature. Let $C = 40$ amperes and $r_i = .125$ ohm; then, $W_i = C^2 r_i = 40^2 \times .125 = 200$ watts.

The per cent. loss in armatures of constant-potential dynamos varies from about 12% of the input to dynamos having a rated capacity of about 1,000 watts to as low as 1.5% to 2% of the input to dynamos having a rated capacity of about 100,000 watts and upwards. For example, suppose that a dynamo was working under a load which required 50 horsepower to run it, and, at this rating, the armature loss alone amounted to 3% of the input; determine the number of watts dissipated or lost in the armature conductors. Changing the input from horsepower to watts gives 50 × 746 = 37,300 watts, since 746 watts are equal to 1 horsepower. The armature $C^2 r$ loss is therefore 3% of the input, or 37,300 × .03 = 1,119 watts.

71. Other Losses.—Aside from the four principal losses mentioned, other small losses occur in some machines when the armature is revolving. If large conductors are used in the winding of the armatures, a difference of potential is sometimes generated between the edges of the conductor in such a manner as to give rise to small eddy or local currents in the conductors themselves, and which do not appear in the external circuit and are useless. In some cases these local currents dissipate considerable energy and heat the armature badly when the machine is not loaded; but in a well-designed dynamo they are too small to be considered.

In an armature in which the conductors are wound in slots cut in the core disks, the teeth between the slots have a tendency to disturb the position of the lines of force where they enter and leave the polar faces. This movement causes local or eddy currents to be generated in the pole pieces, thereby dissipating a certain amount of energy. These eddy currents in the pole pieces are sometimes termed **Foucault currents,** in memory of the man who first recognized their existence. In order to prevent these Foucault currents, the pole pieces of machines having toothed armatures are frequently laminated. Other local currents may occur in various parts of some dynamos on account of bad design, but it is only necessary here to treat specifically upon such losses as are common to all dynamos and impossible to eliminate.

72. From the four previous articles, the following summary will be a help to establish the rules of efficiency and losses:

Input = the power driving the dynamo, which is derived from some outside agency.

Output = input minus the total losses.

Total losses = the sum of the friction, core, field, armature, and other losses.

Per cent. efficiency $= \frac{\text{input minus total losses}}{\text{input}} \times 100$, or $\frac{\text{output}}{\text{input}} \times 100$.

Per cent. loss in friction $= \frac{\text{friction losses}}{\text{input}} \times 100$.

Per cent. loss in core $= \frac{\text{core losses}}{\text{input}} \times 100$.

Per cent. loss in field $= \frac{\text{field losses}}{\text{input}} \times 100$.

Per cent. loss in armature $= \frac{\text{armature losses}}{\text{input}} \times 100$.

THE OUTPUT OF CONSTANT-POTENTIAL DYNAMOS.

73. If a dynamo is so constructed as to give a constant potential at any load, it is evident that the current flowing is inversely proportional to the resistance of the external circuit; that is, if the external resistance is reduced, the amount of current will be correspondingly increased. There is a limit, however, to the amount of current that any given machine can give out, depending on one (or both) of two factors; namely, the **heating** and the **sparking.**

The heat that is being continually generated in the armature and field coils of a dynamo when working under load, due both to the $C^2 r$ *loss* and the *core loss*, is given off from the surface of the armature and of the whole machine to the surrounding air. This giving off of heat can only occur when the dynamo is hotter than the air, for if two bodies [illegible] one cannot give any heat to the other. Consequently, the greater the difference in temperature between two bodies, such as a dynamo armature and the surrounding air, the more heat will be given from the hot [illegible]

[illegible] When a dynamo is first started, it is at about the same temperature as the air, so that when the conductors in the armature begin to generate heat, this heat cannot pass [illegible] it raises the temperature of the armature, until it is enough hotter than the surrounding air [illegible] the heat which is being generated to be [illegible]

[illegible] heat generated is practically constant, [illegible] the load remains constant, the temperature [illegible] armature will also remain constant, because the [illegible] as fast as generated; and if the load is increased [illegible] increase the amount of heat generated, the temperature will again rise until the armature is enough [illegible] give off all this increased amount of [illegible]

[illegible] when other conditions remain the [illegible] a dynamo armature, that is, [illegible] the hotter it will get.

[illegible] temperature, the materials used in [illegible] the armature, such as cotton, [illegible] will become *carbonized*, that is, [illegible] rendered useless as insulating materials. [illegible] these materials will withstand a temperature [illegible] above the boiling point of water [illegible] F. [illegible] found that if they are *continually* subjected to a temperature greater than about 180° F., they will gradually become carbonized; hence, as armatures are expected to last several years, they should never be subjected to a continual temperature greater than about 170° F. Consequently, the amount of current which will cause a dynamo armature to heat to about 170° F. is the limiting amount which that armature can *safely* give.

75. As an armature must be a certain number of degrees hotter than the air in order to give off the heat generated, it is evident that if the air itself were originally of a high temperature, the armature would actually have a higher *temperature* when giving off a certain amount of heat than if the air were cooler; that is, for a certain amount of heat generated, the temperature of the armature will rise to a certain number of degrees *above the temperature of the air*. The average temperature of the air in places where dynamos are installed is often as high as 90° F., so the allowable rise in temperature of the armature above that of the air is about $170 - 90 = 80°$ F., and dynamos are usually rated according to this rise in temperature.

As *still* air is a very poor conductor of heat, most of the heat given off to it is carried away by the motion of the air; this motion is partly due to the air-currents set up by the rise of the heated air and the flowing in of the cooler air to take its place, but mainly to the air-currents set up by the motion of the armature itself. This latter effect is usually greater in ring than drum armatures, due to the more open construction of the former and to the *fan* action of the spider arms.

The heat generated in the field coils is disposed of in the same way as that of the armature; that is, it is given off to the surrounding air. The rise in temperature of the field coils is subject to the same limitations as the rise of the armature; i. e., it is usually limited to about 80° F. above the temperature of the air.

76. By the *sparking* of a dynamo is meant the sparks which appear at the brushes, due to the *reversal of the current in the armature coils*. If the commutator is out of true or has one segment higher or lower than the others, or from other similar causes, there will be flashes or sparks at the brushes; but these are merely *mechanical* faults which can be easily remedied, and this is not what is meant by *sparking*. Referring to Fig. 28, it will be seen that in the armature coil $a' p p'$, when in the position shown, the general direction of the current is from right to left; but as soon as [illegible] E. M. F. [illegible]

[illegible] E. M. F. generated [illegible] E. M. F. [illegible]

[illegible] This is seldom [illegible] E. M. F. [illegible] changes with [illegible] the current; but the effect [illegible] shifting of the brushes will usually so nearly counterbalance the E. M. F. of self-induction that the sparking will be slight at different loads.

77. It has been shown that the current in the armature winding reacts upon the magnetic field, forcing the actual neutral point ahead (in the direction of rotation). Now, if the brushes are moved ahead of this neutral point to avoid sparking, the effect is to move the consequent poles (due to the current circulating in the armature winding) also ahead, which shifts the neutral point still farther ahead, which requires a further slight shifting of the brushes. As long as the field due to the magnetizing coils is much stronger than the reactive effect of the armature, this action is slight, so that only a slight shifting of the brushes is necessary for practically sparkless operation. As the current in the armature increases, its reactive effect grows stronger, and a movement of the brushes is followed by a considerable movement of the neutral plane. Indeed, if the current in the armature is strong enough, the brushes may be shifted more than half way around the commutator without coming to the sparkless position. There is, therefore, a limit to the amount of current which can be taken from an armature (aside from its heating limit), which is reached when any further amount of shifting of the brushes will not afford sparkless commutation.

This amount of shifting is generally confined to the space between the tips of the pole pieces; that is, the brushes may be shifted until the coil short-circuited by a brush is at or just under the tip of a pole piece.

In dynamos of good design, the heating limit and sparking limit are reached with about the same current; that is, a current which will raise the temperature of the armature above that of the air by the amount decided upon as a limit will also necessitate the brushes being shifted to the maximum allowable extent. In some modern railway generators, the output is limited more by the heating than by the sparking. Such machines may stand a 50 per cent. overload without excessive sparking. Moreover, by careful designing

short-circuited coil in a field which will generate an E. M. F. equal to the E. M. F. of self-induction, since the absence of sparking depends mainly on this point. With carbon brushes the absence of sparking depends both on generating an E. M. F. in the coil and on the contact resistance of the brush. On account of the increased resistance of contact, carbon brushes require less shifting for variations in load than do metallic brushes, and are generally used on machines where the variations in load are so frequent and extensive that a great deal of time would be spent in shifting the brushes, if this had to be done for every change in the load.

79. If the brushes are shifted so far forwards that the E. M. F. generated in the short-circuited coil is *greater* than the E. M. F. of self-induction, not only will the latter be neutralized, but a current will be sent around the coil through the commutator bars and the brush which short-circuits the coil. If this current is greater than the current which one-half of the armature is supplying to the external circuit, it is evident that when the short-circuited coil moves over and becomes a part of that half of the armature, its current will be *reduced;* this reduction is opposed by the self-induction of the coil, as before, and sparking results. Since the circuit of the short-circuited coil is partly through the brush and its contact with the commutator bars, it is evident that with metallic brushes of low resistance the liability of the current in this coil becoming excessive is greater than with carbon brushes of (comparatively) high resistance. For the reason, therefore, that they are of higher resistance, carbon brushes will spark less than metallic brushes under the same conditions.

DIRECT-CURRENT MOTORS.

80. It has already been explained that a conductor carrying current at right angles to a magnetic field will move across the field. This is, in brief, the principle on which an electric motor operates. If we take an ordinary direct-current dynamo and, instead of driving it from some outside source of power and making it generate an E. M. F., we supply it with current from some outside source, say another dynamo, the machine will run as a motor and deliver power at its pulley. A direct-current motor, then, is the same as a dynamo, so far as its construction goes. The difference between a dynamo and a motor has more to do with the way in which the machines are used than with any radical difference in their construction. At one time it was thought that motors should be constructed different from dynamos, but it is now known that the machine that makes a good dynamo will nearly always make a good motor. Of course, motors often differ considerably from dynamos in their mechanical details, because motors must often be adapted to special work for which the ordinary mechanical construction of a dynamo would be unsuitable. For example, railway motors must stand all sorts of abuse, and hence must be enclosed as much as possible. As regards their electrical features, however, direct-current dynamos and motors are the same; when a machine is used as a dynamo, it is supplied with power and it converts this power into electrical energy; when it is used as a motor, it is supplied with power in the shape of an electric current, the reaction between the current in the armature conductors and the magnetic field set up by the field magnets forces the armature to revolve, and the machine, therefore, converts the electrical energy supplied at the brushes into mechanical energy delivered at the pulley.

CLASSES OF MOTORS.

81. Motors, like dynamos, may be divided into several classes according to the method used for exciting their field magnets. We may then class them as ***series-wound***, ***shunt-wound***, ***compound-wound***. The last class may be subdivided into *differentially wound* and ***accumulatively wound***, according as the series field coils are connected so as to ***oppose*** or ***aid*** the shunt coils. Nearly all motors in common use are operated at constant potential, and we will confine our remarks here to motors that are so operated.

82. Shunt-Wound Motors.—Shunt-wound motors are very extensively used for stationary work for operating all classes of machinery where a fairly constant speed is required. A constant-potential shunt-wound motor is the same as regards construction as the corresponding shunt-wound dynamo. When supplied with a constant pressure, it will run at nearly constant speed, no matter what may be the load, up to the limit of the machine. The speed drops off slightly with an increase of load, but the falling off is slight if the motor is well designed. A shunt-wound motor will give a good starting effort if the field is fully excited before the current is allowed to pass through the armature. For stationary work, the shunt-wound motor is the one most largely used.

83. Series-Wound Motors.—These motors are used most largely for work when a wide variation in speed is desired, where the motor is to be started and stopped frequently, and where a strong starting effort is needed. For these reasons they are used on street railways and for traction work generally, for hoists, electric cranes, and in some cases for pumps and similar machinery. Thousands of series-wound, direct-current motors are in use for operating street cars alone. Since the field is in series with the armature, the strength of the field depends on the current flowing through the motor. If at starting a large current is allowed to flow, the field is made very strong, and as the current in the armature is also very large, a powerful starting effort is the result. This is a valuable feature if the motor is to be used for traction or hoisting work. The speed of a constant-potential series motor varies widely if the load on it is varied, because every change in load produces a corresponding change in the field strength. If the load is thrown off entirely, the motor will race, and may reach a speed high enough to injure itself.

84. Compound-Wound Motors.—These motors are not rgely used, compared with the other two classes. They are used mostly for special purposes, such as operating printing presses, looms, etc. When a speed that is very nearly constant under all loads is required, the differentially wound motor is used. When a motor is required that will have a fairly good starting effort combined with the property of approximately constant speed, the accumulatively wound motor is employed. In this motor the series coils aid the shunt coils, and when starting, the heavy current through the series coils strengthens the field, and thus gives a strong starting effort, or *torque*, as it is called.

A SERIES

OF

QUESTIONS AND EXAMPLES

RELATING TO THE SUBJECTS
TREATED OF IN THIS VOLUME.

It will be noticed that the Examination Questions that follow have been divided into sections, which have been given the same numbers as the Instruction Papers to which they refer. No attempt should be made to answer any of the questions or to solve any of the examples until that portion of the text having the same section number as the section in which the questions or examples occur has been carefully studied.

ARITHMETIC.

(PART 1.)

EXAMINATION QUESTIONS.

(1) What is arithmetic?

(2) What is a number?

(3) What is the difference between a concrete number and an abstract number?

(4) Define notation and numeration.

(5) Write each of the following numbers in words: 980; 605; 28,284; 9,006,042; 850,317,002; 700,004.

(6) Represent in figures the following expressions:

Seven thousand six hundred. Eighty-one thousand four hundred two. Five million four thousand seven. One hundred eight million ten thousand one. Eighteen million six. Thirty thousand ten.

(7) What is the sum of 3,290, 504, 865,403, 2,074, 81, and 7? Ans. 871,359.

(8) $709 + 8,304,725 + 391 + 100,302 + 300 + 909 =$ what? Ans. 8,407,336.

(9) During a 12-hour test of a steam engine the counter showed the number of revolutions per hour to have been as follows: 12,600, 12,444, 12,467, 12,528, 12,468, 12,590, 12,610, 12,589, 12,576, 12,558, 12,546, 12,532. How many revolutions were made during the test? Ans. 150,508 rev.

For notice of copyright, see page immediately following the title page.

(10) Find the difference between the following: (*a*) 50,962 and 3,338; (*b*) 10,001 and 15,339.

Ans. { (*a*) 47,624.
(*b*) 5,338.

(11) (*a*) 70,968 − 32,975 = ? (*b*) 100,000 − 98,735 = ?

Ans. { (*a*) 37,993.
(*b*) 1,265.

(12) On a certain morning 7,240 gallons of water were drawn from an engine-room tank and 4,780 gallons were pumped in. In the afternoon 7,633 gallons were drawn out and 8,675 gallons pumped in. How many gallons remained in the tank at night, if it contained 3,040 gallons at the beginning of the day? Ans. 1,622 gal.

(13) Find the product of the following: (*a*) 526,387 × 7; (*b*) 700,298 × 17; (*c*) 217 × 103 × 67.

Ans. { (*a*) 3,684,709.
(*b*) 11,905,066.
(*c*) 1,497,517.

(14) If your watch ticks once every second, how many times will it tick in one week? Ans. 604,800.

(15) An engine and boiler in a manufactory are worth $3,246. The building is worth three times as much, plus $1,200, and the tools are worth twice as much as the building, plus $1,875. (*a*) What is the value of the building and tools? (*b*) What is the value of the whole plant?

Ans. { (*a*) $34,689.
(*b*) $37,935.

(16) Divide the following:

(*a*) 962,842 by 84; (*b*) 39,728 by 63; (*c*) 29,714 by 108; (*d*) 406,089 by 135.

Ans. { (*a*) 11,462.4047.
(*b*) 630.603.
(*c*) 275.1296.
(*d*) 3,008.0666.

(17) Suppose that in 1 hour 10 pounds of coal are burned per square foot of grate area in a certain boiler,

and that 9 pounds of water are evaporated per pound of coal burned. If the grate area is 30 square feet, how many pounds of water would be evaporated in a day of 10 hours?
Ans. 27,000 lb.

(18) If a mechanic receives $1,500 a year for his labor and his expenses are $968 per year, in what time can he save enough to buy 28 acres of land at $133 an acre?
Ans. 7 years.

(19) Solve the following by cancelation:

(*a*) (72 × 48 × 28 × 5) ÷ (84 × 15 × 7 × 6).

(*b*) (80 × 60 × 50 × 16 × 14) ÷ (70 × 50 × 24 × 20).

Ans. $\begin{cases} (a) & 9\frac{1}{7}. \\ (b) & 32. \end{cases}$

(20) A freight train ran 365 miles in one week, and 3 times as far, lacking 246 miles, the next week; how far did it run the second week? Ans. 849 miles.

(21) If the driving wheel of a locomotive is 16 feet in circumference, how many revolutions will it make in going from Philadelphia to Pittsburg, the distance between them being 354 miles, and there being 5,280 feet in 1 mile?
Ans. 116,820 rev.

ARITHMETIC.

(PART 2.)

EXAMINATION QUESTIONS.

(1) What is a fraction?

(2) What are the terms of a fraction?

(3) What does the denominator show?

(4) What does the numerator show?

(5) Is $\frac{13}{8}$ a proper or an improper fraction, and why?

(6) Write three mixed numbers.

(7) Reduce the following fractions to their lowest terms: $\frac{4}{8}$, $\frac{4}{16}$, $\frac{8}{32}$, $\frac{32}{64}$. Ans. $\frac{1}{2}$, $\frac{1}{4}$, $\frac{1}{4}$, $\frac{1}{2}$.

(8) Reduce 6 to an improper fraction whose denominator is 4. Ans. $\frac{24}{4}$.

(9) Reduce $7\frac{7}{8}$, $13\frac{5}{16}$, and $10\frac{3}{4}$ to improper fractions.
Ans. $\frac{63}{8}$, $\frac{213}{16}$, $\frac{43}{4}$.

(10) How many feet does the piston of a steam engine pass over in a week of 6 days, running $8\frac{1}{2}$ hours a day, if the length of the stroke of the engine is $1\frac{1}{2}$ feet and the number of revolutions per minute is 160?
Ans. 1,468,800 ft.

(11) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = ?$ Ans. 1.

(12) $\frac{1}{4} + \frac{3}{8} + \frac{5}{16} = ?$ Ans. $\frac{15}{16}$.

(13) $42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Ans. $83\frac{1}{16}$.

For notice of copyright, see page immediately following the title page.

(14) An iron plate is divided into four sections; the first contains $29\frac{3}{4}$ square inches; the second, $50\frac{5}{8}$ square inches; the third, 41 square inches; and the fourth, $69\frac{3}{16}$ square inches. How many square inches are in the plate?

Ans. $190\frac{9}{16}$ sq. in.

(15) What is the difference between $\frac{7}{8}$ and $\frac{7}{16}$? 13 and $7\frac{7}{16}$? $312\frac{9}{16}$ and $229\frac{5}{32}$?

Ans. $\frac{7}{16}$; $5\frac{9}{16}$; $83\frac{13}{32}$.

(16) Solve the following:

(*a*) $35 \div \frac{5}{16}$; (*b*) $\frac{9}{16} \div 3$; (*c*) $\frac{17}{2} \div 9$; (*d*) $\frac{113}{64} \div \frac{7}{16}$; (*e*) $15\frac{3}{4} \div 4\frac{3}{8}$.

Ans.
(*a*) 112.
(*b*) $\frac{3}{16}$.
(*c*) $\frac{17}{18}$.
(*d*) $4\frac{1}{28}$.
(*e*) $3\frac{3}{5}$.

(17) The numerator of a fraction is 28 and the value of the fraction is $\frac{7}{8}$; what is the denominator?

Ans. 32.

(18) Four bolts are required, $2\frac{1}{2}$, $6\frac{7}{8}$, $3\frac{1}{16}$, and 4 inches long. How long a piece of iron will be required from which to cut them, allowing $\frac{7}{16}$ of an inch to each bolt for cutting off and finishing?

Ans. $18\frac{3}{16}$ in.

ARITHMETIC.

(PART 3.)

EXAMINATION QUESTIONS.

(1) State the difference between a common fraction and a decimal fraction.

(2) Reduce the following fractions to equivalent decimals: $\frac{1}{2}$, $\frac{7}{8}$, $\frac{5}{32}$, $\frac{65}{100}$, and $\frac{125}{1000}$.

$$\text{Ans.}\left\{\begin{array}{l} .5. \\ .875. \\ .15625. \\ .65. \\ .125. \end{array}\right.$$

(3) Write out in words the following numbers: .08, .131, .0001, .000027, .0108, and 93.0101.

(4) What is the sum of .125, .7, .089, .4005, .9, and .000027? Ans. 2.214527.

(5) Add 17 thousandths, 2 tenths, and 47 millionths. Ans. .217047.

(6) Work out the following examples:

(*a*) 709.63 − .8514; (*b*) 81.963 − 1.7; (*c*) 18 − .18; (*d*) 1 − .001; (*e*) 872.1 − (.8721 + .008); (*f*) (5.028 + .0073) − (6.704 − 2.38).

$$\text{Ans.}\left\{\begin{array}{ll} (a) & 708.7786. \\ (b) & 80.263. \\ (c) & 17.82. \\ (d) & .999. \\ (e) & 871.2199. \\ (f) & .7113. \end{array}\right.$$

For notice of copyright, see page immediately following the title page.

(7) The cost of the coal consumed under a nest of steam boilers during a week's run was as follows: Monday, \$15.83; Tuesday, \$14.70; Wednesday, \$14.28; Thursday, \$13.87; Friday, \$14.98; Saturday, \$12.65. What was the cost of the week's supply of coal? Ans. \$86.31.

(8) It is desired to increase the capacity of an electric-light plant to 1,500 horsepower by adding a new engine. If the indicated horsepower of the engines already in use is $482\frac{4}{5}$, $316\frac{1}{3}$, and $390\frac{3}{4}$, what power must the new engine develop? Ans. $310.11\frac{2}{3}$ H. P.

(9) The inside diameter of a 6-inch steam pipe is 6.06 inches, and the outside diameter is 6.62 inches. How thick is the pipe? Ans. .28 in.

(10) Find the products of the following expressions: (*a*) $.013 \times .107$; (*b*) $203 \times 2.03 \times .203$; (*c*) $(2.7 \times 31.85) \times (3.16 - .316)$; (*d*) $(107.8 + 6.541 - 31.96) \times 1.742$.

Ans.
- (*a*) .001391.
- (*b*) 83.65427.
- (*c*) 244.56978.
- (*d*) 143.507702.

(11) How many square feet of heating surface are in the tubes of a boiler having sixty 3-inch tubes, each $15\frac{1}{2}$ feet long, if the heating surface of each tube per foot in length is .728 square foot? Ans. 677.04 sq. ft.

(12) Find the values of the following expressions:

(*a*) $\dfrac{7}{\frac{3}{16}}$; (*b*) $\dfrac{\frac{15}{32}}{\frac{5}{8}}$; (*c*) $\dfrac{1.25 \times 20 \times 3}{\dfrac{87 + 88}{459 + 32}}$.

Ans.
- (*a*) $37\frac{1}{3}$.
- (*b*) .75.
- (*c*) $210\frac{3}{7}$.

(13) The distance around a cylindrical boiler is 166.85 inches. If there are 72 rivets in one of the circular seams, find what the pitch (distance between the centers of any two rivets) of the rivets is. Ans. 2.317+ in.

(14) A keg of $\frac{7}{8}'' \times 2\frac{3}{4}''$ boiler rivets weighs 100 pounds and contains 133 rivets. What is the weight of each rivet? Ans. .75+ lb.

(15) How many inches in .875 of a foot? Ans. $10\frac{1}{2}$ in.

(16) What decimal part of a foot is $\frac{3}{16}$ inch?
Ans. .015625 ft.

(17) In a steam-engine test of an hour's duration, the horsepower developed was found to be as follows at 10-minute intervals: 48.63, 45.7, 46.32, 47.9, 48.74, 48.38, 48.59. What was the average horsepower? Ans. 47.75+ H. P.

[redacted] [illegible]

[illegible] [redacted]

ARITHMETIC.

(PART 4.)

EXAMINATION QUESTIONS.

(1) What is 25% of 8,428 lb.? Ans. 2,107 lb.

(2) What is ½% of $35,000? Ans. $175.

(3) What per cent. of 50 is 2? Ans. 4%.

(4) What per cent. of 10 is 10? Ans. 100%.

(5) The coal consumption of a steam plant is 5,500 lb. per day when the condenser is not running, or an increase of 15% over the consumption when the condenser is used. How many pounds are used per day when the condenser is running? Ans. 4,782.61 lb.

(6) An engineer receives a salary of $950. He pays 24% of it for board, 12½% of it for clothing, and 17% of it for other expenses. How much of it does he save a year? Ans. $441.75.

(7) If 37½% of a number is 961.38, what is the number? Ans. 2,563.68.

(8) The speed of an engine running unloaded was 1½% greater than when running loaded. If it made 298 revolutions per minute with the load, what was its speed running unloaded? Ans. 302.47 rev. per min.

(9) Reduce 4 yd. 2 ft. 10 in. to inches. Ans. 178 in.

For notice of copyright, see page immediately following the title page.

[illegible]
Ans. [illegible]

[illegible]
Ans. [illegible]

[illegible] How
[illegible] Ans. [illegible]

[illegible]
Ans. [illegible]

[illegible]
Ans. [illegible]

[illegible] the [illegible]
[illegible] How
Ans. [illegible]

[illegible] would
[illegible] Ans. [illegible]

[illegible] Ans. [illegible]

[illegible] lengths each
[illegible] length
[illegible]
[illegible] shaft.
[illegible] between the
Ans. [illegible]

[illegible] how
[illegible] longitudinal
[illegible] the rivets to
[illegible] rivets to be [illegible] in.
Ans. [illegible] rivets.

ARITHMETIC.

(PART 5.)

EXAMINATION QUESTIONS.

(1) What is the square of 108? Ans. 11,664.

(2) What is the cube of 181.25?

Ans. 5,954,345.703125.

(3) Find the values of the following: (*a*) $.0133^3$; (*b*) 301.011^3; (*c*) $(\frac{1}{8})^3$; (*d*) $(3\frac{3}{4})^3$.

Ans. (*a*) .000002352637.
(*b*) 27,273,890.942264331.
(*c*) $\frac{1}{512}$.
(*d*) $52\frac{47}{64}$, or 52.734375.

Note.—In the answers to the following examples requiring the extraction of a root, a minus sign after a number indicates that the last digit is not quite as large as the number printed. Thus, 12,497— indicates that the number really is 12.496+, and that the 6 has been made a 7 because the next succeeding figure was 5 or greater. The answers to the examples requiring the extraction of cube root are exact to the last digit given. If the student is unable to get the same answer by means of the method described in Arts. **30-34**, he should then use the method described in Arts. **35** and **36**.

(4) Find the square root of the following: (*a*) 3,486,784.401; (*b*) 9,000,099.4009.

Ans. (*a*) 1,867.29+.
(*b*) 3,000.017—.

(5) Extract the cube root of .32768. Ans. .6894+.

(6) Extract the cube root of 2 to four decimal places.

Ans. 1.2599+.

For notice of copyright, see page immediately following the title page.

(7) **Solve the following:** (*a*) $\sqrt{123.21}$; (*b*) $\sqrt{114.921}$.

Ans. $\begin{cases} (a) & 11.1. \\ (b) & 10.72+. \end{cases}$

(8) **Solve the following:** (*a*) $\sqrt[3]{.0065}$; (*b*) $\sqrt[3]{.000021}$.

Ans. $\begin{cases} (a) & .18663-. \\ (b) & .02759-. \end{cases}$

Find the value of x in the following:

(9) $11.7 : 13 :: 20 : x$. Ans. 22.22+.

(10) (*a*) $20 \div 7 : 10 \div 8 :: 3 : x$; (*b*) $12^2 : 100^2 :: 4 : x$.

Ans. $\begin{cases} (a) & 2. \\ (b) & 277.7+. \end{cases}$

(11) (*a*) $\frac{4}{x} = \frac{7}{21}$; (*b*) $\frac{x}{24} = \frac{8}{16}$; (*c*) $\frac{2}{10} = \frac{x}{100}$; (*d*) $\frac{15}{45} = \frac{60}{x}$; (*e*) $\frac{10}{150} = \frac{x}{600}$.

Ans. $\begin{cases} (a) & 12. \\ (b) & 12. \\ (c) & 20. \\ (d) & 180. \\ (e) & 40. \end{cases}$

(12) If a piece of 2-inch shafting 3½ ft. long weighs 37.45 lb., how much would a piece 6¾ ft. long weigh?

Ans. 72.225 lb.

(13) If a railway train runs 444 miles in 8 hr. 40 min., in what time can it run 1,000 miles at the same rate of speed?

Ans. 20 hr. 41.44 min.

(14) If a pump discharging 135 gal. per min. fills a tank in 38 min., how long would it take a pump discharging 85 gal. per min. to fill it? Ans. $60\frac{6}{17}$ min.

(15) The distances around the driving wheels of two locomotives are 12.56 ft. and 15.7 ft., respectively. How many times will the larger turn while the smaller turns 520 times?

Ans. 416 times.

(16) If a cistern 28 ft. long, 12 ft. wide, 10 ft. deep holds 798 bbl. of water, how many barrels of water will a cistern hold that is 20 ft. long, 17 ft. wide, and 6 ft. deep?

Ans. 484½ bbl.

MENSURATION AND USE OF LETTERS IN FORMULAS.

EXAMINATION QUESTIONS.

$A = 5 \qquad h = 200$

$B = 10 \qquad x = 12$

$i = 3.5 \qquad D = 120$

Work out the solutions to the following formulas, using the above values for the letters:

(1) $C = \dfrac{D - x}{B + i}$. Ans. $C = 8$.

(2) $Q = \dfrac{A h + D}{2 x + 6} + D$. Ans. $Q = 157\frac{1}{3}$.

(3) $v = \sqrt{\dfrac{A D}{i B + 1.5}}$. Ans. $v = 4.05+$.

(4) $g = \dfrac{(B - A)^2 - \sqrt{h + 2 B + A}}{A^3 - (1 + D)}$. Ans. $g = 2\frac{1}{2}$.

(5) If one of the angles formed by one straight line meeting another straight line equals 152° 3′, what is the other angle equal to? Ans. 27° 57′.

(6) Draw an obtuse angle, a right angle, and an acute angle. State the name of each angle by using letters to designate them.

For notice of copyright, see page immediately following the title page.

Draw a rhombus and then draw a rectangle having ıe same area.

(8) A sheet of zinc measures 11½ inches by 2½ feet. ·w many square inches does it contain? Ans. 345 sq. in.

(9) How many boards 16 feet long and 5 inches wide ıld be required to lay a floor measuring 15 ft. × 24 ft.?
Ans. 54 boards.

(10) The accompanyin re shows the floor plan of n electric-light station. the dimensions given, cal- te the number of square ı of unoccupied floor space.
Ans. 2,059.08 sq. ft.

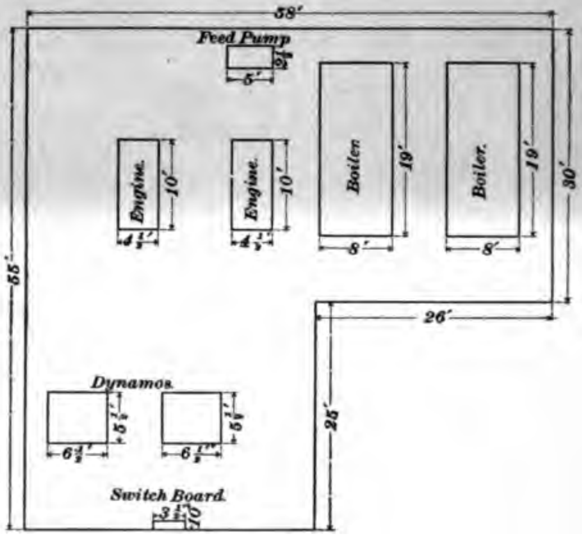


FIG. I.

(11) A triangle has three equal angles; what is it called?

(12) If a triangle has two equal angles, what kind of a triangle is it?

(13) In a triangle $A\ B\ C$, angle $A = 23°$ and $B = 32° 32'$; what does angle C equal? Ans. $C = 124° 28'$.

(14) In the figure, if $AD = 10$ inches, $AB = 24$ inches, and $BC = 13\frac{1}{2}$ inches, how long is DE, DE being parallel to BC? Ans. $DE = 5.625$ in.

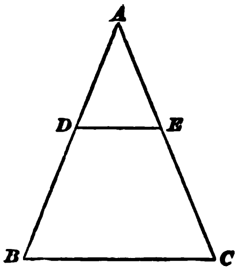


FIG. II.

(15) An engine room is 52 feet long and 39 feet wide. How many feet is it from one corner to a diagonally opposite one, measured in a straight line? Ans. 65 ft.

(16) It is required to make a miter-box in which to cut molding to fit around an octagon post. At what angle with the side of the box should the saw run? Ans. 67½°.

(17) If the distance between two opposite corners of a hexagonal nut is 2 inches, what is the distance between two opposite sides? Ans. 1.732+ in.

(18) In the accompanying figure, if the distance BI is 6 inches and HK 18 inches, what is the diameter of the circle? Ans. 19.5 in.

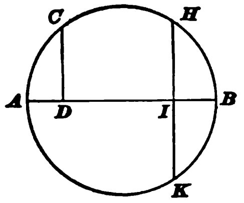


FIG. III.

(19) How many revolutions will a 72-inch locomotive driver make in going 1 mile? Ans. 280.112 revolutions.

(20) A pipe has an internal diameter of 6.06 inches; what is the area of a circle having this diameter? Ans. 28.8427 sq. in.

(21) How long must the arc of a circle be to contain 12°, supposing the radius of the circle to be 6 inches? Ans. 1.25664 in.

(22) What is the area of the sector of a circle 15 inches in diameter, the angle between the two radii forming the sector being 12½°? Ans. 6.1359 sq. in.

(23) (*a*) What would be the length of the side of a square metal plate having an area of 103.8691 square inches?

(*b*) What would be the diameter of a round plate having this area? (*c*) How much shorter is the circumference of the round plate than the perimeter of the square plate?

Ans. $\begin{cases} (a) & 10.1916 \text{ in.} \\ (b) & 11\frac{1}{2} \text{ in.} \\ (c) & 4.638 \text{ in.} \end{cases}$

(24) Find the area in square feet of the entire surface of a hexagonal column 12 feet long, each edge of the ends of the column being 4 inches long. Ans. 24.5774 sq. ft.

(25) Find the cubical contents of the above column in cubic inches. Ans. 5,985.9648 cu. in.

(26) Compute the weight per foot of an iron boiler tube 4 inches outside diameter and 3.73 inches inside diameter, the weight of the iron being taken at .28 pound per cubic inch. Ans. $5\frac{1}{4}$ lb.

(27) The dimensions of a return-tubular boiler are as follows: Diameter, 60 inches; length between heads, 16 feet; outside diameter of tubes, $3\frac{1}{2}$ inches; number of tubes, 64; distance of mean water-line from top of boiler, 18 inches. (*a*) Compute the steam space of the boiler in cubic feet. (*b*) Determine the number of gallons of water that will be required to fill the boiler up to the mean water level.

Ans. $\begin{cases} (a) & 79.2 \text{ cu. ft.} \\ (b) & 1{,}246 \text{ gal., nearly.} \end{cases}$

(28) The length of the circumference of the base of a cone is 18.8496 inches and its slant height is 10 inches. Find the area of the entire surface of the cone.

Ans. 122.5224 sq. in.

(29) If the altitude of the above cone were 9 inches, what would be its volume? Ans. 84.8232 cu. in.

(30) A square vat is 11 feet deep, 15 feet square at the top, and 12 feet square at the bottom. How many gallons will it hold? Ans. 15,058.29 gal.

(31) How many pails of water would be required to fill the vat, the pail having the following dimensions: Depth,

11 inches; diameter at the top, 12 inches; diameter at the bottom, 9 inches? Ans. 3,627.28.

(32) Find (*a*) the area of the surface, and (*b*) the cubical contents of a ball $22\frac{1}{2}$ inches in diameter.

Ans. $\begin{cases} (a) & 1{,}590.435 \text{ sq. in.} \\ (b) & 5{,}964.1313 \text{ cu. in.} \end{cases}$

(33) (*a*) What is the volume and area of a cylindrical ring whose outside diameter is 16 inches and inside diameter 13 inches? (*b*) If made of cast iron, what is its weight? Take the weight of 1 cubic inch of cast iron as .261 pound. Ans. Weight = 21 lb.

14) Why is not the weight of a given body the same at ery point on the surface of the earth?

(15) Determine the mass of a body that weighs 346 ds at a place where *g* is equal to 32.174. Ans. 10.75+.

) How does the position of a body above or below surface of the earth affect its weight?

.7) A locomotive weighing 30 tons has to overcome a stant force of 15 pounds per ton when it is in motion. at total force must the locomotive exert so that its speed y increase at the rate of 3 feet per second?

Ans. 6,047 lb.

(18) What will be the momentum of the locomotive in the eding example when it has attained a velocity of 1 mile hour? Ans. 2,736.3 lb.

9) Define (*a*) work; (*b*) power; (*c*) energy.

) What horsepower is required to raise a body weigh-66,000 pounds through a distance of 80 feet in ½ hour?

Ans. 5⅓ H. P.

(21) A body weighing 6,432 pounds is moving with a constant velocity of 60 feet per second. What horsepower will be required to bring the body to rest in 3 minutes?

Ans. $3\frac{7}{11}$ H. P.

(22) What is the tangential pressure on the crank of an engine when the crank is on either dead center?

(23) What is friction?

(24) A body weighing 5,000 pounds rests on a horizontal surface. In order to slide the body along the surface, a horizontal force of 300 pounds must be exerted. What is the coefficient of friction in this particular case? Ans. .06.

(25) A crosshead weighing 1,000 pounds and having bronze shoes slides on a slightly greased, horizontal wrought-iron guide. If the contact area of the bronze shoe is 100 square inches, what will be (*a*) the total friction and (*b*) the friction per square inch of contact surface?

Ans. $\begin{cases} (a) & 160 \text{ lb.} \\ (b) & 1.6 \text{ lb.} \end{cases}$

(26) What is the center of gravity of a body?

(27) Explain the method of finding the common center of gravity of several bodies whose weights and the distances between whose centers of gravity are known.

(28) Give a practical method of determining the center of gravity of a solid body.

(29) Define (*a*) centrifugal force; (*b*) centripetal force.

(30) What is the relation between the centrifugal and centripetal forces of a revolving body?

(31) A body weighing 10 pounds revolves at a speed of 60 revolutions per minute about a point 6 feet from its center of gravity. What is the centrifugal force tending to pull the body from the point about which it revolves?

Ans. 73.44 lb.

(32) Name the three states of equilibrium and give examples of each.

(33) In the case of a body at rest, what are the conditions of the forces acting on that body?

(34) How is the condition of equilibrium of the forces acting on a body affected by the position of its "line of direction" with respect to the base?

MACHINE ELEMENTS.

EXAMINATION QUESTIONS.

(1) Define (*a*) lever; (*b*) weight arm; (*c*) force arm; (*d*) fulcrum.

(2) What is the condition necessary for the equilibrium of the lever?

(3) State the general rule that expresses the relation existing between the weight, the force, and the distances through which they move.

(4) What must be the length of the weight arm in order that a force of 12 pounds at a distance of 20 inches from the fulcrum will raise a weight of 100 pounds at the end of the weight arm? Ans. 2.4 in.

(5) Into what two classes may pulleys be divided in reference to their construction?

(6) What advantages have split pulleys over solid pulleys?

(7) Explain how a crowned pulley tends to prevent the slipping off of the belt.

(8) What is meant by "balancing pulleys," and how is it accomplished?

(9) Define the terms "driver" and "driven" as applied to pulleys.

[illegible] will be required which, [illegible] will cause the [illegible] revolutions per minute? Ans. 18 in.

[illegible] 12 inches in [illegible] whose diameter is 8 inches, [illegible] per minute?

Ans. [illegible] per min.

The distance between the centers of two pulleys, [illegible] is 40 feet. [illegible] Ans. 83 ft.

[illegible] feet per minute is [illegible] If the arc of contact on [illegible] should the belt be?

Ans. 28 in.

[illegible] will be transmitted by the belt [illegible] is reduced to [illegible] feet per [illegible] Ans. [illegible] H. P.

[illegible] belt were substituted [illegible] Ans. [illegible] in.

[illegible] that [illegible]

[illegible] prevent slipping?

[illegible] slipping belts and their remedies [illegible]

[illegible] methods used in joining belts. [illegible]

[illegible] should be observed when using [illegible]

(21) Define the terms "driver" and "follower" as applied to a train of gear-wheels.

(22) In Fig. [illegible], if the diameter of *A* is 40 inches, that of *F* 30 inches, and if *B* has 12 teeth, *C* 80 teeth, *D* 24 teeth, and *E* 36 teeth, find the weight that a force of 50 pounds at *P* can raise. Ans. 675 lb.

(23) What is (*a*) circular pitch? (*b*) diametral pitch?

(24) What are the most common forms of teeth used in ordinary practice?

(25) What advantages have involute teeth over epicycloidal teeth?

(26) Find the pitch diameter of a gear-wheel having 60 teeth and a circular pitch of 1.152 inches. Ans. 22 in.

(27) What is the circular pitch of a gear-wheel 30 inches in diameter having 60 teeth? Ans. 1.57 in.

(28) What is the over-all diameter of a gear-wheel having 80 teeth with a diametral pitch of 8? Ans. 10¼ in.

(29) Find the number of teeth in a gear-wheel whose outside diameter is 7½ inches and whose diametral pitch is 8. Ans. 58 teeth.

(30) What distinguishes a fixed pulley from a movable pulley?

(31) In a certain combination of pulleys there are seven movable ones. Neglecting losses due to friction, how heavy a weight can a force of 150 pounds raise when applied to the free end of the rope? Ans. 2,100 lb.

(32) How does friction affect the force required to raise a given weight by means of a rope or chain and pulleys?

(33) In an ordinary block and tackle having four movable pulleys, what is the probable actual force that must be applied to raise a weight of 1,500 pounds? Ans. 312.5 lb.

(34) In what respect is the Weston differential pulley block better than the ordinary block and tackle?

(35) An inclined plane is 70 feet long and 12 feet high. What force acting parallel to the plane will be required to sustain a weight of 600 pounds on the plane? Ans. 103 lb.

(36) What weight will a force of 36 pounds acting parallel to the base of the plane be able to sustain on an inclined plane having a base 50 feet long and a height of 15 feet? Ans. 120 lb.

(37) What is the probable actual weight that can be raised by means of a screw jack that has a screw 2¼ inches in diameter with 6 threads to the inch if a force of 50 pounds is applied at the end of a lever 20 inches from the shaft?

Ans. 2,356.2 lb.

(38) Define (*a*) velocity ratio; (*b*) efficiency.

(39) What is the efficiency of the screw jack of question 37?

Ans. 6¼ per cent.

ELEMENTS OF ELECTRICITY AND MAGNETISM.

EXAMINATION QUESTIONS.

(1) Fig. I represents a helix of wire around which an electric current is supposed to be circulating in the direction indicated by the arrows. Which of the two ends, *a* or *b*, is the north pole of the solenoid? Why?

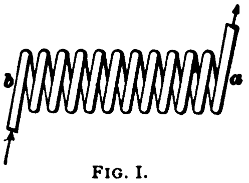


FIG. I.

(2) What will be the sign of the static charge developed (*a*) on a glass rod when rubbed with fur? (*b*) on a piece of sealing-wax when rubbed with silk?

(3) The separate resistances of two branches *A* and *B* of a derived, or shunt, circuit are 16.2 and 14.1 ohms, respectively. If the sum of the currents in the two branches is 6.37 amperes, what is the current in each branch?

Ans. { 2.9643 amperes in branch *A*.
3.4057 amperes in branch *B*.

(4) In a closed circuit, the resistance between two points is 2.3 ohms. (*a*) What current flowing between these points will cause a difference of potential of 58.4 volts?

For notice of the copyright, see page immediately following the title page.

(*b*) What is the power in watts dissipated between these two points? (*c*) Give its equivalent in horsepower.

Ans. (*a*) 25.3913 amperes.
(*b*) 1,482.8521 watts.
(*c*) 1.9877 horsepower.

(5) Fig. II represents a closed circuit consisting of a voltaic battery *B* and two conductors *X* and *Y* connected in series. The internal resistance of the battery is 17.2 ohms, and the separate resistances of the conductors *X* and *Y* are, respectively, 8.2 and 11.3 ohms. What is the total E. M. F. in volts generated by the battery if a current of .75 ampere flows through the circuit? Find the difference of potential in volts between *a* and *b*, between *b* and *c*, and between *c* and *a*.

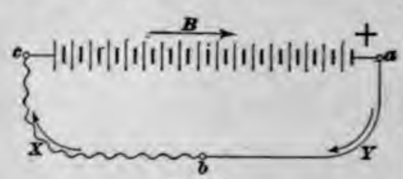


FIG. II.

Ans. Total E. M. F. developed by battery = 27.525 volts.
Difference of potential between *a* and *b* = 8.475 volts.
Difference of potential between *b* and *c* = 6.15 volts.
Difference of potential between *c* and *a* = 14.625 volts.

(6) A voltaic battery whose internal resistance is 36.2 ohms is connected to a copper wire having a resistance of 21.7 ohms. What is the total electromotive force in volts generated in the battery if a current of .127 ampere flows through the circuit? Ans. 7.3533 volts.

(7) How many coulombs of electricity pass through a circuit in 2¼ hours when the strength of current is 8.32 amperes? Ans. 67,392 coulombs.

(8) Given, electromotive force = 112.5 volts and strength of current = 12.2 amperes; find the power in watts.

Ans. 1,372.5 watts.

(9) The resistance of a copper wire is 43.2 ohms at 60° F.; find its resistance at 85° F. Ans. 45.5274 ohms.

(10) The separate resistances of three conductors *A*, *B*, and *C* are, respectively, 37, 45, and 72 ohms. What is their joint resistance when connected in parallel?

Ans. 15.8383 ohms.

(11) The separate resistances of four conductors *A*, *B*, *C*, and *D* are, respectively, 3, 19, 72, and 111 ohms; find their joint resistance when connected in series. Ans. 205 ohms.

(12) How much energy in *joules* is expended in a closed circuit during 1¼ hours in which the current is maintained at 14.2 amperes, the resistance of the circuit being 8 ohms?

Ans. 7,259,040 joules.

(13) In Fig. III, the difference of potential between *a* and *b* is 11.6 volts. If the strength of the current in branch *A* is 6.7 amperes and the strength of the current in *B* is 4.9 amperes, what is the separate resistance of each branch?

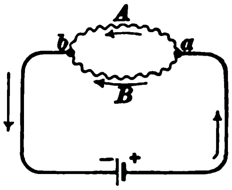


FIG. III.

Ans. $\begin{cases} \text{The separate resistance of branch } A = 1.7313 \text{ ohms.} \\ \text{The separate resistance of branch } B = 2.3673 \text{ ohms.} \end{cases}$

(14) The E. M. F. of a battery is 22.4 volts and its internal resistance is 13.4 ohms. What is the resistance of an external conductor connected to the battery when the current flowing in the circuit is .43 ampere?

Ans. 38.693 ohms.

(15) What must have been the strength of current in amperes in a closed circuit through which 368,422 coulombs of electricity passed in 4½ hours? Ans. 22.7421 amperes.

(16) Find the work done in foot-pounds when a current of 2.4 amperes flows against a resistance of 45 ohms for 50 minutes. Ans. 573,324.48 foot-pounds.

(17) Given, the electromotive force = 525 volts and strength of current = 12.5 amperes; express the number of horsepower. Ans. 8.7969 horsepower.

(18) (*a*) How many watts are dissipated by a current of 110 amperes flowing against a resistance of 4.2 ohms? (*b*) Give its equivalent in horsepower.

Ans. { (*a*) 50,820 watts.
(*b*) 68.1233 horsepower.

(19) If the resistance of 1,000 feet of round copper wire .1 inch in diameter is 1 ohm, find the resistance of 2,000 feet of square copper wire .1 inch on a side. Ans. 1.5708 ohms.

(20) Find the equivalent of 54,200 watts in horsepower.
Ans. 72.6541 horsepower.

(21) The specific resistance of mercury is 37.15 microhms per cubic inch; find the resistance in ohms of a round column of mercury 72.3 inches high and .04 inch in diameter, at 32° F. Ans. 2.1368 ohms.

(22) The total E. M. F. developed within a battery is 45 volts and the internal resistance of the battery is 33 ohms; find the strength of current flowing when the battery is connected in circuit with a resistance of 30 ohms.
Ans. .7143 ampere.

(23) An E. M. F. of 510 volts is consumed in an electric receptive device and a current of 24.3 amperes is flowing in the circuit; calculate the power in watts supplied to the receptive device. Ans. 12,393 watts.

(24) A battery of twenty-four cells is arranged in multiple-series as shown in Fig. IV. There are four groups of six cells each, connected in series, and the four groups are connected in multiple, or parallel, to two main conductors *c* and *c'*. If the E. M. F. developed by each cell is 1.5 volts, what will be the E. M. F. indicated by the voltmeter *V M* when its binding posts are connected to the main conductors *c* and *c'*, as shown in the figure?

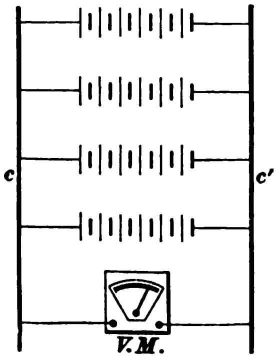


FIG. IV.

(25) The available E. M. F. developed by an electric source is 250 volts and a current of 65.7 amperes is flowing from it; determine its output in horsepower.

Ans. 22.0174 horsepower.

(26) Fig. V represents a horseshoe electromagnet *M* around which is wound an insulated conductor *c c′ c″*. If a current circulates through the conductor in the direction as indicated by the arrows, which of the two ends, *a* or *b*, is the south pole of the magnet?

(27) Give the names of all the known magnetic substances.

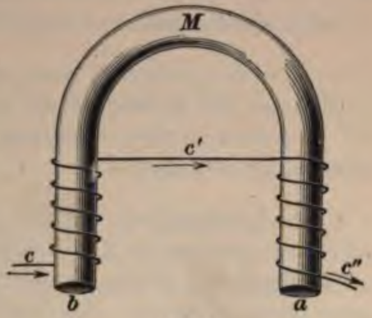


FIG. V.

(28) A compass *C* is placed between the north and the south poles of two magnets, as shown in Fig. VI. Toward which pole will the north pole of the compass needle tend to point, and why?

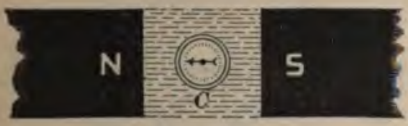


FIG. VI.

(29) A compass *C* is placed alongside a bar magnet opposite the neutral line, as shown in Fig. VII. Toward which pole of the magnet will the south pole of the compass needle tend to point, and why?

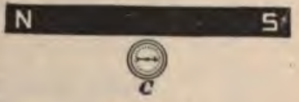


FIG. VII.

(30) In an electromagnet, Fig. VIII, the coil of wire is wound around an iron core in a right-handed spiral. Through which end, *a* or *b*, of the wire must the current enter in order to produce the polarity as represented in the figure? Why?

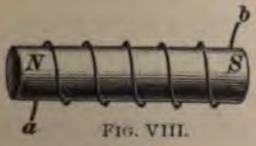


FIG. VIII.

sistance of a platinum wire 112 feet 6 inches ong is 100 hms; calculate the resistance of 11.7 inches of he same w other conditions remaining unchanged.
Ans. .8736 ohm.

(32) The r sistance of a German-silver wire is 91.8 ohms at 45° F.; calculate its resistance when its temperature is 72° F., other conditions remaining unchanged.
Ans. 92.4048 ohms.

(33) If the resis per wire is .144 ohm at 87° F., what is its 41° F., other conditions remaining unchange
Ans. .131 ohm.

(34) The diagram, sents a particular pattern of resistance box for bridge, with battery and galvanometer circuits pr nected for taking resistance measurements. An own resistance *X* is connected

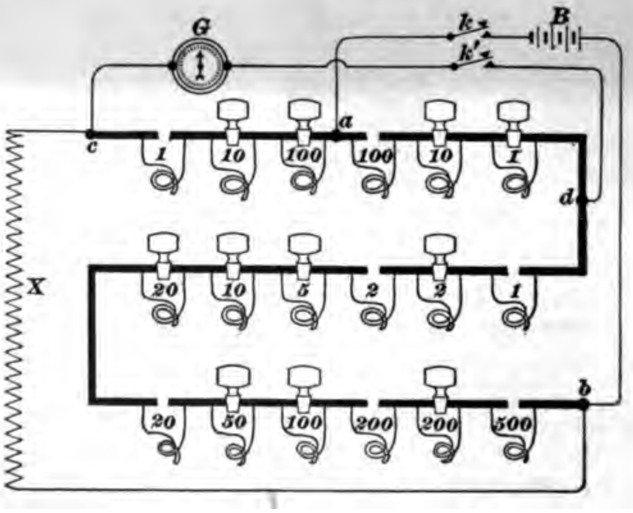


FIG. IX.

to the terminals *c* and *b*. After adjusting the resistances of the same by withdrawing the plugs, as represented by the open spaces between the contacts, the galvanometer shows no deflection when the keys *k* and *k'* are pressed and the battery and galvanometer circuits are closed. Under these conditions, what is the resistance of *X* ? Ans. 7.23 ohms.

(35) The total E. M. F. developed in a closed circuit is 36 volts, the internal resistance is 18 ohms, and the external resistance is 24 ohms; determine the strength of current in amperes flowing in the circuit. Ans. .8571 ampere.

(36) A current of 2.7 amperes is flowing in a closed circuit. If the total E. M. F. developed in the circuit is 12.6 volts, what is the total resistance of the circuit ?

Ans. 4.6667 ohms.

(37) The external resistance of a closed circuit is 31.5 ohms and the internal is 11 ohms. If a current of .8 ampere is flowing through the circuit, what is the total E. M. F. in volts developed ? Ans. 34 volts.

(38) The total E. M. F. developed in an electric source is 250 volts. If 10 per cent. of this E. M. F. is required to transmit a current of 80 amperes to and from a receptive device situated 600 feet from the source, (*a*) what is the total resistance of the two conductors, and (*b*) what is their resistance per foot, considering each to be 600 feet long ?

Ans. { (*a*) .3125 ohm.
(*b*) .00026 ohm per foot.

(39) The internal resistance of a battery is 8.1 ohms and the total E. M. F. developed in it is 24 volts. What is the available or external E. M. F. of the battery when the circuit is completed by an external conductor offering a resistance of 15.9 ohms ? Ans. 15.9 volts.

(40) The separate resistances of two branches *A* and *B* of a derived circuit are 1.2 and 2.2 ohms, respectively. If the sum of the currents in the two branches is 45 amperes, what is the current in each branch ?

Ans. { The current in branch *A* is 29.1176 amperes.
The current in branch *B* is 15.8824 amperes.

(41) (*a*) What is a kilowatt? (*b*) If a machine has a capacity of 150 kilowatts, what is its capacity in horsepower?
Ans. (*b*) 201.072 horsepower.

(42) A 250-volt dynamo has a capacity of 300 kilowatts, t will be its full-load current? Ans. 1,200 amperes.

(43) (*a*) What is a kilowatt-hour? (*b*) If an average of 0 amperes flows in a 110-volt circuit for 9 hours and 30 min-tes, what is the amount of work accomplished, expressed n kilowatt-hours? Ans. (*b*) 41.8 kilowatt-hours.

DYNAMOS AND MOTORS.

EXAMINATION QUESTIONS.

(1) Suppose that a ring-core armature of a bipolar dynamo is wound with 200 complete turns of wire that are properly connected to the segments of a commutator for generating a continuous current, and that there are 6,250,000 lines of force passing through the armature from the poles of the field magnets. If the strength of the field remains constant and the armature is rotated at a uniform speed of 1,200 revolutions per minute, what is the total electromotive force in volts generated in the armature?

Ans. 250 volts.

(2) If the resistance of the field coils in a shunt-wound dynamo is 440 ohms, and the difference of potential between the brushes when the external circuit is open is 220 volts, what is the strength of current in the field coils?

Ans. .5 ampere.

(3) What is the distinction between an alternating current and a continuous current?

(4) Fig. I shows a cross-sectional view of a uniform magnetic field taken at right angles to the direction of the lines of force; that is, the dots represent the ends of the lines of force, their direction being downwards, piercing the paper. *C* represents a closed coil of some conducting material, such as copper, that is placed in the magnetic field with its plane

For notice of the copyright, see page immediately following the title page.

es to the direction of the lines of force. If the coil is suddenly moved from its original position to another position in the field, as to C, as shown by the dotted coil, without changing the relative position of its plane with the direction of the lines of force, state whether or not a momentary current will circulate around the coil when the movement is made, and give the reason.

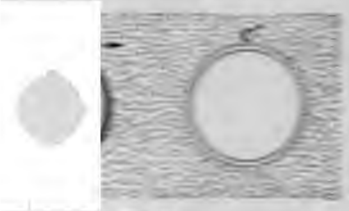


FIG. I.

(5) The output from a certain dynamo is 17,500 watts, and its efficiency at this output is 87.5 per cent. If 2.6 per cent. of the input is used to excite the field magnets, state the field loss in watts. Ans. 520 watts.

(6) The resistance of the shunt field coils of a constant-potential dynamo is 55 ohms and the difference of potential between the brushes when the armature is revolving at normal speed is 110 volts. How many watts are required to excite the field magnets? Ans. 220 watts.

(7) What is a commutator and for what is it used?

(8) The output of a dynamo is 65,000 watts and its efficiency at this output is 90.5 per cent.; determine the input to the armature and express the same in horsepower. Ans. 96.28 horsepower.

(9) Fig. II shows the connections of a shunt-wound dynamo and the direction in which the field coils are wound. If the current flows in the direction indicated by the arrowheads, which of the two pole pieces, P or P', is the north pole? Suppose that the winding of the right-hand coil were reversed, which pole piece would then be the north pole?

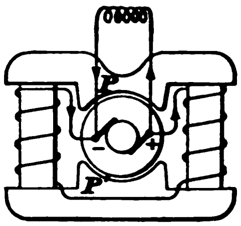


FIG. II.

(10) The input to a dynamo is 10 horsepower and its output is 6,341 watts. What is its efficiency at this load?
Ans. 85 per cent.

(11) Fig. III represents a cross-sectional view of a uniform magnetic field. The dots represent an end view of the lines of force, their direction being downwards, piercing the paper; or, in other words, the observer is looking along the lines of force toward the face of a south pole; *c* represents a moving conductor placed in the magnetic field with its length at right angles to the direction of the lines of force; its two ends are connected to an external circuit consisting of the resistance *R*. If the conductor is moving upwards across the magnetic field in the direction shown by the arrows, in which direction will the current tend to flow in the circuit?

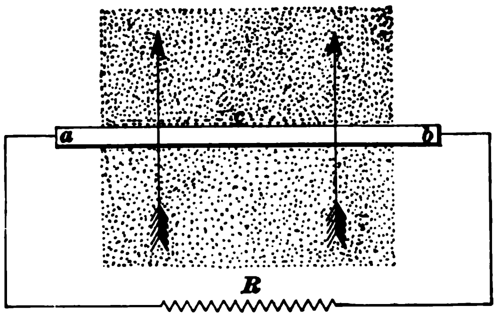


FIG. III.

(12) A dynamo shows an efficiency of 85 per cent. when its output is 11,900 watts, and 1.8 per cent. of the input is lost in the core by eddy currents and hysteresis. What is the core loss in watts?
Ans. 252 watts.

(13) (*a*) What is meant by the *counter torque* of a dynamo? (*b*) What causes it?

(14) A dynamo generates 125 volts at a normal load of 120 amperes; if the resistance of the armature from brush to

onductors, armature reactions, etc., remain nchanged ? Ans. $333\frac{1}{3}$ volts.

(26) To hat are the following losses in a dynamo due: (*a*) core loss ? (*b*) armature loss ? (*c*) field loss ?

(27) In Fig. V, the observer is looking at the face of a north magnetic pole *N* and a straight conductor *C* is placed in a vertical position in front of the ith its length at right to the direction of the force as they pass from . If the two ends of the or are connected to the ls of the battery *B*, and a flows through the circuit rmed in the direction d by the arrowheads, toward which side, *a* or *b*, of pole face will the conductor tend to move ?

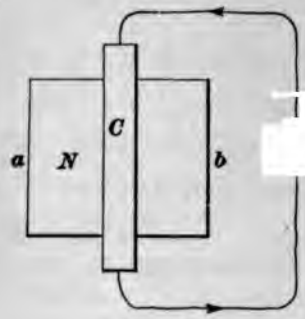


Fig. V.

(28) In a shunt-wound dynamo, if the resistance of the field coil is 650 ohms and the difference of potential between the brushes remains constant at 525 volts when the armature is rotated at a constant speed, what is the strength of current in the field coil under these conditions ?

Ans. .8076 ampere.

(29) A compound-wound dynamo generates 115 volts between its terminals when no current is flowing into the external circuit. At full load, however, the difference of potential between its terminals is 124.2 volts. What percentage of over-compounding do these figures represent ?

Ans. 8 per cent.

(30) What becomes of the heat generated in a dynamo armature ?

(31) State the differences between separately excited, shunt-wound, and series-wound dynamos.

(32) The core losses in a particular generator amount to 800 watts and the input to the generator is 64 horsepower at full load. Determine the percentage of loss in the core at this input. Ans. 1.6756 per cent.

(33) Why must the brushes of the dynamo be shifted ahead of the neutral point when operating under load?

(34) What is the difference between a consequent pole and a salient pole?

A KEY

TO ALL THE

QUESTIONS AND EXAMPLES

CONTAINED IN THE

EXAMINATION QUESTIONS

INCLUDED IN THIS VOLUME.

The Keys that follow have been divided into sections corresponding to the Examination Questions to which they refer, and have been given corresponding section numbers. The answers and solutions have been numbered to correspond with the questions. When the answer to a question involves a repetition of statements given in the Instruction Paper, the reader has been referred to a numbered article, the reading of which will enable him to answer the question himself.

To be of the greatest benefit, the Keys should be used sparingly. They should be used much in the same manner as a pupil would go to a teacher for instruction with regard to answering some example he was unable to solve. If used in this manner, the Keys will be of great help and assistance to the student, and will be a source of encouragement to him in studying the various papers composing the Course.

ARITHMETIC.

(PART 1.)

(1) See Art. **1.**

(2) See Art. **3.**

(3) See Arts. **5** and **6.**

(4) See Arts. **10** and **11.**

(5) 980 = Nine hundred eighty.
605 = Six hundred five.
28,284 = Twenty-eight thousand two hundred eighty-four.
9,006,042 = Nine million six thousand forty-two.
850,317,002 = Eight hundred fifty million three hundred seventeen thousand two.
700,004 = Seven hundred thousand four.

(6) Seven thousand six hundred = 7,600.
Eighty-one thousand four hundred two = 81,402.
Five million four thousand seven = 5,004,007.
One hundred and eight million ten thousand one = 108,010,001.
Eighteen million six = 18,000,006.
Thirty thousand ten = 30,010.

For notice of copyright, see page immediately following the title page.

adding whole numbers, place the numbers to be directly under each other, so that the extreme right-

$$\begin{array}{rl} 3290 & \\ 504 & \\ 865403 & \\ 2074 & \\ 81 & \\ 7 & \\ \hline 871359 & \text{Ans.} \end{array}$$

hand figures will stand in the same column, regardless of the position of those at the left. Add the first column of figures at the extreme right, which equals 19 units, or 1 ten and 9 units. We place 9 units under the units column and reserve 1 ten for the column of tens, 8 + 7 + 9 + 1 = 25 tens, or 2 hundreds and tens. Place 5 tens under the tens column and reserve hundreds for the hundreds column. 4 + 5 + 2 + 2 = 13 reds, or 1 thousand and 3 hundreds. Place 3 hundreds the hundreds column and reserve the 1 thousand for usands column. 2 + 5 + 3 + 1 = 11 thousands, or ousand and 1 thousand. Place the 1 thousand in of thousands and reserve the 1 ten-thousand column of ten-thousands. 6 + 1 = 7 ten-thousands. this 7 ten-thousands in the ten-thousands column. There is but one figure, 8, in the hundreds of thousands place in the numbers to be added, so it is placed in the hundreds of thousands column of the sum.

A simpler (though less scientific) explanation of the same problem is the following: 7 + 1 + 4 + 3 + 4 + 0 = 19; write the 9 and reserve the 1. 8 + 7 + 0 + 0 + 9 + 1 reserved = 25; write the 5 and reserve the 2. 0 + 4 + 5 + 2 + 2 reserved = 13; write the 3 and reserve the 1. 2 + 5 + 3 + 1 reserved = 11; write the 1 and reserve 1. 6 + 1 reserved = 7; write the 7. Bring down the 8 to its place in the sum.

(8)

$$\begin{array}{rl} 709 & \\ 8304725 & \\ 391 & \\ 100302 & \\ 300 & \\ 909 & \\ \hline 8407336 & \text{Ans.} \end{array}$$

(9) The steam engine, during the 12-hour test, showed that the number of revolutions made were 150,508, since 12,600 + 12,444 + 12,467 + 12,528 + 12,468 + 12,590 + 12,610 + 12,589 + 12,576 + 12,558 + 12,546 + 12,532 = 150,508 rev.

$$\begin{array}{rl}
12600 & \text{revolutions.} \\
12444 & \text{revolutions.} \\
12467 & \text{revolutions.} \\
12528 & \text{revolutions.} \\
12468 & \text{revolutions.} \\
12590 & \text{revolutions.} \\
12610 & \text{revolutions.} \\
12589 & \text{revolutions.} \\
12576 & \text{revolutions.} \\
12558 & \text{revolutions.} \\
12546 & \text{revolutions.} \\
12532 & \text{revolutions.} \\
\hline
150508 & \text{revolutions.} \quad \text{Ans.}
\end{array}$$

(10) In subtracting whole numbers, place the subtrahend, or smaller number, under the minuend, or larger number, so that the right-hand figures stand directly under each other. Begin *at the right* to subtract. We cannot subtract 8 units from 2 units, so we take 1 ten from the 6 tens and add it to the 2 units. 1 *ten* = 10 *units*, so we have 10 units + 2 units = 12 units. Then, 8 units from 12 units leaves 4 units. We took 1 ten from 6 tens, so only 5 tens remain. 3 tens from 5 tens leaves 2 tens. In the hundreds column we have 3 hundreds from 9 hundreds leaves 6 hundreds. We cannot subtract 3 thousands from 0 thousands, so we take 1 ten-thousand from 5 ten-thousands and add it to the 0 thousands. 1 *ten-thousand* = 10 *thousands*, and 10 thousands + 0 thousands = 10 thousands. Subtracting, we have 3 thousands from 10 thousands leaves 7 thousands. We took 1 ten-thousand from 5 ten-thousands and have 4 ten-thousands remaining. Since there are no ten-thousands in the subtrahend, the 4 in the

$$(a) \quad \begin{array}{r}
50962 \\
3338 \\
\hline
47624
\end{array} \quad \text{Ans.}$$

[illegible] brought down into [illegible] 4 from 4 leaves 4.

[illegible] Ans.

[illegible] Ans. [illegible] Ans.

[illegible] gallons in the tank at the beginning of the [illegible]

[illegible] gallons pumped in during the morning.
[illegible] gallons in the tank after [illegible] gallons were [illegible]

[illegible] gallons drawn out during the morning.
[illegible] gallons in the tank at the beginning of the afternoon.

[illegible] gallons pumped in during the afternoon.
[illegible] gallons in the tank after [illegible]75 gallons were [illegible]

[illegible] gallons drawn out during the afternoon.
[illegible] gallons remaining in the tank at night. Ans.

[illegible] numbers, place the [illegible] and multiply each term [illegible] of the multiplier, writing [illegible] obtained under the [illegible]

[illegible] Ans.

7 times 7 units = 49 units, or 4 tens and 9 units. We write the 9 units and reserve the 4 tens. 7 times 8 tens = 56 tens; 56 + 4 tens reserved = 60 tens, or 6 hundreds and 0 tens. Write the 0 tens and reserve the 6 hundreds. 7 × 3 hundreds = 21 hundreds; 21 + 6 hundreds reserved = 27 hundreds, or 2 thousands and 7 hundreds. Write the 7 hundreds and reserve the 2 thousands. 7 × 6 thousands =

42 thousands; 42 + 2 thousands reserved = 44 thousands, or 4 ten-thousands and 4 thousands. Write the 4 thousands and reserve the 4 ten-thousands. 7 × 2 ten-thousands = 14 ten-thousands; 14 + 4 ten-thousands reserved = 18 ten-thousands, or 1 hundred-thousand and 8 ten-thousands. Write the 8 ten-thousands and reserve the 1 hundred-thousand. 7 × 5 hundred-thousands = 35 hundred-thousands; 35 + 1 hundred-thousand reserved = 36 hundred-thousands. Since there are no more figures in the multiplicand to be multiplied, we write the 36 hundred-thousands in the product. This completes the multiplication.

A simpler (though less scientific) explanation of the same problem is the following:

7 times 7 = 49; write the 9 and reserve the 4. 7 times 8 = 56; 56 + 4 reserved = 60; write the 0 and reserve the 6. 7 times 3 = 21; 21 + 6 reserved = 27; write the 7 and reserve the 2. 7 × 6 = 42; 42 + 2 reserved = 44; write the 4 and reserve 4. 7 × 2 = 14; 14 + 4 reserved = 18; write the 8 and reserve 1. 7 × 5 = 35; 35 + 1 reserved = 36; write the 36.

$$\text{(}b\text{)}\quad\begin{array}{r}
700298\\
17\\
\hline
4902086\\
700298\\
\hline
11905066
\end{array}\ \text{Ans.}$$

In this case the multiplier is 17 *units*, or 1 *ten* and 7 *units*, so that the product is obtained by adding two partial products, namely, 7 × 700,298 and 10 × 700,298. The actual operation is performed as follows:

7 times 8 = 56; write the 6 and reserve the 5. 7 times 9 = 63; 63 + 5 reserved = 68; write the 8 and reserve the 6. 7 times 2 = 14; 14 + 6 reserved = 20; write the 0 and reserve the 2. 7 times 0 = 0; 0 + 2 reserved = 2; write the 2. 7 times 0 = 0; 0 + 0 reserved = 0; write the 0. 7 times 7 = 49; 49 + 0 reserved = 49; write the 49.

To multiply by the 1 ten we have 1 ten times 8 units = 8 tens and 0 units. We do not write the 0 units, but write the 8 tens under the 8 tens in the first partial product above. We next multiply 1 ten and 9 tens = 90 tens = 9 hundreds + 0 tens; write the 9 hundreds under the 0 hundreds of the first partial product above. Again, 1 ten times 2 hundreds = 2 thousands; write the 2 thousands under the 2 thousands above. 1 ten times 0 thousands = 0 ten-thousands; write the 0 in the ten-thousands place. 1 ten times 0 ten-thousands = 0 hundred-thousands; write the 0 in the hundred-thousands place. 1 ten times 7 hundred-thousands = 7 millions; write the 7 in the millions place. This completes the second partial product. Add the two partial products; their sum equals the entire product.

(*c*) Multiply any two of the numbers together, and multiply their product by the third number.

```
    217
    103
    ---
    651
   000
  217
  -----
  22351
     67
  ------
 156457
134106
-------
1497517 Ans.
```

(14) If your watch ticks every second, to find how many times it ticks in 1 week, it is necessary to find the number of seconds in 1 week.

```
      60 seconds = 1 minute.
      60 minutes = 1 hour.
    3600 seconds = 1 hour.
      24 hours = 1 day.
   14400
   7200
   -----
   86400 seconds = 1 day.
       7 days = 1 week.
  ------
  604800 seconds in 1 week, or the number of times that
         your watch ticks in a week.   Ans.
```

(15) (*a*) If an engine and boiler are worth $3,246, and a building is worth three times as much, plus $1,200, then [illegible] building is worth

$$\begin{array}{rr} & 3246 \\ & 3 \text{ times} \\ \hline & 9738 \\ \text{plus} & 1200 \\ \hline & \$10938 \end{array} = \text{value of building.}$$

If the tools are worth twice as much as the building, plus $1,875, then the tools are worth

$$\begin{array}{rr} & 10938 \\ & 2 \\ \hline & 21876 \\ \text{plus} & 1875 \\ \hline & \$23751 \end{array} = \text{value of tools.}$$

$$\begin{array}{lr} \text{Value of building} = & 10938 \\ \text{Value of tools} = & 23751 \\ \hline & \$34689 \end{array} = \text{value of the building and tools. Ans.}$$

(*b*)

$$\begin{array}{lr} \text{Value of engine and boiler} = & 3246 \\ \text{Value of building and tools} = & 34689 \\ \hline & \$37935 \end{array} = \text{value of the whole plant. Ans.}$$

(16) (*a*)

$$\begin{array}{r} 84)\,962842.0000\,(11462.4047+ \\ 84 \\ \hline 122 \\ 84 \\ \hline 388 \\ 336 \\ \hline 524 \\ 504 \\ \hline 202 \\ 168 \\ \hline 340 \\ 336 \\ \hline 400 \\ 336 \\ \hline 640 \\ 588 \\ \hline 52 \end{array}$$

Ans.

84 is contained once in 96. Place 1 as the first figure in the quotient and multiply the divisor 84 by it. Subtract the product, which is 84, from 96, leaving a remainder of 12. Bring down the next figure in the dividend, which is 2, and annex it to 12, making a new dividend of 122.

84 is contained in 122 once. Place 1 as the second figure in the quotient and multiply the divisor 84 by it. Subtract the product (84) from 122, leaving a remainder of 38. Bring down the next figure in the dividend, which is 8, and annex it to 38, making a new dividend of 388.

84 is contained in 388 4 times. Place 4 as the third figure in the quotient and multiply the divisor 84 by it. The product is 336. Subtract the product from 388, leaving a remainder of 52. Bring down the next figure, which is 4, and annex it to 52, making a new dividend of 524.

84 is contained in 524 6 times. Place 6 as the fourth figure in the quotient. Multiply the divisor 84 by it and subtract the product (504) from 524, leaving a remainder of 20. Bring down the next figure, which is 2, and annex it to 20, making a new dividend of 202.

84 is contained in 202 2 times. Place 2 as the fifth figure in the quotient. Multiply the divisor 84 by it and subtract the product (168) from 202, leaving a remainder of 34. If it is desired to carry the quotient to 4 decimal places, annex 4 ciphers to the dividend and continue in the same way. In the quotient point off as many decimal places as there are ciphers annexed, or, in this case, 4 decimal places.

(*b*)

$$\begin{array}{r@{\,}ccccccccl}
63) & 3 & 9 & 7 & 2 & 8. & 0 & 0 & 0 & (630.603+\quad\text{Ans.} \\
 & 3 & 7 & 8 & & & & & & \\ \cline{2-4}
 & & 1 & 9 & 2 & & & & & \\
 & & 1 & 8 & 9 & & & & & \\ \cline{3-6}
 & & & & 3 & 8 & 0^{*} & & & \\
 & & & & 3 & 7 & 8 & & & \\ \cline{5-9}
 & & & & & & 2 & 0 & 0 & \\
 & & & & & & 1 & 8 & 9 & \\ \cline{8-9}
 & & & & & & & 1 & 1 &
\end{array}$$

*63 is not contained in 38, so we place a cipher in the quotient and bring down the next figure in the dividend, which is a cipher that has been annexed, making a new dividend of 380. 63 is contained in 380 6 times. 6 × 63 = 378. Subtracting 378 from 380 leaves 2. Bringing down the next figure in the dividend, we have 20 for a new dividend. 63 is not contained in 20, so we place a cipher in the quotient and bring down the next cipher in the dividend, making a new dividend of 200. 63 is contained 3 times in 200.

```
(c) 108)29714.0000(275.1296  Ans.
        216
        ---
        811   (d) 135)406089.0000(3008.0666
        756           405                Ans.
        ---           ----
         554            1089*
         540            1080
         ---            ----
          140              900
          108              810
          ---              ---
           320              900
           216              810
           ---              ---
          1040               900
           972               810
           ---               ---
            680               90
            648
            ---
             32
```

* 135 is not contained in 10, so we place a cipher as the second figure in the quotient. Bringing down the next figure, 8, and annexing it to 10, we have a new dividend of 108. 135 is not contained in 108, so we place a cipher as the third figure in the quotient and bring down the next figure in the dividend, or 9, and annex it to 108, making a new dividend of 1,089. 135 is contained in 1,089 8 times. Write 8 as the fourth figure in the quotient. Multiply 135 by 8 and subtract the product (1,080) from 1,089, which leaves a remainder of 9. Bring down the next figure in the dividend, which is a cipher that has been annexed, and annex

it to the remainder 9, making a new dividend of 90. As 135 is not contained in 90, we place a 0 in the quotient and bring down another cipher from the dividend, making a new dividend of 900. 135 is contained in 900 6 times. Write 6 as the next figure in the quotient, multiply 135 by 6, and subtract the product (810) from 900, which leaves a remainder of 90. Bring down the next figure (0) in the dividend and annex it to the remainder 90, making a new dividend of 900. 135 is contained in 900 6 times. Place 6 as the next figure in the quotient and multiply the divisor by it. It is plain that each succeeding figure of the quotient will be 6. Point off *four* decimal places in the quotient, since four ciphers were annexed.

(17) If in 1 hour 10 pounds of coal are burned for every square foot of grate area and 9 pounds of water are evaporated for every pound of coal burned, then in 1 hour there would be 9 × 10 or 90 pounds of water evaporated for 1 sq. ft. of grate area; and since the grate area is 30 sq. ft., the amount of water evaporated would be 30 × 90 = 2,700 lb. Since 2,700 lb. of water are evaporated in 1 hour, in a day of 10 hours there would be 10 × 2,700 lb., or 27,000 lb. of water evaporated.

(18) If a mechanic earns $1,500 a year and his expenses are $968 per year, then he would save $1500 − $968, or $532 per year.

$$\begin{array}{r} 1500 \\ 968 \\ \hline \$532 \end{array}$$

If he saves $532 in 1 year, to save $3,724 it would take as many years as $532 is contained times in $3,724, or 7 years.

$$\begin{array}{r} 532\,)\,3724\,(\,7 \text{ years. Ans.} \\ 3724 \\ \hline \end{array}$$

(19) (*a*) (72 × 48 × 28 × 5) ÷ (84 × 15 × 7 × 6).

Placing the numerator over the denominator, the problem

$$\frac{72 \times 48 \times 28 \times 5}{84 \times 15 \times 7 \times 6} = ?$$

The 5 in the numerator and 15 in the denominator are both *divisible* by 5, since 5 divided by 5 equals 1 and 15 divided by 5 equals 3. *Cross off* the 5; also *cross off* the 15 and write the 3 *under* it. Thus,

$$\frac{72 \times 48 \times 28 \times \cancel{5}}{84 \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

72 in the numerator and 84 in the denominator are *divisible* by 12, since 72 divided by 12 equals 6 and 84 divided by 12 equals 7. *Cross off* the 72 and write the 6 *over* it; also, *cross off* the 84 and write the 7 *under* it. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times 28 \times \cancel{5}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times 7 \times 6} =$$

Again, 28 in the numerator and 7 in the denominator are *divisible* by 7, since 28 divided by 7 equals 4 and 7 divided by 7 equals 1. *Cross off* the 28 and write the 4 *over* it; also, *cross off* the 7. Thus,

$$\frac{\overset{6}{\cancel{72}} \times 48 \times \overset{4}{\cancel{28}} \times \cancel{5}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times \cancel{7} \times 6} =$$

Again, 48 in the numerator and 6 in the denominator are *divisible* by 6, since 48 divided by 6 equals 8 and 6 divided by 6 equals 1. *Cross off* the 48 and write the 8 *over* it; also, *cross off* the 6. Thus,

$$\frac{\overset{6}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \cancel{5}}{\underset{7}{\cancel{84}} \times \underset{3}{\cancel{15}} \times \cancel{7} \times \cancel{6}} =$$

Again, 6 in the numerator and 3 in the denominator are *divisible* by 3, since 6 divided by 3 equals 2 and 3 divided by 3 equals 1. *Cross off* the 6 and write the 2 *over* it; also, *cross off* the 3. Thus,

$$\frac{\overset{\overset{2}{\cancel{6}}}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \cancel{5}}{\underset{7}{\cancel{84}} \times \underset{\cancel{3}}{\cancel{15}} \times \cancel{7} \times \cancel{6}} =$$

Since there are *no two remaining numbers* (one in the numerator and one in the denominator) *divisible* by *any number* except 1, without a remainder, it is *impossible* to cancel further.

Multiply all the *uncanceled numbers* in the numerator together and divide their *product* by the *product* of all the *uncanceled* numbers in the denominator. The *result* will be the *quotient*. The *product* of all the *uncanceled numbers* in the numerator equals $2 \times 8 \times 4 = 64$; the product of all the *uncanceled* numbers in the denominator equals 7.

Hence, $\dfrac{\overset{\overset{2}{\cancel{6}}}{\cancel{72}} \times \overset{8}{\cancel{48}} \times \overset{4}{\cancel{28}} \times \cancel{5}}{\underset{7}{\cancel{84}} \times \underset{\cancel{3}}{\cancel{15}} \times \cancel{7} \times \cancel{6}} = \dfrac{2 \times 8 \times 4}{7} = \dfrac{64}{7} = 9\tfrac{1}{7}$. Ans.

(*b*) $(80 \times 60 \times 50 \times 16 \times 14) \div (70 \times 50 \times 24 \times 20)$.

Placing the numerator over the denominator, the problem becomes

$$\frac{80 \times 60 \times 50 \times 16 \times 14}{70 \times 50 \times 24 \times 20} = ?$$

The 50 in the numerator and 70 in the denominator are both *divisible* by 10, since 50 divided by 10 equals 5 and 70 divided by 10 equals 7. *Cross off* the 50 and write the 5 *over* it; also, *cross off* the 70 and write the 7 *under* it. Thus,

$$\frac{80 \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times 20} =$$

Also, 80 in the numerator and 20 in the denominator are *divisible* by 20, since 80 divided by 20 equals 4 and 20 divided by 20 equals 1. *Cross off* the 80 and write the 4 *over* it; also, cross off the 20. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times \overset{5}{\cancel{50}} \times 16 \times 14}{\underset{7}{\cancel{70}} \times 50 \times 24 \times \cancel{20}} =$$

Again, 16 in the numerator and 24 in the denominator are *divisible* by 8, since 16 divided by 8 equals 2 and 24 divided by 8 equals 3. *Cross off* the 16 and write the 2 *over* it; also, cross off the 24 and write the 3 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times 60 \times \overset{5}{\cancel{50}} \times \overset{2}{\cancel{16}} \times 14}{\underset{7}{\cancel{70}} \times 50 \times \underset{3}{\cancel{24}} \times \cancel{20}} =$$

Again, 60 in the numerator and 50 in the denominator are *divisible* by 10, since 60 divided by 10 equals 6 and 50 divided by 10 equals 5. *Cross off* the 60 and write the 6 *over* it; also, cross off the 50 and write the 5 *under* it. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{6}{\cancel{60}} \times \overset{5}{\cancel{50}} \times \overset{2}{\cancel{16}} \times 14}{\underset{7}{\cancel{70}} \times \underset{5}{\cancel{50}} \times \underset{3}{\cancel{24}} \times \cancel{20}} =$$

The 14 in the numerator and 7 in the denominator are *divisible* by 7, since 14 divided by 7 equals 2 and 7 divided by 7 equals 1. *Cross off* the 14 and write the 2 *over* it; also, cross off the 7. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{6}{\cancel{60}} \times \overset{5}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\cancel{7}}{\cancel{70}} \times \underset{5}{\cancel{50}} \times \underset{3}{\cancel{24}} \times \cancel{20}} =$$

The 5 in the numerator and 5 in the denominator are *divisible* by 5, since 5 divided by 5 equals 1. *Cross off* the 5 of the *dividend;* also, cross off the 5 of the *divisor*. Thus,

$$\frac{\overset{4}{\cancel{80}} \times \overset{6}{\cancel{60}} \times \overset{\cancel{5}}{\cancel{50}} \times \overset{2}{\cancel{16}} \times \overset{2}{\cancel{14}}}{\underset{\cancel{7}}{\cancel{70}} \times \underset{\cancel{5}}{\cancel{50}} \times \underset{3}{\cancel{24}} \times \cancel{20}} =$$

The 6 in the numerator and 3 in the denominator are *divisible* by 3, since 6 divided by 3 equals 2, and 3 divided by 3 equals 1. *Cross off* the 6 and place 2 *over* it; also, cross off the 3. Thus,

$$\frac{\text{[illegible]} \times \text{[illegible]} \times \text{[illegible]} \times 16 \times 14}{\text{[illegible]} \times \text{[illegible]} \times 24 \times 20} =$$

[illegible] $$\frac{\text{[illegible]} \times \text{[illegible]} \times \text{[illegible]} \times 16 \times 14}{\text{[illegible]} \times \text{[illegible]} \times \text{[illegible]} \times \text{[illegible]}} = 4 \times 2 \times 2 \times 2 = 32.$$

Ans.

[illegible] miles in one week, and [illegible] miles the next week, then it ran [illegible] miles − [illegible] miles, or [illegible] miles the second week.

[illegible]

[illegible] miles. Ans.

[illegible] Philadelphia to Pittsburg is [illegible] in 1 mile, in 354 miles [illegible] If the driving [illegible] in circumference, in going [illegible] a distance of 1,869,120 feet, [illegible] revolutions.

[illegible] rev. Ans.

0

ARITHMETIC.

(PART 2.)

(1) See Art. **1.**

(2) See Art. **6.**

(3) See Art. **3.**

(4) See Art. **3.**

(5) $\frac{13}{8}$ is an improper fraction, since its numerator 13 is greater than its denominator 8.

(6) $4\frac{1}{2}$; $14\frac{3}{10}$; $85\frac{4}{19}$.

(7) To reduce a fraction to its lowest terms means to change its form without changing its value. In order to do this, we must divide both numerator and denominator by the same number until we can no longer find any number (except 1) which will divide both of these terms without a remainder.

To reduce the fraction $\frac{4}{8}$ to its lowest terms, we divide both numerator and denominator by 4 and obtain as a result the fraction $\frac{1}{2}$. Thus, $\frac{4 \div 4}{8 \div 4} = \frac{1}{2}$; similarly, $\frac{4 \div 4}{16 \div 4} = \frac{1}{4}$; $\frac{8 \div 4}{32 \div 4} = \frac{2 \div 2}{8 \div 2} = \frac{1}{4}$; $\frac{32 \div 8}{64 \div 8} = \frac{4 \div 4}{8 \div 4} = \frac{1}{2}$.

(8) When the denominator of any number is not expressed, it is understood to be 1, so that $\frac{6}{1}$ is the same as $6 \div 1$, or 6.

For notice of copyright, see page immediately following the title page.

To reduce $\frac{6}{1}$ to an improper fraction whose denominator is 4, we must multiply both numerator and denominator by some number which will make the denominator of 6 equal to 4. Since the denominator is 1, by multiplying both terms of $\frac{6}{1}$ by 4, we will have $\frac{6 \times 4}{1 \times 4} = \frac{24}{4}$, which has the *same value* as 6, but has a *different form.*

(9) In order to reduce a mixed number to an improper fraction, we must multiply the whole number by the denominator of the fraction and add the numerator of the fraction to the product. This result is the *numerator* of the improper fraction, of which the *denominator* is the denominator of the fractional part of the mixed number.

$7\frac{3}{8}$ means the same as $7 + \frac{3}{8}$. In 1 there are $\frac{8}{8}$; hence, in 7 there are $7 \times \frac{8}{8} = \frac{56}{8}$. Add the $\frac{3}{8}$ of the mixed number and we obtain $\frac{56}{8} + \frac{3}{8} = \frac{59}{8}$, which is the required improper fraction.

$$13\tfrac{5}{16} = \frac{(13 \times 16) + 5}{16} = \tfrac{213}{16};\ 10\tfrac{3}{4} = \frac{(10 \times 4) + 3}{4} = \tfrac{43}{4}.$$

(10) Each stroke of the engine is 18 inches in length. Since the piston makes 2 strokes for each revolution, it would pass over a distance of 2 × 18 inches = 36 inches, or 3 feet, in 1 minute, and in making 160 revolutions it would pass over 160 × 3, or 480 ft. Since 480 ft. are passed over in 1 minute, in 1 hour, or 60 minutes, the distance passed over equals 60 × 480 = 28,800 ft. Since the steam engine runs 6 days a week and $8\frac{1}{2}$ hours per day, the total number of hours it runs per week = $6 \times 8\frac{1}{2}$, or 51 hours. If the piston passes over a distance of 28,800 ft. in 1 hour, in 51 hours it would pass over 51 × 28,800 ft., or 1,468,800 ft. Ans.

$$\begin{array}{r l} 480 & \text{ft.} \\ \times\ \ 60 & \\ \hline 28800 & \text{ft.} \\ \\ 28800 & \\ \times\ \ \ 51 & \\ \hline 28800 & \\ 144000\ & \\ \hline 1468800 & \end{array}$$

(11) $\frac{1}{8} + \frac{2}{8} + \frac{5}{8} = \frac{1 + 2 + 5}{8} = \frac{8}{8} = 1.$ Ans.

When the denominators of the fractions to be added are alike, we know that the units are divided into the same number of parts (in this case eighths); we therefore add the numerators of the fractions to find the number of parts (eighths) taken or considered, thereby obtaining $\frac{8}{8}$, or 1, as the sum.

(12) When the denominators are not alike, we know that the units are divided into unequal parts, so before adding them we must find a common denominator for the denominators of all the fractions. Reduce the fractions to fractions having this common denominator, add the numerators, and write the sum over the common denominator.

In this case, the least common denominator, or the least number that will contain all the denominators, is 16; hence, we must reduce all these fractions to 16ths and then add their numerators.

$\frac{1}{4} + \frac{3}{8} + \frac{5}{16} = ?$ To reduce the fraction $\frac{1}{4}$ to a fraction having 16 for a denominator, we must multiply both terms of the fraction by some number which will make the denominator 16. This number evidently is 4, hence, $\frac{1 \times 4}{4 \times 4} = \frac{4}{16}$.

Similarly, both terms of the fraction $\frac{3}{8}$ must be multiplied by 2 to make the denominator 16, and we have $\frac{3 \times 2}{8 \times 2} = \frac{6}{16}$. The fractions now have a common denominator 16; hence, we find their sum by adding the numerators and placing their sum over the common denominator, thus: $\frac{4}{16} + \frac{6}{16} + \frac{5}{16} = \frac{4 + 6 + 5}{16} = \frac{15}{16}$. Ans.

(13) When mixed numbers and whole numbers are to be added, add the fractional parts of the mixed numbers separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction.

$42 + 31\frac{5}{8} + 9\frac{7}{16} = ?$ Reducing $\frac{5}{8}$ to a fraction having a denominator of 16, we have $\frac{5 \times 2}{8 \times 2} = \frac{10}{16}$. Adding the two fractional parts of the mixed numbers, we have $\frac{10}{16} + \frac{7}{16} = \frac{10 + 7}{16} = \frac{17}{16} = 1\frac{1}{16}$.

The problem now becomes $42 + 31 + 9 + 1\frac{1}{16} = ?$

Adding all the whole numbers and the number obtained from adding the fractional parts of the mixed numbers, we obtain $83\frac{1}{16}$ as their sum.

$$\begin{array}{r} 42 \\ 31 \\ 9 \\ 1\frac{1}{16} \\ \hline 83\frac{1}{16} \end{array} \text{ Ans.}$$

(14) $29\frac{3}{4} + 50\frac{5}{8} + 41 + 62\frac{3}{16} = ?$ $\frac{3}{4} = \frac{3 \times 4}{4 \times 4} = \frac{12}{16}$; $\frac{5}{8} = \frac{5 \times 2}{8 \times 2} = \frac{10}{16}$. $\frac{12}{16} + \frac{10}{16} + \frac{3}{16} = \frac{12 + 10 + 3}{16} = \frac{25}{16} = 1\frac{9}{16}$.

The problem now becomes $29 + 50 + 41 + 62 + 1\frac{9}{16} = ?$

$$\begin{array}{rl} 29 & \text{square inches.} \\ 50 & \text{square inches.} \\ 41 & \text{square inches.} \\ 62 & \text{square inches.} \\ 1\frac{9}{16} & \text{square inches.} \\ \hline 183\frac{9}{16} & \text{square inches.} \quad \text{Ans.} \end{array}$$

(15) $\frac{7}{8} - \frac{7}{16} = ?$ When the denominators of fractions are not alike, it is evident that the units are divided into unequal parts; therefore, before subtracting, reduce the fractions to fractions having a common denominator. Then, subtract the numerators and place the remainder over the common denominator.

$$\frac{7 \times 2}{8 \times 2} = \frac{14}{16}. \quad \frac{14}{16} - \frac{7}{16} = \frac{14 - 7}{16} = \frac{7}{16}. \quad \text{Ans.}$$

$13 - 7\frac{7}{16} = ?$ This problem may be solved in two ways:

First: $13 = 12\frac{16}{16}$, since $\frac{16}{16} = 1$, and $12\frac{16}{16} = 12 + \frac{16}{16} = 12 + 1 = 13$.

$$\begin{array}{r} 12\frac{16}{16} \\ 7\frac{7}{16} \\ \hline 5\frac{9}{16} \end{array}$$

We can now subtract the whole numbers separately and the fractions separately, and obtain $12 - 7 = 5$ and $\frac{16}{16} - \frac{7}{16} = \frac{16-7}{16} = \frac{9}{16}$. $5 + \frac{9}{16} = 5\frac{9}{16}$. Ans.

Second: By reducing both numbers to improper fractions having a denominator of 16.

$$13 = \frac{13}{1} = \frac{13 \times 16}{1 \times 16} = \frac{208}{16}. \quad 7\frac{7}{16} = \frac{(7 \times 16) + 7}{16} = \frac{112 + 7}{16} = \frac{119}{16}.$$

Subtracting, we have $\frac{208}{16} - \frac{119}{16} = \frac{208 - 119}{16} = \frac{89}{16}$, and

$$\frac{89}{16} = 16)89(5\frac{9}{16}$$
$$\begin{array}{r} 80 \\ \hline 9 \\ \hline 16 \end{array}$$

the same result that was obtained by the first method.

$312\frac{9}{16} - 229\frac{5}{32} = ?$ We first reduce the fractions of the two mixed numbers to fractions having a common denominator. Doing this, we have $\frac{9}{16} = \frac{9 \times 2}{16 \times 2} = \frac{18}{32}$. We can now subtract the whole numbers and fractions separately, and have $312 - 229 = 83$ and $\frac{18}{32} - \frac{5}{32} = \frac{18-5}{32} = \frac{13}{32}$. $83 + \frac{13}{32} = 83\frac{13}{32}$. Ans.

$$\begin{array}{r} 312\frac{18}{32} \\ 229\frac{5}{32} \\ \hline 83\frac{13}{32} \end{array}$$

(16) In division of fractions, invert the divisor (or, in other words, turn it upside down) and proceed as in multiplication.

(*a*) $35 \div \frac{5}{16} = \frac{35}{1} \times \frac{16}{5} = \frac{35 \times 16}{1 \times 5} = \frac{560}{5} = 112$. Ans.

(*b*) $\frac{9}{16} \div 3 = \frac{9}{16} \div \frac{3}{1} = \frac{9}{16} \times \frac{1}{3} = \frac{9 \times 1}{16 \times 3} = \frac{9}{48} = \frac{3}{16}$. Ans.

(*c*) $\frac{17}{2} \div 9 = \frac{17}{2} \div \frac{9}{1} = \frac{17}{2} \times \frac{1}{9} = \frac{17 \times 1}{2 \times 9} = \frac{17}{18}$. Ans.

denominator, prefixing ciphers to the numerator if necessary, and then insert the decimal point. Proceeding thus, $\frac{45}{100} = .45$ and $\frac{135}{1000} = .135$. Similarly, $\frac{24}{100000} = .00024$.

		tenths.	hundredths.	
(3)	.	0	8	= *Eight* hundredths.

	tenths.	hundredths.	thousandths.	
.	1	3	1	= *One hundred thirty-one* thousandths.

	tenths.	hundredths.	thousandths.	ten-thousandths.	
.	0	0	0	1	= *One* ten-thousandths.

	tenths.	hundredths.	thousandths.	ten-thousandths.	hundred thousandths.	millionths.	
.	0	0	0	0	2	7	= *Twenty-seven* millionths.

	tenths.	hundredths.	thousandths.	ten-thousandths.	
.	0	1	0	8	= *One hundred eight* ten-thousandths.

	tenths.	hundredths.	thousandths.	ten-thousandths.	
9 3 .	0	1	0	1	= Ninety-three and *one hundred one* ten-thousandths.

In reading decimals, read the number just as you would if there were no ciphers before it. Then count from the decimal point towards the right, beginning with tenths, to as many places as there are figures, and the name of the last figure must be annexed to the previous reading of the figures to give the decimal reading. Thus, in the first example given, the simple reading of the figure is eight and the name of its position in the decimal scale is hundredths, so that the decimal reading is eight hundredths. Similarly, the figures in the fourth example are ordinarily read twenty-seven; the name of the position of the figure 7 in the decimal scale is millionths, giving, therefore, the decimal reading as twenty-seven millionths.

If there should be a whole number before the decimal point, read it as you would read any whole number and read the decimal as you would if the whole number were not there; or, read the whole number and then say "and" so many hundredths, thousandths, or whatever it may be, as " ninety-three and *one* hundred one ten-thousandths."

(4) In addition of decimals, the decimal points must be placed directly under each other, so that tenths will come under tenths, hundredths under hundredths, thousandths under thousandths, etc. The addition is then performed as in whole numbers, the decimal point of the sum being placed directly under the decimal points above.

.1 2 5
.7
.0 8 9
.4 0 0 5
.9
.0 0 0 0 2 7
2.2 1 4 5 2 7 Ans.

(5)

tenths. hundredths. thousandths. ten-thousandths. hundred-thousandths. millionths.

.0 1 7
.2
.0 0 0 0 4 7
.2 1 7 0 4 7 = *Two hundred and seventeen thousand and forty-seven* **millionths.** Ans.

(6) (*a*) In subtraction of decimals, place the decimal points directly under each other and proceed as in the subtraction of whole numbers, placing the decimal point in the remainder directly under the decimal points above.

$$\begin{array}{r} 709.6300 \\ .8514 \\ \hline 708.7786 \end{array} \text{ Ans.}$$

In the above example, we proceed as follows: We cannot subtract 4 ten-thousandths from 0 ten-thousandths, and as there are no thousandths, we take 1 hundredth from the 3 hundredths. 1 hundredth = 10 thousandths = 100 ten-thousandths. 4 ten-thousandths from 100 ten-thousandths leaves 96 ten-thousandths. 96 ten-thousandths = 9 thousandths + 6 ten-thousandths. Write the 6 ten-thousandths in the ten-thousandths place in the remainder. The next figure in the subtrahend is 1 thousandth. This must be subtracted from the 9 thousandths which is a part of the 1 hundredth taken previously from the 3 hundredths. Subtracting, we have 1 thousandth from 9 thousandths leaves 8 thousandths, the 8 being written in its place in the remainder. Next we have to subtract 5 hundredths from 2 hundredths (1 hundredth having been taken from the 3 hundredths leaves but 2 hundredths now). Since we cannot do this, we take 1 tenth from 6 tenths. 1 tenth = 10 hundredths, and 10 hundredths + 2 hundredths = 12 hundredths. 5 hundredths from 12 hundredths leaves 7 hundredths. Write the 7 in the hundredths place in the remainder. Next we have to subtract 8 tenths from 5 tenths (5 tenths now, because 1 tenth was taken from the 6 tenths). Since this cannot be done, we take 1 unit from the 9 units. 1 unit = 10 tenths. 10 tenths + 5 tenths = 15 tenths, and 8 tenths from 15 tenths leaves 7 tenths. Write the 7 in the tenths place in the remainder. In the minuend we now have 708 units (1 unit having been taken away) and 0 units in the subtrahend. 0 units from 708 units leaves 708 units; hence, we write 708 in the remainder.

(*b*)
$$\begin{array}{r} 81.963 \\ 1.700 \\ \hline 80.263 \end{array} \text{ Ans.}$$

(*c*)
$$\begin{array}{r} 18.00 \\ .18 \\ \hline 17.82 \end{array} \text{ Ans.}$$

(*d*)
$$\begin{array}{r} 1.000 \\ .001 \\ \hline .999 \end{array} \text{ Ans.}$$

(*e*) $872.1 - (.8721 + .008) = ?$ In this problem we are to subtract $(.8721 + .008)$ from 872.1. First perform the operation as indicated by the sign between the decimals enclosed by the parenthesis.

$$\begin{array}{rl} .8721 & \\ .0080 & \\ \hline .8801 & \textit{sum.} \end{array}$$

Subtracting the sum (obtained by adding the decimals enclosed within the parenthesis) from the number 872.1 (as required by the minus sign before the parenthesis), we obtain the required remainder.

$$\begin{array}{rl} 872.1000 & \\ .8801 & \\ \hline 871.2199 & \text{Ans.} \end{array}$$

(*f*) $(5.028 + .0073) - (6.704 - 2.38) = ?$ First perform the operations as indicated by the signs between the numbers enclosed by the parentheses. The first parenthesis shows that 5.028 and .0073 are to be added. This gives 5.0353 as their sum.

$$\begin{array}{rl} 5.0280 & \\ .0073 & \\ \hline 5.0353 & \textit{sum.} \end{array}$$

The second parenthesis shows that 2.38 is to be subtracted from 6.704. The difference is found to be 4.324.

$$\begin{array}{rl} 6.704 & \\ 2.380 & \\ \hline 4.324 & \textit{diff.} \end{array}$$

The sign between the parentheses indicates that the quantities obtained by performing the above operations are to be subtracted—namely, that 4.324 is to be subtracted from 5.0353. Performing this operation, we obtain .7113 as the final result.

$$\begin{array}{rl} 5.0353 & \\ 4.3240 & \\ \hline .7113 & \text{Ans.} \end{array}$$

(7) If the cost of the coal consumed by a nest of steam boilers amounts to \$15.83 on Monday, to \$14.70 on Tuesday, to \$14.28 on Wednesday, to \$13.87 on Thursday, to \$14.98 on Friday, and to \$12.65 on Saturday, then we find the total cost of the week's supply by adding the different amounts together; hence, \$15.83 + \$14.70 + \$14.28 + \$13.87 + \$14.98 + \$12.65 = \$86.31.

$$\begin{array}{rl} \$\,15.83 & \\ 14.70 & \\ 14.28 & \\ 13.87 & \\ 14.98 & \\ 12.65 & \\ \hline \$\,86.31 & \text{Ans.} \end{array}$$

(8) $482\frac{4}{5} + 316\frac{1}{3} + 390\frac{3}{4} =$ what ?

When mixed numbers are to be added, add the fractional parts of the mixed numbers separately, and if the resulting fraction is an improper fraction, reduce it to a whole or mixed number. Next, add all the whole numbers, including the one obtained from the addition of the fractional parts, and annex to their sum the fraction of the mixed number obtained from reducing the improper fraction. First, we will reduce the fractional parts $\frac{4}{5}$, $\frac{1}{3}$, and $\frac{3}{4}$ to equivalent fractions having the least common denominator. In this case the least common denominator equals the product of the denominators 5, 3, and 4, since we cannot divide any two of them by any number (except 1) without having a remainder, as can be done in the examples in Art. 26, Part 2. Hence, the least common denominator $= 5 \times 3 \times 4 = 60$. Reducing $\frac{4}{5}$, $\frac{1}{3}$, and $\frac{3}{4}$ to fractions having this least common denominator, we have 60 divided by the first denominator, 5, equals 12. Then, $\frac{4}{5} \times \frac{12}{12} = \frac{48}{60}$. 60 divided by the second denominator, 3, equals 20. Then, $\frac{1}{3} \times \frac{20}{20} = \frac{20}{60}$. 60 divided by the third denominator, 4, equals 15. Then, $\frac{3}{4} \times \frac{15}{15} = \frac{45}{60}$. The sum of these fractions equals

$$\frac{48}{60} + \frac{20}{60} + \frac{45}{60} = \frac{48 + 20 + 45}{60} = \frac{113}{60}, \text{ or } 1\tfrac{53}{60}.$$

The problem now becomes $482 + 316 + 390 + 1\frac{53}{60}$, the sum of which equals $1,189\frac{53}{60}$.

$$\begin{array}{r} 482 \\ 316 \\ 390 \\ 1\frac{53}{60} \\ \hline 1189\frac{53}{60} \end{array}$$

1500 represents the actual horsepower required.
$1189\frac{53}{60}$ represents the indicated horsepower of the engines in use.

$310\frac{7}{60}$, or $310.11\frac{2}{3}$ = the H. P. to be developed by the new engine. Ans.

$\frac{7}{60}$ reduced to its equivalent decimal $= \frac{7}{60}$

$$\begin{array}{r} 60\,)\,7.00\,(.11\tfrac{2}{3} \\ 60 \\ \hline 100 \\ 60 \\ \hline \frac{4\cancel{0}}{6\cancel{0}} = \frac{2}{3}. \end{array}$$

(9) Since the inside diameter of the steam pipe is 6.06 inches and the outside diameter is 6.62 inches, there is a difference of 6.62 − 6.06, or .56 of an inch, in both diameters. But .56 of an inch is just twice the thickness of the pipe; hence, the pipe is ½ of .56, or .28 of an inch thick.

(10) (*a*) There are 3 decimal places in the multiplicand and 3 in the multiplier; hence, there are 3 + 3, or 6, decimal places in the product. Since the product contains but four figures, we prefix two ciphers in order to obtain the necessary 6 decimal places.

$$\begin{array}{r} .107 \\ .013 \\ \hline 321 \\ 107 \\ \hline .001391 \end{array}\quad \text{Ans.}$$

(*b*) There are 2 decimal places in the multiplier and none in the multiplicand; hence, there are 2 + 0, or 2, decimal places in the first product.

Since in the second multiplication there are 2 decimal places in the multiplicand and 3 decimal places in the multiplier, there are 3 + 2, or 5, decimal places in the second product.

When there are one or more ciphers in the multiplier, multiply just the same as with the other figures.

$$\begin{array}{r} 203 \\ 2.03 \\ \hline 609 \\ 000 \\ 406 \\ \hline 412.09 \\ .203 \\ \hline 123627 \\ 00000 \\ 82418 \\ \hline 83.65427 \end{array}\quad \text{Ans.}$$

(*c*) First perform the operations indicated by the signs between the numbers enclosed by the parentheses, and then whatever may be required by the sign between the parentheses.

$$\begin{array}{r}
31.85 \\
\underline{2.7} \\
22295 \\
\underline{6370} \\
85.995
\end{array}$$

The first parenthesis shows that the numbers 2.7 and 31.85 are to be multiplied together.

The second parenthesis shows that .316 is to be taken from 3.16.

$$\begin{array}{r}
3.160 \\
\underline{.316} \\
2.844
\end{array}$$

The product obtained by performing the operation indicated by the signs within the first parenthesis is now multiplied by the remainder obtained by performing the operation indicated by the signs within the second parenthesis.

$$\begin{array}{r}
85.995 \\
\underline{2.844} \\
343980 \\
343980 \\
687960 \\
\underline{171990} \\
244.569780 \text{ Ans.}
\end{array}$$

(*d*) (107.8 + 6.541 − 31.96) × 1.742 = ?

$$\begin{array}{rr}
& 107.8 \\
+ & \underline{6.541} \\
& 114.341 \\
- & \underline{31.96} \\
& 82.381
\end{array}
\qquad
\begin{array}{r}
82.381 \\
\underline{\times \; 1.742} \\
164762 \\
329524 \\
576667 \\
\underline{82381} \\
143.507702 \text{ Ans.}
\end{array}$$

(11) If one 3-inch tube measures 15½ ft. in length, 60 of these tubes would measure 60 × 15½ ft., or 930 ft. in length. If 1 foot of tubing heats a surface of .728 sq. ft., then it is evident that 930 ft. of tubing would heat a surface of 930 × .728 ft., or 677.04 sq. ft.

$$\begin{array}{r}
15\tfrac{1}{2} \\
\underline{60} \\
900 \\
\underline{30} \\
930 \text{ ft.}
\end{array}
\qquad
\begin{array}{r}
.728 \\
\underline{930} \\
21840 \\
\underline{6552} \\
677.040 \text{ sq. ft.}
\end{array}$$

(12) (*a*) $\frac{7}{\frac{3}{16}} = 7 \div \frac{3}{16} = 7 \times \frac{16}{3} = \frac{7 \times 16}{3} = \frac{112}{3} = 37\frac{1}{3}$. Ans.

The heavy line indicates that 7 is to be divided by $\frac{3}{16}$.

(*b*) $\frac{\frac{15}{32}}{\frac{5}{8}} = \frac{15}{32} \div \frac{5}{8} = \frac{15}{32} \times \frac{8}{5} = \frac{\overset{3}{\cancel{15}} \times \cancel{8}}{\underset{4}{\cancel{32}} \times \cancel{5}} = \frac{3}{4} = .75$. Ans.

(*c*) $\frac{1.25 \times 20 \times 3}{\frac{87 + 88}{459 + 32}} = ?$ In this problem $1.25 \times 20 \times 3$ constitutes the numerator of the complex fraction.

$$\begin{array}{lr} & 1.25 \\ \times & \underline{20} \\ & 25.00 \\ \times & \underline{3} \\ & 75 \end{array}$$

Multiplying the factors of the numerator together, we find their product to be 75.

The fraction $\frac{87 + 88}{459 + 32}$ constitutes the denominator of the complex fraction. The value of the numerator of this fraction equals $87 + 88 = 175$.

The value of the denominator of this fraction is equal to $459 + 32 = 491$. The problem then becomes

$$\frac{75}{\frac{175}{491}} = \frac{\frac{75}{1}}{\frac{175}{491}} = \frac{75}{1} \div \frac{175}{491} = \frac{75}{1} \times \frac{491}{175} = \frac{\overset{3}{\cancel{75}} \times 491}{\underset{7}{\cancel{175}}} = \frac{1,473}{7} = 210\frac{3}{7}. \text{ Ans.}$$

(13) The pitch of the rivets is the distance between the centers of the rivets. Hence, since the distance around the cylindrical boiler is 166.85 in., and there are 72 rivets in one of the seams, the pitch of the rivets equals $166.85 \div 72 = 2.317+$ in. Ans.

$$\begin{array}{rl} 72\,)\,166.850 & (\,2.317+ \\ \underline{144} & \\ 228 & \\ \underline{216} & \\ 125 & \\ \underline{72} & \\ 530 & \\ \underline{504} & \\ 26 & \end{array}$$

(14) If a keg containing 133 boiler rivets weighs 100 pounds, then each rivet must weigh as much as 133 is contained times in 100, or .75 of a pound.

$$\begin{array}{r l}
133\,)\,100.00\,(.75+ & \text{Ans.} \\
\underline{931} & \\
690 & \\
\underline{665} & \\
25 &
\end{array}$$

Since there are 2 decimal places in the dividend and 0 decimal places in the divisor, we must point off $2-0=2$ decimal places in the quotient, or answer.

(15) $.875=\frac{875}{1,000}=\frac{175}{200}=\frac{7}{8}$ of a foot.

1 foot = 12 inches.

$$\frac{7}{8} \text{ of 1 foot} = \frac{7}{\underset{2}{\cancel{8}}} \times \frac{\overset{3}{\cancel{12}}}{1} = \frac{21}{2} = 10\tfrac{1}{2} \text{ inches. Ans.}$$

(16) 12 inches = 1 foot.

$$\frac{3}{16} \text{ of an inch} = \frac{3}{16} \div 12 = \frac{\cancel{3}}{16} \times \frac{1}{\underset{4}{\cancel{12}}} = \frac{1}{64} \text{ of a foot.}$$

$$\begin{array}{r l}
\frac{1}{64}\,)\,1.000000\,(.015625 & \textbf{Ans.} \\
\underline{64} & \\
360 & \\
\underline{320} & \\
400 & \\
\underline{384} & \\
160 & \\
\underline{128} & \\
320 & \\
\underline{320} & \\
0 &
\end{array}$$

Point off 6 decimal places in the quotient, since we annexed six ciphers to the dividend, the divisor containing no decimal places; hence, $6-0=6$ places to be pointed off.

(17) The total horsepower developed equals $48.63 + 45.7 + 46.32 + 47.9 + 48.74 + 48.38 + 48.59 = 334.26$.

Since the horsepower developed equals 334.26, then the average horsepower developed must equal $334.26 \div 7$, or 47.75+ H. P.

$$\begin{array}{rl}
 & 48.63 \\
 & 45.7 \\
 & 46.32 \\
 & 47.9 \\
 & 48.74 \\
 & 48.38 \\
 & 48.59 \\
\cline{2-2}
7\,) & \underline{334.26} \\
 & 47.75+ \quad \text{Ans.}
\end{array}$$

ARITHMETIC.

(PART 4.)

(1) A certain per cent. of a number means so many hundredths of that number.

25% of 8,428 lb. means 25 hundredths of 8,428 lb. $\frac{25}{100} = .25$. Hence, 8,428 lb. × .25 = 2,107 lb. Ans.

(2) ½% means one-half of 1 per cent. Since 1% is .01, ½% is .005, for $\begin{array}{r}2\,)\,.0\,1\,0\\ \hline 0\,0\,5\end{array}$. And \$35,000 × .005 = \$175. Ans.

$$\begin{array}{r}\$\,3\,5\,0\,0\,0\\ .0\,0\,5\\ \hline \$\,1\,7\,5.0\,0\,0\end{array}$$

(3) If 2 is a certain per cent. of 50, then 50 multiplied by a certain rate gives a product of 2, and that rate is equal to 2 divided by 50. Dividing 2 by 50, the quotient is .04, which means that 2 is 4% of 50, or, since percentage = base × rate,

$$\begin{array}{l}5\,0\,)\,2.0\,0\,(\,.0\,4 \text{ Ans.}\\ \quad\;\; \underline{2\,0\,0}\end{array}$$

rate = percentage ÷ base
= 2 ÷ 50 = .04, or 4%. Ans.

(4) Since percentage = base × rate, rate = percentage ÷ base.

As percentage = 10 and base = 10, we have rate = 10 ÷ 10 = 1.

For notice of copyright, see page immediately following the title page.

But $1 = \frac{100}{100}$ and $\frac{100}{100} = 100\%$; hence, the rate (1) means that 10 is 100% of 10.

(5) Since 5,500 lb. represent an increase of 15% over the consumption when the condenser is used, 5,500 lb. must be the amount, .15 the rate, and the number of pounds consumed when the condenser is running (to be found) the base.

$$\begin{aligned} \text{Base} &= \text{amount} \div (1 + \text{rate}) = 5{,}500 \div (1 + .15) \\ &= 5{,}500 \div 1.15 = 4{,}782.61 \text{ lb., nearly. Ans.} \end{aligned}$$

```
1.15 ) 5500.0000 ( 4782.61
       460
       ---
        900
        805
        ---
         950
         920
         ---
          300
          230
          ---
           700
           690
           ---
            100
            115
            ---
```

Or, this problem could also have been solved as follows: 100% = the number of pounds consumed when the condenser is running. If there is a gain of 15%, then 100% + 15%, or 115% = 5,500 lb., the amount used when the condenser is not running. If 115% = 5,500 lb., $1\% = \frac{1}{115}$ of 5,500 = 47.8261 lb., and 100% = 100 × 47.8261 = 4,782.61 lb. Ans.

(6)

$$\begin{array}{llr} 24\ \% \text{ of } \$950 = 950 \times .24 & = & \$228 \\ 12\frac{1}{2}\% \text{ of } \$950 = 950 \times .125 & = & 118.75 \\ \underline{17}\ \% \text{ of } \$950 = 950 \times .17 & = & \underline{161.50} \\ 53\frac{1}{2}\% \text{ of } \$950 & = & \$508.25 \end{array}$$

The total amount of his yearly expenses, then, is $508.25; hence, his savings are $950 − $508.25 = $441.75. Ans.

Or, as above, 24% + 12½% + 17% = 53½%, the total percentage of expenditures; hence,

$950 × .535 = $508.25, and
$950 − $508.25 = $441.75 = his yearly savings. Ans.

(7) The percentage is 961.38 and the rate is .37½.

Base = percentage ÷ rate
= 961.38 ÷ .375 = 2,563.68, the number. Ans.

$$\begin{array}{l}
.375)\,961.38000\,(2563.68 \\
750 \\
\hline
2113 \\
1875 \\
\hline
2388 \\
2250 \\
\hline
1380 \\
1125 \\
\hline
2550 \\
2250 \\
\hline
3000 \\
3000 \\
\hline
\end{array}$$

Another method of solving is the following:
If 37½% of a number is 961.38, then .37½ times the number = 961.38 and the number = 961.38 ÷ .37½, which, as above = 2,563.68. Ans.

(8) 298 revolutions per minute with the load = base, .01½ = rate, and the amount (to be found) will equal the speed of the engine when running unloaded.

Amount = base × (1 + rate)
= 298 × (1 + .015) = 302.47 rev. per min. Ans.

$$\begin{array}{r}
298 \\
\times\ 1.015 \\
\hline
1490 \\
298 \\
000 \\
298 \\
\hline
302.470
\end{array}$$

(9) 4 yd. 2 ft. 10 in. to inches.

$$\begin{array}{rrl}
 & 4 & \\
\times & 3 & \\ \hline
 & 12 & \\
+ & 2 & \\ \hline
 & 14 & \text{feet} \\
\times & 12 & \\ \hline
 & 28 & \\
 & 14 & \\ \hline
 & 168 & \\
+ & 10 & \\ \hline
 & 178 & \text{inches. Ans.}
\end{array}$$

Since there are 3 feet in 1 yard, in 4 yards there are 4 × 3 feet, or 12 feet. 12 feet plus 2 feet = 14 feet.

There are 12 inches in 1 foot; therefore, in 14 feet there are 12 × 14, or 168 inches. 168 inches plus 10 inches = 178 inches.

(10)

$$\begin{array}{r}
12\,\underline{)\,3722}\ \text{inches.} \\
3\,\underline{)\,310} + 2\ \text{inches.} \\
103 + 1\ \text{foot.}
\end{array}$$

Ans. = 103 yd. 1 ft. 2 in.

Explanation.—There are 12 inches in 1 foot; hence, in 3,722 inches there are as many feet as 12 is contained times in 3,722, or 310 feet and 2 inches remaining. Write 2 inches as a remainder. There are 3 feet in 1 yard; hence, in 310 yards there are as many feet as 3 is contained times in 310, or 103 yards and 1 foot remaining. Hence, in 3,722 inches there are 103 yd. 1 ft. 2 in.

(11)

$$\begin{array}{r}
1728\,\underline{)\,764325}\ \text{cu. in.} \\
27\,\underline{)\,442} + 549\ \text{cu. in.} \\
16\ \text{cu. yd.} + 10\ \text{cu. ft.}
\end{array}$$

Ans. = 16 cu. yd. 10 cu. ft. 549 cu. in.

Explanation.—There are 1,728 cubic inches in 1 cubic foot; hence, in 764,325 cu. in. there are as many cubic feet as 1,728 is contained times in 764,325, or 442 cubic feet and 549 cubic inches remaining. Write the 549 cubic inches as a remainder. There are 27 cubic feet in 1 cubic yard; hence, in 442 cubic feet there are as many cubic yards as 27 is contained times in 442 cubic feet, or 16 cubic yards and 10 cubic feet remaining. Then, in 764,325 cubic inches there are 16 cu. yd. 10 cu. ft. 549 cu. in.

(12)

$$\begin{array}{rlll} & \text{T.} & \text{cwt.} & \text{lb.} \\ & 16 & 8 & 75 \\ \times & 20 & & \\ \hline & 320 & & \\ + & 8 & & \\ \hline & 328\ \text{cwt.} & & \\ \times & 100 & & \\ \hline & 32800 & & \\ + & 75 & & \\ \hline & 32875\ \text{lb.} & \text{Ans.} & \end{array}$$

Since in 1 ton there are 20 cwt., in 16 tons there are 16 × 20 = 320 cwt. 320 cwt. + 8 cwt. = 328 cwt. There are 100 lb. in 1 cwt.; hence, in 328 cwt. there are 328 × 100 = 32,800 lb. 32,800 lb. + 75 lb. = 32,875 lb. Ans.

(13)

$$\begin{array}{r} 100\,)\,\underline{25396}\ \text{lb.} \\ 20\,)\,\underline{253}\ \text{cwt.} + 96\ \text{lb.} \\ 12\ \text{T.} + 13\ \text{cwt.} \end{array}$$

There are 100 lb. in 1 cwt.; hence, in 25,396 lb. there are as many cwt. as 100 is contained times in 25,396, or 253 cwt. and 96 lb. remaining.

There are 20 cwt. in 1 ton, and in 253 cwt. there are as many tons as 20 is contained times in 253, or 12 tons and 13 cwt. remaining. Hence, 25,396 lb. = 12 T. 13 cwt. 96 lb. Ans.

(14) Arrange the different terms in columns, taking care to have like denominations in the same column. We begin to add at the right-hand column. 7 + 9 + 3 = 19 in.; since 12 in. = 1 ft., 19 in. = 1 ft. and 7 in. Place the 7 in. in the inches column and reserve the 1 ft. to add to the sum of the feet. 2 + 1 + 2 + 1 (reserved) = 6 ft. Since 3 ft. = 1 yd., 6 ft. = 2 yd. and 0 ft. remaining. Place the 0 in the feet column and reserve the 2 yd. to add to the sum of the yards. 4 + 2 + 2 (reserved) = 8 yd., which we place in yards column. Ans. = 8 yd. 7 in.

$$\begin{array}{cccl} \text{yd.} & \text{ft.} & \text{in.} & \\ 2 & 2 & 3 & \\ 4 & 1 & 9 & \\ & 2 & 7 & \\ \hline 8 & 0 & 7 & \text{Ans.} \end{array}$$

13. Since 12 gal. 2 qt. 1 pt. of machine oil is sold at one time and 14 gal. 3 qt. at another time, together there was sold 27 gal. 1 qt. 1 pt. 0 + 1 = 1 pt. We cannot reduce 1 pt. to any higher denomination, so place it under pints column. 3 qt. + 2 qt. = 5 qt. Since 4 qt. = 1 gal., 5 qt. = 1 gal. and 1 qt. remaining. Place 1 qt. under quarts column and reserve the 1 gal. to add to the gallons. 14 gal. + 12 gal. + 1 gal. (reserved) = 27 gal.

gal.	qt.	pt.
12	2	1
14	3	0
27	1	1

Since the barrel contained 31½, or 31.5 gal., and 27 gal. 1 qt. 1 pt. were sold, there remained the difference, or 4 gal. 1 pt. 31.5 gal. = 31 gal. 2 qt., since .5 = ½, and ½ of 1 gal. = ½ of 4 qt. = 2 qt.

gal.	qt.	pt.	
31	2	0	
27	1	1	
4	0	1	Ans.

1 pt. cannot be taken from 0 pt., so we take 1 qt. from the 2 qt. The 1 qt. taken = 2 pt. 1 pt. from 2 pt. = 1 pt. Place 1 pt. under pints column. Since we took 1 qt. from the quarts column, there remains 2 − 1, or 1 qt. 1 qt. from 1 qt. [illegible]. Place 0 qt. under the quarts column. [illegible] 4 gal. Place 4 gal. under the [illegible]; therefore we find that 4 gal. 1 pt. of [illegible] the barrel.

[illegible] denominate numbers, we place [illegible] lowest denomination of the multiplicand, [illegible]

$$\begin{array}{r}
17\text{ ft. } 3\text{ in.}\\
51\\
\hline
879\text{ ft. } 9\text{ in.}
\end{array}$$

and begin at the right to multiply. 51 × 3 = 153 in. Since there are 12 in. in 1 ft., in 153 in. there are as many feet as 12 is contained times in 153, or 12 ft. and 9 in. remaining. Place the 9 in. under the inches and reserve the 12 ft. 51 × 17 ft. = 867 ft.; 867 ft. + 12 ft. (reserved) = 879 ft. 879 ft.

can be reduced to higher denominations by dividing by 3 ft. to find the number of yards, and by $5\frac{1}{2}$ to find the number of rods.

$$\begin{array}{r l} 3\,) & \underline{879}\text{ ft. } 9\text{ in.} \\ 5.5\,) & \underline{293}\text{ yd.} \\ & 53\text{ rd. } 1\tfrac{1}{2}\text{ yd.} \end{array}$$

Then, 879 ft. 9 in. = 53 rd. $1\frac{1}{2}$ yd. 0 ft. 9 in., or 53 rd. 1 yd. 2 ft. 3 in.

(17) Since 2 pt. = 1 qt., 3 qt. = 3 × 2, or 6 pt. 6 pt. + 1 pt. = 7 pt. 4.7 × 7 = 32.9 pt. Ans.

qt.	pt.
3	1
× 2	
6	
+ 1	
7 pt.	

$$\begin{array}{r l} 7 & \text{pt.} \\ \underline{4.7} & \\ 49 & \\ \underline{28} & \\ 32.9 & \text{pt.} \end{array}$$

(18) If there are four lengths, each 15 ft. 5 in., 15 ft. 5 in. × 4 = 60 ft. 20 in., or the length of the four pieces.

ft.	in.
60	20
14	8
8	10
82	38, or 85 ft. 2 in. = the length of the shaft.

$$\begin{array}{r r} 15\text{ ft.} & 5\text{ in.} \\ & \underline{4} \\ 60\text{ ft.} & 20\text{ in.} \end{array}$$

From the length of the shaft we must subtract 8 in. × 2 = 16 in. to get the distance between the end hangers.

ft.	in.
82	38
	16
82	22, or 83 ft. 10 in.

Since there are six hangers, there are *five* spaces. The length of one space is 83 ft. 10 in ÷ 5 = 16 ft. $9\frac{1}{5}$ in. Ans.

[illegible] Reducing 18 ft. 11¾ in. to inches, we have 227¾ in., or 227.75 in.

$$\begin{array}{rl} \text{ft.} & \text{in.} \\ 18 & 11\tfrac{3}{4} \\ 12 & \\ \hline 36 & \\ 18 & \\ \hline 216 & \\ 11\tfrac{3}{4} & \\ \hline 227\tfrac{3}{4} & \text{in.} \end{array}$$

1⅜ × 2, or 2¾ in., for the two end rivets is deducted from the length, leaving 225 in., which is divided into equal spaces by the rivets.

$$\begin{array}{rl} 227.75 & \text{in.} \\ -\quad 2.75 & \text{in.} \\ \hline 225 & \text{in.} \end{array}$$

The pitch of the rivets (or the distance between their centers) is 1¼ in., or 1.25 in.; hence, 225 ÷ 1.25 = 180 spaces between the rivets. But since there will be one more rivet than the number of spaces, the number of rivets required for this boiler shell will be 180 + 1 = 181. Ans.

1.25)225.00(180
125
1000
1000
0

ARITHMETIC.

(PART 5.)

(1) To find the second power of a number, we must multiply the number by itself once; that is, use the number twice as a factor. Thus, the second power of 108 is 108 × 108 = 11,664.

```
  108
  108
  ---
  864
 108
-----
11664  Ans.
```

```
        181.25
        181.25
        ------
         90625
        36250
       18125
     145000
     18125
    ----------
    32851.5625
        181.25
    ----------
    1642578125
    657031250
   328515625
 2628125000
 328515625
--------------
5954345.703125  Ans.
```

(2) The third power of 181.25 equals the number obtained by using 181.25 as a factor three times. Thus, the third power of 181.25 is 181.25 × 181.25 × 181.25 = 5,954,345.703125.

Since there are 2 decimal places in the multiplier and 2 in the multiplicand, there are $2 + 2 = 4$ decimal places in the first product.

Since there are 4 decimal places in the multiplicand and 2 in the multiplier, there are $4 + 2 = 6$ decimal places in the final product.

For notice of copyright, see page immediately following the title page.

[illegible] = [illegible] = .000002352637.

[illegible]

Since there are 4 decimal places in the multiplicand and 4 in the multiplier, we should point off 4 + 4 = 8 decimal places in the product; but as there are only 5 figures in the product, we prefix 3 ciphers to form the 8 necessary decimal places in the first product.

Since there are 8 decimal places in the multiplicand and 4 in the multiplier, we should point off 8 + 4 = 12 decimal places in the product; but as there are only 7 figures in the product, we prefix 5 ciphers to make the necessary 12 decimal places in the final product.

[illegible] = [illegible]

= [illegible] 942264331.

[illegible]

[illegible] 4331 Ans.

Since there are 3 decimal places in the multiplicand and 3 in the multiplier, we should point off 3 + 3 = 6 decimal places in the first product.

Since there are 6 decimal places in the multiplicand and 3 in the multiplier, we should point off $6 + 3 = 9$ decimal places in the final product.

(*c*) $(\frac{1}{8})^3 = \frac{1}{8} \times \frac{1}{8} \times \frac{1}{8} = \frac{1 \times 1 \times 1}{8 \times 8 \times 8} = \frac{1}{512}$. Ans.

$$\begin{array}{r} 8 \\ \times\ \underline{8} \\ 64 \\ \times\ \underline{8} \\ 512 \end{array}$$

(*d*) To find any power of a mixed number, first reduce it to an improper fraction, and then multiply the numerators together for the numerator of the answer and multiply the denominators together for the denominator of the answer.

$$(3\tfrac{3}{4})^3 = \tfrac{15}{4} \times \tfrac{15}{4} \times \tfrac{15}{4} = \frac{15 \times 15 \times 15}{4 \times 4 \times 4} = \frac{3,375}{64} = 52.734375.$$

Ans.

$$3\tfrac{3}{4} = \frac{(3 \times 4) + 3}{4} = \frac{12 + 3}{4} = \tfrac{15}{4}.$$

$$\begin{array}{r} 15 \\ \underline{15} \\ 75 \\ \underline{15} \\ 225 \\ \underline{15} \\ 1125 \\ \underline{225} \\ 3375 \end{array}
\qquad
\begin{array}{l} 64)\,3375.000000(52.734375 \\ \underline{320} \\ 175 \\ \underline{128} \\ 470 \\ \underline{448} \\ 220 \\ \underline{192} \\ 280 \\ \underline{256} \\ 240 \\ \underline{192} \\ 480 \\ \underline{448} \\ 320 \\ \underline{320} \end{array}$$

Since 6 ciphers were annexed to the dividend, 6 decimal places must be pointed off in the quotient.

```
(4)  (a)

(a)  1                √3'4 8'6 7'8 4.4 0'1 0 = 1 8 6 7.2 9+   Ans.
     1            (b) 1
(d)  2 0          (c) 2 4 8
       8              2 2 4
     2 8          (e)   2 4 6 7
       8                2 1 9 6
     3 6 0                2 7 1 8 4
         6                2 6 0 8 9
     3 6 6                  1 0 9 5 4 0
         6                    7 4 6 8 4
     3 7 2 0                  3 4 8 5 6 1 0
           7                  3 3 6 1 0 4 1
     3 7 2 7                    1 2 4 5 6 9
           7
     3 7 3 4 0
             2
     3 7 3 4 2
             2
     3 7 3 4 4 0
               9
     3 7 3 4 4 9
```

EXPLANATION.—(I) Divide the number into periods. In the above case, where the number consists of a whole number and a decimal, we begin at the decimal point and proceed to the *left* in pointing off the *whole* number and towards the *right* in pointing off the *decimal*, annexing a 0 to complete the last decimal period. In *square root two* figures constitute a period.

Find the greatest single number whose square is *less* than or *equal* to 3, the first period. This is evidently 1, since $2^2 = 4$, which is greater than 3. Write 1 as the first figure of the root; also write it to the left, as shown at (*a*). Now, multiply the 1 at (*a*) by the 1 in the root and write the result under the first period, or 3, as shown at (*b*). Subtract, bring down the next period, 48, and annex it to the remainder 2, thus making 248 the *dividend*, as shown at (*c*). Add the root already found to the 1 at (*a*), thereby obtaining 2, and annex a cipher to this 2, thus making it 20, which we call the *trial divisor*.

Divide the dividend 248 at (*c*) by the trial divisor 20 at (*d*) and obtain 8, which is probably the next figure of the root. Write 8 in the root, as shown, and also add it to 20, the trial divisor, making it 28. This is called the *complete divisor*.

(II) Multiply the complete divisor 28 by 8, the second figure of the root, and subtract the result from the dividend (*c*). The remainder is 24, to which annex the next period, making 2,467, as shown at (*e*), which call the *new dividend*.

Add the second figure of the root, or 8, to the divisor 28 and annex a cipher, thus obtaining 360. Dividing 2467 by 360, we find 6 to be the next figure of the root. Adding this last figure of the root, or 6, to 360, we get 366, and multiplying by this last figure of the root, or 6, gives 2196, which we write under 2467 and subtract.

(III) Annexing the next period, 84, to the remainder 271 gives 27184 as the next new dividend. Now, adding the third figure of the root to 366 and annexing a cipher, as before, we have 3720. Dividing 27184 by 3720, the result is 7, which write as the next figure of the root. Adding the fourth figure of the root, or 7, to 3720, we get 3727, and multiplying by 7 of the root gives 26089, which write under 27184 and subtract, obtaining 1095 as a remainder.

(IV) Annexing the next period, or 40, to the remainder 1095 gives 109540 as the next new dividend. Adding the last figure of the root to 3727 and annexing a cipher, as before, the result is 37340. Dividing 109540 by 37340, the result is 2, which write as the next figure of the root. Adding the fifth figure of the root, or 2, to 37340, we obtain 37342, and multiplying by 2 of the root gives 74684, which write under 109540 and subtract, obtaining 34856 as a remainder.

(V) Annexing the next period, or 10, to the remainder 34856 gives 3485610 as the next new dividend. Now, adding the last figure of the root to 37342 and annexing a cipher, as before, the result is 373440. Dividing 3485610 by 373440 gives 9 as a result, which write as the next, or last figure, of the root. Adding the last figure of the root, or 9, to 373440, we get 373449, and multiplying by 9 of the root gives 3361041, which write under 3485610 and subtract. Since there is a remainder, we know the given power is not a perfect square, so we place + after the root.

In this problem there are *six* periods—four in the whole number and two in the decimal—hence, there will be *six* figures in the root (since we obtain one figure of the root for each period), four figures constituting the whole number and two figures the decimal of the root. Hence,

$$\sqrt{3{,}486{,}784.401} = 1{,}867.29+.$$

$$\begin{array}{lrlr}
(b)\ (a) & 3 & & \sqrt{9'0\,0'0\,0'9\,9.4\,0'0\,9'0\,0} = 3\,0\,0\,0.0\,1\,6+ \\
 & \underline{3} & (b) & \underline{9\qquad\qquad\qquad} \\
(d) & 6\,0 & (c) & 0\,0\,0\,0\,9\,9\,4\,0\,0\,9 \\
 & \underline{0} & & \underline{6\,0\,0\,0\,0\,1} \\
 & 6\,0\,0 & & 3\,9\,4\,0\,0\,8\,0\,0 \\
 & \underline{0} & & \underline{3\,6\,0\,0\,0\,1\,5\,6} \\
 & 6\,0\,0\,0 & & 3\,4\,0\,0\,6\,4\,4 \\
 & \underline{0} & & \\
 & 6\,0\,0\,0\,0 & & \\
 & \underline{0} & & \\
 & 6\,0\,0\,0\,0\,0 & & \\
 & \underline{1} & & \\
 & 6\,0\,0\,0\,0\,1 & & \\
 & \underline{1} & & \\
 & 6\,0\,0\,0\,0\,2\,0 & & \\
 & \underline{6} & & \\
 & 6\,0\,0\,0\,0\,2\,6 & &
\end{array}$$

Explanation.—Beginning at the decimal point, we point off the whole number into periods of two figures each, proceeding from right to left; also point off the decimals into periods of two figures each, proceeding from left to right. The largest number whose square is contained in the first period, 9, is 3; hence, 3 is the first figure of the root. Place 3 at the left, as shown at (*a*), and multiply it by the first figure in the root, or 3. The result is 9. Write 9 under the first period, 9, as at (*b*), subtract, and there is no remainder: Bring down the next period, which is 00, as shown at (*c*). Add the root already found to the 3 at (*a*), obtaining 6, and annex a cipher to this 6, thus making it 60, which is the *trial divisor*, as shown at (*d*). Divide the dividend (*c*) by the trial divisor and obtain 0 as the next figure in the root. Write 0 in the root as shown, and also add it to the trial divisor 60, and annex a cipher, thereby making the next trial divisor 600. Bring down the next period, 00, annex it to the dividend already obtained, and divide it by the trial divisor. 600 is contained in 0000 no times, so we place another cipher in the root. Write 0 in the root, as shown, and also add it to the trial divisor 600, and annex a cipher, thereby making the next trial divisor 6000. Bring down the next period, 99. The trial divisor 6000 is contained in 000099 no times, so we place 0 as the next figure in the root, as shown, and also add it to the trial divisor 6000 and annex a cipher, thereby making the next trial divisor 60000. Bring down the next period, 40, and annex it to the dividend already obtained to form the new dividend, 00009940, and divide it by the trial divisor 60000. 60000 is contained in 00009940 no times, so we place another cipher in the root, as shown, and also add it to the trial divisor 60000 and annex one cipher, thereby making the next trial divisor 600000. Bring down the next period, 09, and annex it to the dividend already obtained to form the new dividend, 0000994009, and divide it by the trial divisor 600000. 600000 is contained in 0000994009 once, so we place 1 as the next figure in the root and also add it to the trial divisor 600000, thereby making the complete divisor 600001. Multiply the complete divisor 600001 by 1, the sixth figure in the root, and subtract the result obtained from the dividend. The remainder is 394008, to which we annex the next period, 00, to form the next new dividend, or 39400800. Add the sixth figure of the root, or 1, to the divisor 600001 and annex a cipher, thus obtaining 6000020 as the next trial divisor. Dividing 39400800 by 6000020, we find 6 to be the next figure of the root. Adding this last figure, 6, to the trial divisor, we obtain 6000026 for our next omplete divisor, and multiplying by the last figure of the ot, or 6, gives 36000156, which write under 39400800 and subtract. Since there is a remainder, it is clearly evident that the given power is not a perfect square; and since the next figure of the root is 5, we increase the last figure just found by 1 and write — after the root.

In this problem there are *seven* periods—four in the whole umber and three in the decimal; hence, there will be *seven* ures in the root, *four* figures constituting the whole ber and three figures the decimal of the root. Hence, 00,099.4009 = 3,000.017—. Ans.

(5) Beginning at the decimal point and pointing off into iods of three figures each, we have .327′680, annexing a r to complete the right-hand period. Since the nums entirely decimal, the root is entirely decimal. Neglect-decimal point for the present, we find the cube ,680. Referring to the table in Art. **30,** 327,680 en 314,432, the cube of 68, and 328,509, the cube of st two figures of the root are, therefore, 68. The difference is 328,509 — 314,432 = 14,077; the second difference is 327,680 — 314,432 = 13,248; and 13,248 ÷ 14,077 =.94+. Therefore, the first four figures of the root are 6894. Locating the decimal point according to Art. **32,** we have the $\sqrt[3]{.32768} = .6894+$. Ans.

		14077)13248.00(.94+
		126693
328509	327680	57870
314432	314432	56308
14077	13248	1562

(6) Instead of finding the cube root of 2, we annex a cipher period and find the cube root of 2,000, in order to obtain two figures of the root from the table (see Art. **32**). Referring to the table, 2,000 lies between 1,728, the cube of 12, and 2,197, the cube of 13; the first two figures of the root are, therefore, 12. The first difference is 2,197 — 1,728 = 469; the second difference is 2,000 — 1,728 = 272. and 272 ÷ 469 = .58—. Therefore, the first four figures of the

root are 1258. Since there is only one period in the number, the whole-number part of the root will contain only one figure; hence, the root is 1.258−. We need one more figure, however; hence, proceeding as in Art. **35,** $2 \div 1.258 = 1.58983$; $1.58983 \div 1.258 = 1.26377$; $1.258 + 1.258 + 1.26377 = 3.77977$; $3.77977 \div 3 = 1.25992$. Therefore, $\sqrt[3]{2} = 1.2599$ to four decimal places.

```
2197    2000    469)272.000(.579+, or .58−
1728    1728        2345
----    ----        ----
 469     272         3750
                     3283
                     ----
                      4670
                      4221
                      ----

1.258)2.000000000(1.589825+, or 1.58983−
      1258
      ----
       7420
       6290
       ----
       11300     1.258)1.58983000(1.26377
       10064           1258
       -----           ----
        12360           3318
        11322           2516
        -----           ----
         10380           8023
         10064           7548
         -----           ----
           3160           4750
           2516           3774
           ----           ----
            6440           9760
            6290           8806
            ----           ----
                            9540
   1.258                    8806
   1.258                    ----
   1.26377
   -------
3.)3.77977
   1.25992, or 1.2599  Ans.
```

```
(7) (a)                              (b)
    1   √1'2 3.2 1 = 11.1 Ans.      1   √1'1 4.9 2'1 0 = 10.72+
    1    1                          1    1                    Ans.
    2 0   2 3                       2 0 0   1 4 9 2
      1   2 1                           7   1 4 4 9
    2 1     2 2 1                   2 0 7     4 3 1 0
      1     2 2 1                       7     4 2 8 4
    2 2 0                           2 1 4 0       2 6
        1                                 2
    2 2 1                           2 1 4 2
```

(8) (*a*) Since the number is entirely decimal, the root is entirely decimal. Adding two ciphers to complete the second period, and neglecting the decimal point, we find the cube root of 6,500. Referring to the table, 6,500 lies between 5,832, the cube of 18, and 6,859, the cube of 19; the first two figures of the root are 18. The first difference is 6,859 − 5,832 = 1,027; the second difference is 6,500 − 5,832 = 668; and 668 ÷ 1,027 = .65+. Therefore, the first four figures of the root are .1865+. Since five figures are required, we proceed as in Art. **35**; .0065 ÷ .1865 = .03485254; .03485254 ÷ .1865 = .186876; .1865 + .1865 + .186876 = .559876; .559876 ÷ 3 = .186625, or .18663−, correct to five figures.

```
  6 8 5 9       6 5 0 0         1 0 2 7 ) 6 6 8.0 0 ( .6 5+
  5 8 3 2       5 8 3 2                   6 1 6 2
  1 0 2 7         6 6 8                     5 1 8 0
                                            5 1 3 5

   .1 8 6 5 ).0 0 6 5 0 0 0 0 0 0 0 0 (.0 3 4 8 5 2 5 4+
                  5 5 9 5
                    9 0 5 0
                    7 4 6 0
                    1 5 9 0 0
                    1 4 9 2 0
                        9 8 0 0
                        9 3 2 5
                          4 7 5 0
                          3 7 3 0
                          1 0 2 0 0
                            9 3 2 5
                              8 7 5 0
                              7 4 6 0
                              1 2 9 0
```

```
            .1865).0348525400(.186876+
                    1865
                    16202
                    14920
                     12825
                     11190
                      16354
                      14920
                       14340
                       13055
  1865                  12850
 .1865                  11190
 .186876                 1660
3).559876
 .186625+, or .18663−.  Ans.
```

(*b*) Since the number is entirely decimal, the root is entirely decimal. Pointing off into periods, we obtain .000′021. The figures of the root will be the same as for the cube root of 21. Hence, consider the given number as 21 and annex a cipher period so that two figures of the root may be obtained from the table; in other words, we extract the cube root of 21,000. Referring to the table, 21,000 lies between 19,683, the cube of 27, and 21,952, the cube of 28; the first two figures of the root are, therefore, 27. The first difference is 21,952 − 19,683 = 2,269; the second difference is 21,000 − 19,683 = 1,317; and 1,317 ÷ 2,269 = .58+. Hence, the first four figures of the root are 2758. Locating the decimal point according to the principle given in Art. **32**, the root is .02758+. The result does not agree with the printed answer. Hence, proceeding as directed in the note following example 3 (see Examination Questions), we apply the method described in Arts. **35** and **36**. .000021 ÷ .02758 = .000761421; .000761421 ÷ .02758 = .027607; .02758 + .02758 + .027607 = .082767; .082767 ÷ 3 = .027589, or .02759−, correct to four significant figures.

```
21952      21000      2269)1317.00(.58+
19683      19683           11345
-----      -----           -----
 2269       1317            18250
                            18152
                            -----
                               98


.02758).000021000000000(.000761421+
             19306
             -----
              16940
              16548
              -----
                3920
                2758
                ----
                11620
                11032
                -----
                  5880
                  5516
                  ----
                   3640
                   2758
                   ----
                    882


02758).00076142100(.027607+
            5516
            ----
            20982
            19306
            -----
             16761           .02758
             16548           .02758
             -----           .027607
               21300         -------
               19306      3).082767
               -----         -------
                1994         .027589, or
                                .02759—   Ans.
```

(9) $11.7 : 13 :: 20 : x$

$11.7\,x = 13 \times 20$

$11.7\,x = 260$

The product of the means equals the product of the extremes.

$x = \frac{260}{11.7}$) 2 6 0.0 0 0 (2 2.2 2+ Ans.

2 3 4

2 6 0

2 3 4

2 6 0

2 3 4

2 6 0

2 3 4

2 6

(10) (*a*) $20 + 7 : 10 + 8 :: 3 : x.$

$27 : 18 :: 3 : x$

$27\,x = 18 \times 3$

$27\,x = 54$

$x = \frac{54}{27} = 2.$ Ans.

(*b*) $12^2 : 100^2 :: 4 : x$

$144 : 10{,}000 :: 4 : x$

$144\,x = 10{,}000 \times 4$

$144\,x = 40{,}000$

$$\begin{array}{r} 12 \\ 12 \\ \hline 144 \end{array} \qquad \begin{array}{r} 100 \\ 100 \\ \hline 10000 \end{array}$$

$x = \frac{40{,}000}{144}$) 4 0 0 0 0.0 (2 7 7.7+ Ans.

2 8 8

1 1 2 0

1 0 0 8

1 1 2 0

1 0 0 8

1 1 2 0

1 0 0 8

(*a*) 1,060 miles : 444 miles :: x min. : 520 min.,

$$\text{or } x = \frac{1{,}060 \times 520}{444} = \frac{551{,}200}{444} =$$

```
  1060         444)551200.00(1241.44+ min.
   520             444
------             ---
 21200             1072
 5300               888
------             ----
551200              1840
                    1776
                    ----
                      640
                      444
                      ---
                      1960
                      1776
                      ----
                       1840
                       1776
                       ----
                         64
```

Reducing 1,241.44 min. to hours by dividing by 60, we have

```
60)1241.44(20 hr. 41.44 min.   Ans.
   120
   ---
     41
```

(*b*) 444 miles : 1,060 miles :: 520 min. : x min.

$$x = \frac{1{,}060 \times 520}{444} = 1{,}241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min.} \quad \text{Ans.}$$

(*c*) x min. : 520 min. :: 1,060 miles : 444 miles.

$$x = \frac{1{,}060 \times 520}{444} = 1{,}241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min.} \quad \text{Ans.}$$

(*d*) 520 min. : x min. :: 444 miles : 1,060 miles.

$$x = \frac{1{,}060 \times 520}{444} = 1{,}241.44 \text{ min., or } 20 \text{ hr. } 41.44 \text{ min.} \quad \text{Ans.}$$

(14) A pump discharging 135 gal. per min. fills the tank in 38 min. Therefore, a pump discharging 1 gal. per

min. fills it in 135 × 38 min. Hence, a pump discharging 85 gal. per min. fills it in $\frac{135 \times 38}{85} = 60\frac{6}{17}$ min. Ans.

$$\begin{array}{r}
135\\
\times\quad 38\\
\hline
1080\\
405\\
\hline
\end{array}$$

$$\begin{array}{rl}
85\,)\,5130 & (\,60\tfrac{6}{17}\\
510 & \\
\hline
30 & \\
\end{array}$$

$$\frac{30}{85} = \frac{6}{17}$$

(15) If a wheel measuring 12.56 ft. around it turns 520 times, a wheel measuring 1 ft. around it turns 520 × 12.56 times. Hence, a wheel measuring 15.7 ft. around it turns $\frac{520 \times 12.56}{15.7} = 416$ times. Ans.

$$\begin{array}{r}
520\\
12.56\\
\hline
3120\\
2600\\
1040\\
520\\
\hline
\end{array}$$

$$\begin{array}{rl}
15.7\,)\,6531.20 & (\,416 \text{ times.}\\
628 & \\
\hline
251 & \\
157 & \\
\hline
942 & \\
942 & \\
\hline
\end{array}$$

(16) If a cistern 28 ft. by 12 ft. by 10 ft. holds 798 bbl. of water,

a cistern 1 ft. by 12 ft. by 10 ft. holds $\frac{798}{28}$ bbl.;

a cistern 1 ft. by 1 ft. by 10 ft. holds $\frac{798}{28 \times 12}$ bbl.;

a cistern 1 ft. by 1 ft. by 1 ft. holds $\frac{798}{28 \times 12 \times 10}$ bbl.

Therefore, by a similar course of reasoning, a cistern 20 ft. by 17 ft. by 6 ft. holds

$$\frac{798 \times 20 \times 17 \times 6}{28 \times 12 \times 10} = \frac{\overset{\overset{57}{\cancel{114}}}{\cancel{798}} \times \overset{2}{\cancel{20}} \times 17 \times \cancel{6}}{\underset{\underset{2}{\cancel{4}}}{\cancel{28}} \times \underset{\cancel{2}}{\cancel{12}} \times \cancel{10}} = \frac{969}{2} = 484\tfrac{1}{2} \text{ bbl.}$$

Ans.

$$\begin{array}{r} 57 \\ 17 \\ \hline 399 \\ 57 \\ \hline 2\,\underline{)\,969} \\ 484\tfrac{1}{2} \end{array}$$

MENSURATION AND USE OF LETTERS IN FORMULAS.

---

(1) Substituting for D, x, B, and i their values,

$$C = \frac{D - x}{B + i} = \frac{120 - 12}{10 + 3.5} = \frac{108}{13.5} = 8. \quad \text{Ans.}$$

A line between two numbers signifies that the one above the line is to be divided by the one below the line.

(2) Substituting for A, h, D, and x their values,

$$\frac{A\,h + D}{2\,x + 6} = \frac{(5 \times 200) + 120}{(2 \times 12) + 6} = \frac{1{,}000 + 120}{24 + 6} = \frac{1{,}120}{30} = 37\tfrac{1}{3}$$

$$37\tfrac{1}{3} + D = 37\tfrac{1}{3} + 120 = 157\tfrac{1}{3}. \quad \text{Ans.}$$

When there is no sign between the letters, multiplication is understood.

(3) Substituting for A, D, i, and B their values,

$$v = \sqrt{\frac{A\,D}{i\,B + 1.5}} = \sqrt{\frac{5 \times 120}{(3.5 \times 10) + 1.5}} = \sqrt{\frac{600}{36.5}}$$

$$= \sqrt{16.4383} = 4.05+. \quad \text{Ans.}$$

The square root sign extends over both numerator and denominator, thus indicating that the square root of the entire fraction is to be extracted.

(4) Substituting for A, B, D, and h their values,

$$g = \frac{(B - A)^2 - \sqrt{h + 2\,B + A}}{A^3 - (1 + D)} = \frac{(10 - 5)^2 - \sqrt{200 + 2 \times 10 + 5}}{5^3 - (1 + 120)}$$

$$= \frac{5^2 - \sqrt{225}}{125 - 121} = \frac{25 - 15}{4} = \tfrac{10}{4} = 2\tfrac{1}{2}. \quad \text{Ans.}$$

For notice of copyright, see page immediately following the title page.

(5) When one straight line meets another straight line, two angles are formed which together equal 180°. Hence, if one of the angles = 152° 3′, the other angle = 180° − 152° 3′, or

$$\begin{array}{rl} & 180^\circ = 179^\circ\ 60' \\ \text{subtracting,} & 152^\circ\ \ 3' \\ \hline & 27^\circ\ 57'\quad \text{Ans.} \end{array}$$

(6) See Arts. **26-28.**

(7) See Art. **41.** A rectangle with the same area would have the same base and altitude.

(8) Since the area is to be found in square inches, the $2\frac{1}{2}$ feet must be reduced to inches. $2\frac{1}{2}$ ft. = 30 in. Area $= 30 \times 11\frac{1}{2} = 345$ sq. in. Ans.

(9) It will take $1\frac{1}{2}$ boards to reach lengthways of the room. Since the room is 15 feet wide and each board is 5 *inches* wide, it will take $15 \div \frac{5}{12} = 36$ boards, laid side by side, to extend across the width of the room. Hence, number of boards required $= 36 \times 1\frac{1}{2} = 54$. Ans.

(10) The total area of the floor of the station $= 55 \times 58$ ft. $= 3{,}190$ sq. ft. $- 25 \times 26$ ft. $= 650$ sq. ft., the area represented by the lower right-hand corner of the figure. Hence, total area of floor $= 3{,}190 - 650 = 2{,}540$ sq. ft.

From this we have to deduct the following areas:

$$\begin{array}{lcll} \text{2 boilers} & = 2 \times 8 \times 19 = & 304 & \text{sq. ft.} \\ \text{Feed-pump} & = \ 2\frac{1}{2} \times 5 \ = & 12.5 & \text{sq. ft.} \\ \text{2 engines} & = 2 \times 4\frac{1}{2} \times 10 = & 90 & \text{sq. ft.} \\ \text{2 dynamos} & = 2 \times 5\frac{1}{2} \times 6\frac{1}{2} = & 71.5 & \text{sq. ft.} \\ \text{Switchboard} & = \dfrac{10 \times 3.5}{12} = & 2.92 & \text{sq. ft.} \\ \hline & & 480.92 & \text{sq. ft.} \end{array}$$

The unoccupied floor space, therefore, equals

$$2{,}540 - 480.92 = 2{,}059.08 \text{ sq. ft.} \quad \text{Ans.}$$

(11) A triangle with three equal angles has three equal sides, and is therefore an equilateral triangle.

(12) A triangle with two equal angles has two equal sides, and is therefore an isosceles triangle.

(13) The sum of the three angles in any triangle = 2 right angles, or 180°. In the given triangle, the sum of two angles $= 23° + 32° 32' = 55° 32'$, and the third angle $= 180° - 55° 32'$, or

$$\begin{array}{rr} 180° = & 179° \ 60' \\ \text{subtracting,} & \underline{55° \ 32'} \\ & 124° \ 28' \end{array} \quad \text{Ans.}$$

(14) In Fig. I we have the proportion $A D : D E :: A B : B C$, in which $A D = 10$ in., $A B = 24$ in., and $B C = 13\frac{1}{2}$ in., to find $D E$.

Substituting the given values,

$$10 : D E :: 24 : 13\tfrac{1}{2}, \text{ or}$$

$$D E = \frac{10 \times 13.5}{24} = 5.625 \text{ in.} \quad \text{Ans.}$$

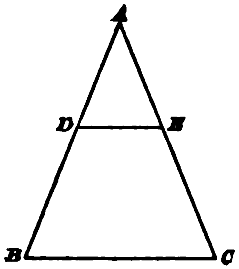


FIG. I.

(15) A line drawn diagonally from one corner to the opposite one would form the hypotenuse of a right triangle, whose two sides are 39 and 52 feet. By rule **6**, Art. **58**, the length of the diagonal $= \sqrt{52^2 + 39^2} = 65$ ft. Ans.

(16) See example, Art. **64.** The process is simply to find one of the angles of the polygon, and then to divide it by 2. By rule **10**, Art. **64**, one of the interior angles $= \frac{180 \times (8 - 2)}{8} = 135°$. This divided by $2 = 67\frac{1}{2}°$. Ans.

(17) Since this is a regular hexagon, it may be inscribed in a circle (Fig. II), and the radius of the inscribing circle will be equal to one side of the hexagon. Since the diameter $E F = 2$ inches, the radii $A B$ and $A C$, and the side $B C$ each = 1 inch, and the triangle $A B C$ is equilateral. Draw the line $A D$ perpendicular to the side $B C$; it will bisect $B C$. Then, in the right-angled triangle $A D B$, $A B = 1''$ and $B D = \frac{1}{2}''$,

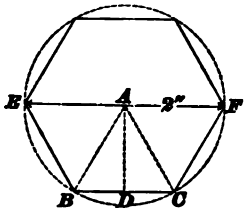


FIG. II.

to find $A\,D$. According to rule **7**, Art. **59**, $A\,D = \sqrt{1^2 - .5^2} = \sqrt{.75} = .866''$. Hence, the distance between two opposite sides of the hexagon $= A\,D \times 2 = .866 \times 2 = 1.732''$. Ans.

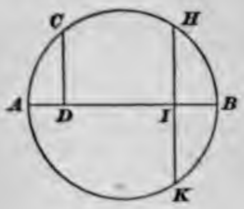


FIG. III.

(18) In Fig. III, we have the proportion $B\,I : H\,I :: H\,I : I\,A$, in which $B\,I = 6$ and $H\,I = \frac{1}{2}$ of $H\,K = \frac{18}{2} = 9$. Substituting, $6 : 9 :: 9 : I\,A$, or $I\,A = \frac{81}{6} = 13.5$ in. Hence, the diameter $A\,B = I\,A + B\,I = 13.5 + 6 = 19.5$ in. Ans.

(19) One mile = 5,280 feet. The circumference of the wheel in feet $= \frac{72 \times 3.1416}{12} = 18.8496$. (See rule **12**, Art. **77**.) Number of revolutions in going 1 mile $= 5{,}280 \div 18.8496 = 280.112$. Ans.

(20) Using rule **15**, Art. **80**, area = diameter squared $\times .7854$. $6.06^2 = 36.7236$; $36.7236 \times .7854 = 28.8427$ sq. in. Ans.

(21) Since the radius of the circle = 6 in., its diameter = 12 in., and its circumference $= 12 \times 3.1416 = 37.6992$ in. There are 360° in the circumference, and the length of an arc of $12° = 37.6992 \times \frac{12}{360} = 1.25664$ in. Ans.

(22) The area of a circle 15 in. in diameter $= 15^2 \times .7854 = 176.715$ sq. in. Hence, the area of a sector of this circle whose angle is $12\frac{1}{2}° = 176.715 \times \frac{12\frac{1}{2}}{360} = \frac{2{,}208.937}{360} = 6.1359$ sq. in. Ans. (See rule **17**, Art. **82**.)

(23) (*a*) The side of a square whose area = 103.8691 sq. in. $= \sqrt{103.8691} = 10.1916$ in. Ans.

(*b*) By rule **16**, Art. **81**, the diameter of a circle having the same area $= \sqrt{\frac{103.8691}{.7854}} = 11\frac{1}{2}$ in. Ans.

(*c*) Perimeter of the square = 10.1916 × 4 = 40.7664 in.; circumference of the circle = 11.5 × 3.1416 = 36.1284 in.; difference = 40.7664 − 36.1284 = 4.638 in. Ans.

(24) The perimeter of the base = 4 × 6 = 24 in. = 2 ft. Convex area = 2 × 12 = 24 sq. ft. The area of the bases is found as follows: In Fig. IV, $AB = 4$ in. and $AC = 2$ in.; since this is a regular hexagon, $AO = AB = 4$ in. By rule **7**, Art. **59**, $OC = \sqrt{4^2 - 2^2} = \sqrt{12} = 3.4641$ in.; area of triangle $AOB = \frac{4 \times 3.4641}{2} = 6.9282$ sq. in.; area of base = 6.9282 × 6 = 41.5692; and the area of both bases = 41.5692 × 2 = 83.1384 sq. in. This reduced to square feet $= \frac{83.1384}{144} = .5774$. Hence, the area of the entire surface of the column is 24 + .5774 = 24.5774 sq. ft. Ans.

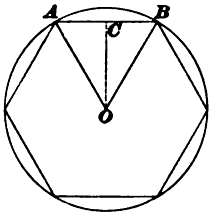


FIG. IV.

(25) The cubical contents in cubic inches = area of base in square inches × altitude in inches. The area of the base in the last example was found to be 41.5692 sq. in.; altitude = 12 × 12 = 144 in. Hence, the cubical contents = 41.5692 × 144 = 5,985.9648 cu. in. Ans.

(26) This example is solved by combining the rules for the circular ring (see example, Art. **81**) and for the cylinder. To obtain the area of one end of the tube, we have $4^2 \times .7854 = 12.5664$ = area of a circle 4 inches in diameter; $3.73^2 \times .7854 = 10.9272$ = area of a circle 3.73 inches in diameter; difference = 12.5664 − 10.9272 = 1.6392 = area of one end of the tube. The cubical contents = 1.6392 × 12 = 19.6704 cu. in.; the weight = 19.6704 × .28 = 5.5, or 5½ lb. Ans.

(27) This example is done exactly like the one in Art. **92**, and the solution is given here without explanation.

(*a*) In the formula of rule **18**, Art. **83**, $\frac{4h^2}{3}\sqrt{\frac{D}{h} - .608}$, h in this case = 18, and $D = 60$.

Substituting, area =

$$\frac{4 \times 18^2}{3}\sqrt{\frac{60}{18} - .608} = \frac{4 \times 324}{3}\sqrt{3.333 - .608} = 432 \times \sqrt{2.725}$$
$= 432 \times 1.65 = 712.8$ sq. in. This reduced to square feet $= 712.8 \div 144 = 4.95$. Hence, the steam space $= 4.95 \times 16 = 79.2$ cu. ft. Ans.

(*b*) Total area of one end of boiler in square inches $= 60^2 \times .7854 = 2{,}827.44$. From this is to be subtracted the area of the tube ends and of the segment found above.

Area of ends of tubes $= 3.5^2 \times .7854 \times 64 = 615.75$ sq. in.
Area of segment = 712.8 sq. in.
1,328.55 sq. in.

Area of water space $= 2{,}827.44 - 1{,}328.55 = 1{,}498.89$ sq. in.

Contents of water space $= 1{,}498.89 \times 16 \times 12 = 287{,}786.88$ cu. in., and $287{,}786.88 \div 231 = 1{,}245.83$, number of gallons, or say 1,246 gal. Ans.

(28) The area of the convex surface = circumference of base × ½ slant height $= 18.8496 \times \frac{10}{2} = 94.248$ sq. in. (See rule **21**, Art. **97**.) The area of the entire surface = 94.248 sq. in. + the area of the base. The diameter of the base $= \frac{18.8496}{3.1416} = 6$ in.; hence, the area of the base $= 6^2 \times .7854 = 28.2744$ (rules **13** and **15**, Arts. **78** and **80**); therefore, the area of the entire surface $= 94.248 + 28.2744 = 122.5224$ sq. in. Ans.

(29) Using rule **22**, Art. **98**, volume = area of base × ⅓ altitude $= 28.2744 \times \frac{9}{3} = 84.8232$ cu. in. Ans.

(30) The vat has the form of an inverted frustum of a pyramid. Area of larger base $= 15^2 = 225$ sq. ft.; area of smaller base $= 12^2 = 144$ sq. ft. Hence, by rule **24**, Art. **102**, the contents of the vat in cubic feet $=$
$$(225 + 144 + \sqrt{225 \times 144})\frac{11}{3} = (369 + 180) \times \frac{11}{3} = 549 \times \frac{11}{3}$$
$= 2{,}013$ cu. ft. This should be reduced to cubic inches by

multiplying by 1,728, the number of cubic inches in a cubic foot. $2{,}013 \times 1{,}728 = 3{,}478{,}464$ cu. in. Since there are 231 cubic inches in a gallon, the number of gallons that the vat will hold $= \frac{3{,}478{,}464}{231} = 15{,}058.29$. Ans.

(31) The pail is in the form of a frustum of a cone. Area of larger base $= 12^2 \times .7854 = 113.0976$ cu. in. Area of smaller base $= 63.6174$ cu. in. Hence, the contents in cubic inches =

$$(113.0976 + 63.6174 + \sqrt{113.0976 \times 63.6174}) \times \frac{11}{3}$$

$$= (176.715 + \sqrt{7{,}194.9753})\frac{11}{3} = (176.715 + 84.8232) \times \frac{11}{3}$$

$$= 261.5382 \times \frac{11}{3} = 958.9734.$$

The contents of the vat in cubic inches were found in the last example to be 3,478,464. Hence, the number of pails of water required to fill the vat $= 3{,}478{,}464 \div 958.9734 = 3{,}627.28$. Ans.

(32) (*a*) By rule **25**, Art. **104,** area of the surface $= 22.5^2 \times 3.1416 = 506.25 \times 3.1416 = 1{,}590.435$ sq. in. Ans.

(*b*) Using rule **26**, Art. **105,** the cubical contents = the cube of the diameter $\times .5236 = 11{,}390.625 \times .5236 = 5{,}964.1313$ cu. in. Ans.

(33) (*a*) Given $OB = \frac{16}{2}$, or 8 inches, and $OA = \frac{13}{2}$, or $6\frac{1}{2}$ inches, to find the volume, area, and weight (see Fig. V):

Radius of center circle equals $\frac{8 + 6.5}{2}$, or $7\frac{1}{4}$ inches.

Length of center line $= 2 \times 3.1416 \times 7\frac{1}{4} = 45.5532$ inches.

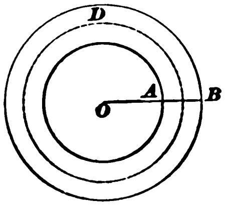


FIG. V.

The radius of the inner circle is $6\frac{1}{2}$ inches and of the outer circle 8 inches; therefore, the diameter of the cross-section on the line AB is $1\frac{1}{2}$ inches. Then, according to rule **27**, Art. **106,** the area of the ring is $1\frac{1}{2} \times 3.1416 \times 45.553 = 214.665$ square inches. Ans.

Diameter of cross-section of ring $= 1\frac{1}{2}$ inches.

Area of cross-section of ring $= (1\frac{1}{2})^2 \times .7854 = 1.76715$ sq. in. Ans.

By rule **28,** Art. **107,** volume of ring $= 1.76715 \times 45.553 = 80.499$ cu. in. Ans.

(*b*) Weight of ring $= 80.499 \times .261 = 21$ lb. Ans.

PRINCIPLES OF MECHANICS.

(1) Broadly speaking, mechanics is the science that treats of forces and their effects on material bodies. See Art. **1.**

(2) See Arts. **2, 3,** and **4.**

(3) Solid, liquid, and gaseous. See Arts. **6, 7,** and **8.**

(4) General properties are such as are common to all substances; special properties are such as are possessed by certain substances only. See Arts. **12** and **24.**

(5) (*a*) Motion is that condition of a body that causes it to change its position in relation to some other body. See Art. **30.**

(*b*) Velocity is the rate of motion, that is, the distance passed through in a unit of time. See Art. **32.**

(*c*) Uniform velocity means passing over equal distances in equal intervals of time. See Art. **32.**

(*d*) Variable velocity means passing over equal distances in unequal intervals of time, or passing over unequal distances in equal intervals of time. See Art. **33.**

(6) (*a*) Acceleration is the rate of change of velocity. See Arts. **35** and **57.**

(*b*) Retardation denotes the rate of decrease of velocity. See Arts. **36** and **58.**

(*c*) The average velocity is that uniform velocity that carries the body over the same distance in the same time as the variable velocity. See Art. **37.**

(7) Since 1 day contains 24 hours, the total number of hours consumed in the trip was $6 \times 24 + 16 = 160$ hours. The average speed per hour was, therefore, $3{,}240 \div 160 = 20\frac{1}{4}$ mi. Ans. See Art. **38.**

(8) Reducing the 25,000 miles to feet, we have $25{,}000 \times 5{,}280 = 132{,}000{,}000$ feet. The time in minutes would, therefore, be $132{,}000{,}000 \div 3{,}000 = 44{,}000$ minutes. Reducing the time to days, hours, and minutes, we get 30 da. 13 hr. 20 min. Ans. See Art. **40.**

(9) By the effects it produces on matter. See Art. **41.**

(10) The point of application, the direction of action, and the magnitude of each force must be known. See Art. **44.**

(11) See Art. **46.**

(12) Inertia is that property of a body by virtue of which the body always tends to remain in the particular state of rest or motion that it has at the moment considered. See Art. **48.**

(13) See Arts. **52** and **53.**

(14) The weight of a body is proportional to the force of gravity; and since the force of gravity is different for different localities, the weight of any body will differ likewise. See Art. **56.**

(15) By rule 4, Art. **60,**

$$m = \frac{W}{g} = \frac{346}{32.174} = 10.75+. \quad \text{Ans.}$$

(16) See Art. **62.**

(17) The constant opposing force is $30 \times 15 = 450$ pounds. 30 tons $= 30 \times 2{,}000 = 60{,}000$ pounds. The force f to produce the required acceleration will, by rule **5,** Art. **63,** be $\frac{W}{g}a = \frac{60{,}000}{32.16} \times 3 = 5{,}597+$ pounds. Hence, the total force required will be $5{,}597 + 450 = 6{,}047$ lb. Ans.

(18) 1 mile per hour $= 5{,}280$ feet per hour. $\frac{5{,}280}{60 \times 60} = \frac{22}{15} = 1\frac{7}{15}$ feet per second. Momentum, according to Art. **67**, $= \frac{60{,}000}{32.16} \times \frac{22}{15} = 2{,}736.3$ lb. Ans.

(19) (*a*) Work is the overcoming of a resistance through a certain distance. See Arts. **69**, **70**, and **71.**

(*b*) Power is the rate of doing work. See Arts. **72** and **73.**

(*c*) Energy denotes ability to do work. See Arts. **74** and **75.**

(20) $\frac{66{,}000 \times 80}{33{,}000 \times 30} = 5\frac{1}{3}$ H. P. Ans. See Art. **73.**

(21) The kinetic energy of the body by rule **7**, Art. **76**, is $\frac{w v^2}{2g} = \frac{6{,}432 \times 60^2}{2 \times 32.16} = 360{,}000$ foot-pounds. This amount of work must be done in 3 minutes. Hence, the horsepower required to stop the body will be $\frac{360{,}000}{3} \div 33{,}000 = 3\frac{7}{11}$ H. P. Ans.

(22) Zero. See Art. **86.**

(23) Friction is the resistance that a moving body encounters from the surface of another body along which or through which it moves. See Art. **88.**

(24) According to Art. **89**, the coefficient of friction will be $\frac{300}{5000} = .06$. Ans.

(25) (*a*) From Table I we find the coefficient of friction to be .16. Hence, the total friction will be $1{,}000 \times .16 = 160$ lb. Ans. See Art. **92.**

(*b*) $\frac{160}{100} = 1.6$ lb. per sq. in. Ans.

(26) The center of gravity of a body is a point at which the whole weight of the body may be considered as concentrated. This point may be either inside or outside the body. See Art. **98.**

(27) See Art. **99.**

(28) Suspend the body successively from two different points and find the point of intersection of two plumb-lines from the points of suspension. The center of gravity will be on the line passing through the point of intersection and perpendicular to the plumb-lines. See Art. **103.**

(29) (*a*) Centrifugal force is that force that tends to pull a revolving body away from the point about which the body revolves. See Art. **104.**

(*b*) Centripetal force is that force that tends to pull a revolving body toward the point about which it revolves. See Art. **105.**

(30) The centrifugal and centripetal forces of a given revolving body are always equal and opposite. See Art. **104.**

(31) Applying rule **14**, given in Art. **106,**

$F = .00034 \times 10 \times 6 \times 60^2 = 73.44$ lb. Ans.

(32) See Arts. **110, 111,** and **112.**

(33) The forces acting on a body at rest are in equilibrium. See Art. **109.**

(34) As long as the line of direction falls within the base, the body will stand; if it falls outside the base, the body will fall. In the former case the forces will be in equilibrium; in the latter they will not be in equilibrium. See Art. **113.**

MACHINE ELEMENTS.

(1) (*a*) A lever is a rigid bar or rod capable of being turned about a pivot.

(*b*) The weight arm is that part of the lever between the fulcrum and the weight.

(*c*) The force arm is that part of the lever between the fulcrum and the force.

(*d*) The fulcrum is the point or pivot about which the lever turns. See Arts. **1** and **2.**

(2) The product of the weight and the weight arm must be equal to the product of the force and the force arm. See Art. **2.**

(3) See rule 1, Art. **3.**

(4) $\frac{12 \times 20}{100} = 2.4$ in. Ans. See Art. **3.**

(5) Solid pulleys and split pulleys. See Art. **9.**

(6) Split pulleys can be more easily put on and removed from shafts than solid pulleys. See Art. **9.**

(7) As explained in Art. **11,** the belt climbs toward the highest part of the pulley; and since in a crowned pulley the highest portion is in the center, the belt will tend to stay on the pulley.

(8) "Balancing pulleys" is the operation of making opposite sides of pulleys equal in weight, so that the centrifugal

forces of the opposite sides of a revolving pulley will be equal. A pulley is balanced by first finding which side is the lighter and then adding weights to the lighter side until it is as heavy as the other. See Art. 12.

(9) The "driver" is the pulley that imparts motion to the belt. The "driven" is the pulley that receives motion from the belt. See Art. 14.

(10) Applying rule 3, Art. 15,

$$D = \frac{6 \times 1{,}800}{600} = 18 \text{ in.} \quad \text{Ans.}$$

(11) Applying rule 6, Art. 18,

$$N = \frac{5 \times 1{,}600}{12} = 666\tfrac{2}{3} \text{ rev. per min.} \quad \text{Ans.}$$

(12) Applying rule 7, Art. 23,

$$B = 3\tfrac{1}{4}\left(\frac{6+2}{2}\right) + 2 \times 40 = 93 \text{ ft.} \quad \text{Ans.}$$

(13) From Table I we find that the allowable effective pull C for an arc of contact of 120° is 28.8 pounds. Applying rule 8, Art. 27,

$$W = \frac{33{,}000 \times 40}{1{,}650 \times 28.8} = 27.7, \text{ say } 28 \text{ in.} \quad \text{Ans.}$$

(14) Applying rule 9, Art. 28,

$$H = \frac{28.8 \times 28 \times 1{,}200}{33{,}000} = 29.3+ \text{ H. P.} \quad \text{Ans.}$$

(15) Applying rule 10, Art. 31,

$$W = \frac{23{,}100 \times 40}{1{,}650 \times 28.8} = 19.44, \text{ say } 20 \text{ in.} \quad \text{Ans.}$$

The hair or grain side is commonly considered to be
er side to be in contact with the pulley face. For
ee Art. 32.

osin makes the belt gummy and causes it to
t. 32.

(18) Pulleys run out of true; they should be turned true. Pulleys out of line; they should be lined up. Belts sometimes flap if they are run at speeds over 4,000 feet per minute; perforating the belt with a series of small holes is said to remedy the defect in this case. Lack of steadiness in running; take steps to insure steady running. A defective joint; unlace the joint and relace it properly. Too great a distance between pulleys; reduce the distance if possible or substitute a wider belt. See Art. **33.**

(19) Lacing, sewing, riveting, and cementing. The cemented joint is considered the best.

(20) See Art. **38.**

(21) A wheel that imparts motion to another is called a "driver." A "follower" is a wheel that receives motion from another.

(22) Letting W represent the weight we have, by rule **12**, Art. **39,**

$$50 \times 90 \times 30 \times 36 = W \times 30 \times 12 \times 20,$$

or

$$4{,}860{,}000 = 7{,}200\ W.$$

Hence, $W = \frac{4{,}860{,}000}{7{,}200} = 675$ lb. Ans.

(23) (*a*) Circular pitch is the distance measured along the pitch circle from a point on one tooth to the corresponding point on the next tooth. See Art. **47.**

(*b*) Diametral pitch is the number of teeth per inch of pitch diameter. See Art. **47.**

(24) The epicycloidal and the involute. See Arts. **51** and **52.**

(25) Involute teeth are stronger and their action is more satisfactory. See Art. **52.**

(26) Applying rule **13,** Art. **54,**

$$D = \frac{1.152 \times 60}{3.1416} = 22 \text{ in.} \quad \text{Ans.}$$

(27) Applying rule **15**, Art. **56**,

$$P = \frac{3.1416 \times 30}{60} = 1.57 \text{ in.} \quad \text{Ans.}$$

(28) Applying rule **17**, Art. **59**,

$$D_o = \frac{80 + 2}{8} = 10\tfrac{1}{4} \text{ in.} \quad \text{Ans.}$$

(29) Applying rule **21**, Art. **63**,

$$N = 7.5 \times 8 - 2 = 58 \text{ teeth.} \quad \text{Ans.}$$

(30) A fixed pulley has a stationary block. A movable pulley has a movable block. See Arts. **71** and **72**.

(31) Calling W the weight that can be raised, we have, by applying rule **27**, Art. **74**,

$$W = 150 \times 14 = 2{,}100 \text{ lb.} \quad \text{Ans.}$$

(32) The force that must be applied is greater than if there were no frictional losses. See Art. **75**.

(33) Applying rule **28**, Art. **75**,

$$f_a = \frac{100 \times 1{,}500}{2 \times 4 \times 60} = 312.5 \text{ lb.} \quad \text{Ans.}$$

(34) With the Weston differential pulley block a much greater weight can be raised with a given force than with the ordinary pulley; and the load can be stopped anywhere by ceasing to pull on the chain. See Art. **78**.

(35) Applying rule **30**, Art. **82**,

$$P = \frac{600 \times 12}{70} = 102.8+, \text{ say } 103 \text{ lb.} \quad \text{Ans.}$$

(36) Applying rule **31**, Art. **83**,

$$W = \frac{36 \times 50}{15} = 120 \text{ lb.} \quad \text{Ans.}$$

(37) First applying rule **34**, Art. **89**, we find the factor by which the theoretical weight is to be multiplied.

Thus, $$E = \frac{\frac{1}{6}}{\frac{1}{6} + 2\frac{1}{2}} = \tfrac{1}{16}.$$

Then applying rule **35,** Art. **90,** to obtain the theoretical weight, we find

$$W = \frac{6.2832 \times 50 \times 20}{\frac{1}{6}} = 37,699.2 \text{ pounds.}$$

Finally applying the principle of Art. **92,** we find the probable actual weight to be

$$37,699.2 \times \tfrac{1}{16} = 2,356.2 \text{ lb.} \quad \text{Ans.}$$

(38) (*a*) The velocity ratio is the ratio between the distance through which the force acts and that through which the weight moves. See Art. **103.**

(*b*) The efficiency is the ratio of the actual work to the theoretical work. See Art. **105.**

(39) By Art. **105,** the efficiency would be $\frac{2,356.2}{37,699.2}$ $= .0625$, or $6\frac{1}{4}$ per cent. Ans.

ELEMENTS OF ELECTRICITY AND MAGNETISM.

---

(1) The end *b*; because in looking at that end the current circulates around the helix in an opposite direction to the hands of a watch. See Art. **29.**

(2) (*a*) Negative. See Art. **7.**
(*b*) Negative. See Art. **7.**

(3) Let *A* represent the first branch and *B* the second; then $r_1 = 16.2$ ohms, $r_2 = 14.1$ ohms, and $C = 6.37$ amperes.

The current c_1 in branch *A* is found by using formula **10**; substituting gives

$$c_1 = \frac{C r_2}{r_1 + r_2} = \frac{6.37 \times 14.1}{16.2 + 14.1} = 2.9643 \text{ amperes. Ans.}$$

The current c_2 in branch *B* is found by using formula **11**; substituting gives

$$c_2 = \frac{C r_1}{r_1 + r_2} = \frac{6.37 \times 16.2}{16.2 + 14.1} = 3.4057 \text{ amperes. Ans.}$$

(4) (*a*) From Art. **64** and formula **6**, $C = \frac{E}{R}$, where C is the current in amperes, E is the difference of potential in volts between two points, and R is the resistance in ohms between them. In this example, $E = 58.4$ volts and $R = 2.3$ ohms; hence, $C = \frac{E}{R} = \frac{58.4}{2.3} = 25.3913$ amperes. Ans.

For notice of copyright, see page immediately following the title page.

(*b*) From formula **21**, $W = \frac{E^2}{R}$, where W is the power in watts, E is the E. M. F., or difference of potential in volts, and R is the resistance in ohms. In this example, $E = 58.4$ volts and $R = 2.3$ ohms; hence,

$$W = \frac{E^2}{R} = \frac{58.4^2}{2.3} = \frac{3{,}410.56}{2.3} = 1{,}482.8521 \text{ watts. Ans.}$$

(*c*) By formula **22**, $\text{H. P.} = \frac{W}{746}$; by formula **21**, $W = \frac{E^2}{R}$; therefore (see Art. **81**), $\text{H. P.} = \frac{E^2}{746\,R}$, where H. P. is the horsepower, E the E. M. F., or difference of potential in volts, and R the resistance in ohms.

Hence, $\text{H. P.} = \frac{58.4^2}{746 \times 2.3} = \frac{3{,}410.56}{1{,}715.8} = 1.9877$ horsepower. Ans.

(5) By formula **8**, $E = C\,R$, where E is the total E. M. F. in volts developed in a closed circuit, C is the current in amperes that is flowing, and R is the total resistance in ohms of the circuit. In this example, $C = .75$ ampere and $R = 17.2 + 8.2 + 11.3 = 36.7$ ohms; hence, $E = C\,R = .75 \times 36.7 = 27.525$ volts, the total E. M. F. developed in the battery.

By derivation from formula **8**, $E' = C\,R'$, where E' is the difference of potential in volts between two points, C is the current in amperes flowing, and R' is the resistance in ohms between the two points.

Between *a* and *b*, $R' = 11.3$ ohms and $C = .75$ ampere; hence, $E' = C\,R' = .75 \times 11.3 = 8.475$ volts. Ans.

Between *b* and *c*, $R' = 8.2$ ohms and $C = .75$ ampere; hence, $E' = C\,R' = .75 \times 8.2 = 6.15$ volts. Ans.

Between *a* and *c*, the difference of potential is the difference of potential between *a* and *b* plus that between *b* and *c*, which is $8.475 + 6.15 = 14.625$ volts. Or, since the difference of potential between *a* and *c* is the available E. M. F. of the battery, when a current of .75 ampere is flowing, it can be calculated by using formula **9,** $E' = E - C\,r_i$, where E' is the available E. M. F., E is the total E. M. F. developed in the battery, C is the current that is flowing, and r_i is the internal resistance of the battery. In this case, $E = 27.525$ volts, $C = .75$ ampere, and $r_i = 17.2$ ohms; hence, $E' = E - C\,r_i = 27.525 - (.75 \times 17.2) = 14.625$ volts. Ans.

(6) By formula **8,** $E = C\,R$, where E is the total E. M. F. in volts developed in a closed circuit, C is the current in amperes flowing, and R is the total resistance in ohms of the circuit. In this example, $C = .127$ ampere and $R = 36.2 + 21.7 = 57.9$ ohms. Hence, by substituting, $E = C\,R = .127 \times 57.9 = 7.3533$ volts. Ans.

(7) By formula **14,** $Q = C\,t$, where Q is the quantity of electricity in coulombs that passes through a circuit, C is the current in amperes flowing in that circuit, and t is the time in seconds during which the current flows. In this example, $C = 8.32$ amperes and $t = 2.25 \times 60 \times 60 = 8{,}100$ seconds. Hence, by substituting, $Q = C\,t = 8.32 \times 8{,}100 = 67{,}392$ coulombs. Ans.

(8) By formula **19,** $W = C\,E$, where W is the power in watts, E is the E. M. F. in volts, and C is the current in amperes. In this example, $E = 112.5$ volts and $C = 12.2$ amperes. Hence, by substituting, $W = C\,E = 12.2 \times 112.5 = 1{,}372.5$ watts. Ans.

(9) By formula **4,** $r_2 = r_1\,(1 + t\,k)$, where r_1 is the original resistance of a conductor, r_2 is the resistance after a rise in temperature, k is the temperature coefficient, and t is the rise of temperature in degrees F. In this example, $r_1 = 43.2$ ohms, $t = 85 - 60 = 25°$ F., and $k = .002155$, from Table I. Hence, $r_2 = r_1\,(1 + t\,k) = 43.2\,(1 + 25 \times .002155) = 43.2 \times 1.053875 = 45.5274$ ohms. Ans.

(10) By formula **13,** the joint resistance of three conductors in parallel $R''' = \dfrac{r_1\,r_2\,r_3}{r_2\,r_3 + r_1\,r_3 + r_1\,r_2}$, where r_1, r_2, and r_3 are the separate resistance of the three conductors,

respectively. In this example, let $r_1 = 37$ ohms, the resistance of A; $r_2 = 45$ ohms, the resistance of B; and $r_3 = 72$ ohms, the resistance of C. Substituting gives

$$\frac{r_1 r_2 r_3}{r_2 r_3 + r_1 r_3 + r_1 r_2} = \frac{37 \times 45 \times 72}{45 \times 72 + 37 \times 72 + 37 \times 45} = \frac{119{,}880}{7{,}569}$$

$= 15.8383$ ohms, the joint resistance of the three conductors A, B, and C connected in parallel. Ans.

(11) From Art. **43**, the joint resistance of several conductors connected in series is equal to the sum of their separate resistances; hence, in this example the joint resistance of the four conductors A, B, C, and D in series is $3 + 19 + 72 + 111 = 205$ ohms. Ans.

(12) We here use formula **16**, $J = C^2 R t$, where J is the work in joules, C is the current in amperes, R is the resistance in ohms, and t is the time in seconds. In this case $C = 14.2$ amperes, $R = 8$ ohms, $t = 4{,}500$ seconds. Then, the work done $= 14.2 \times 14.2 \times 8 \times 4{,}500 = 7{,}259{,}040$ joules. Ans.

(13) From Art. **75**, the separate resistance of any branch of a derived circuit is equal to the difference of potential between where all the branches divide and where they unite, divided by the current in that branch.

Hence, the separate resistance of branch A is $\frac{11.6}{6.7} = 1.7313$ ohms. Ans.

The separate resistance of branch B is $\frac{11.6}{4.9} = 2.3673$ ohms. Ans.

(14) By formula **7**, $R = \frac{E}{C}$, where R is the total resistance in ohms of a closed circuit, E is the total E. M. F. in volts developed in the circuit, and C is the current in amperes flowing in the circuit. In this example, $E = 22.4$ volts, and $C = .43$ ampere; hence, $R = \frac{E}{C} = \frac{22.4}{.43} = 52.093$ ohms, the total resistance of the circuit. Since the total resistance of a closed circuit is equal to the sum of

the external and internal resistances, the external resistance must be the difference between the total resistance and the internal resistance. Hence, the external resistance $= 52.093 - 13.4 = 38.693$ ohms. Ans.

(15) By transposition of terms in formula **14**, $C = \frac{Q}{t}$, where C is the current in amperes, Q is the quantity of electricity in coulombs, and t is the time in seconds. In this example, $Q = 368{,}422$ coulombs and $t = 4.5 \times 60 \times 60 = 16{,}200$ seconds; hence, $C = \frac{Q}{t} = \frac{368{,}422}{16{,}200} = 22.7421$ amperes. Ans.

(16) By formula **16**, $J = C^2 R t$, where J is the work done in joules, C is the current in amperes, R is the resistance in ohms, and t is the time in seconds. In this example, $C = 2.4$ amperes, $R = 45$ ohms, $t = 3{,}000$ seconds. Then the electrical work done $= 2.4 \times 2.4 \times 45 \times 3{,}000 = 777{,}600$ joules. By formula **18**, the mechanical work done $=$ F. P. $= .7373 J = .7373 \times 777{,}600 = 573{,}324.48$ foot-pounds. Ans.

(17) By formula **22**, H. P. $= \frac{W}{746}$; by formula **19**, $W = C E$; therefore (see Art. **81**), H. P. $= \frac{E C}{746}$, where H. P. is the horsepower, E is the E. M. F. in volts, and C is the current in amperes. In this example, $E = 525$ volts, and $C = 12.5$ amperes; hence,

$$\text{H. P.} = \frac{E C}{746} = \frac{525 \times 12.5}{746} = 8.7969 \text{ horsepower. Ans.}$$

(18) (*a*) By formula **20**, $W = C^2 R$, where W is the power in watts, C is the current in amperes, and R is the resistance in ohms. In this example, $C = 110$ amperes and $R = 4.2$ ohms; hence, $W = C^2 R = 110^2 \times 4.2 = 50{,}820$ watts. Ans.

(*b*) H. P. $= \frac{W}{746} = \frac{50{,}820}{746} = 68.123$ horsepower. Ans.

(19) **By formula 1, the changed resistance for variation in length, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance, l_1 is the original length, and l_2 is the changed length. In this case, $r_1 = 1$ ohm, $l_1 = 1,000$ feet, and $l_2 = 2,000$ feet. Then, the changed resistance $r_2 = \frac{1 \times 2,000}{1,000} = 2$ ohms. The next operation is to determine the resistance of the wire when its sectional area is changed. A round wire 1 inch in diameter has a sectional area of $.1^2 \times .7854 = .007854$ square inch, and a square wire .1 inch on a side has a sectional area of $.1 \times .1 = .01$ square inch. By formula 2, $r_2 = \frac{r_1 a_1}{a_2}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its sectional area is changed, a_1 is the original sectional area, and a_2 is the changed sectional area. At this stage of the example, $r_1 = 2$ ohms, $a_1 = .007854$ square inch, and $a_2 = .01$ square inch. Hence, $r_2 = \frac{r_1 a_1}{a_2} = \frac{2 \times .007854}{.01} = 1.5708$ ohms. Ans.**

(20) By formula **22**, H. P. $= \frac{W}{746}$, where H. P. is the horsepower and W is the power in watts. In this example, $W = 54,200$ watts; hence, H. P. $= \frac{W}{746} = \frac{54,200}{746} = 72.6541$ horsepower. Ans.

(21) The sectional area of a round column .04 inch in diameter is $.04^2 \times .7854 = .00125664$ square inch, or .001257 square inch, nearly.

Reduce the specific resistance in microhms to the resistance in ohms by dividing by 1,000,000, Art. **44**, which gives $\frac{37.15}{1,000,000} = .00003715$ ohm; or, in other words, the resistance of a quantity of mercury 1 inch long and whose sectional area is 1 square inch is .00003715 ohm. Next, from this resistance and length, calculate the resistance of a column of mercury 72.3 inches high, with a sectional area of 1 square inch, by using formula **1**, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its length has been changed, l_1 is its original length, and l_2 is its changed length. In this example, $r_1 = .00003715$ ohm, $l_1 = 1$ inch, and $l_2 = 72.3$ inches. Hence, $r_2 = \frac{r_1 l_2}{l_1} = \frac{.00003715 \times 72.3}{1} = .002685945$, or .002686 ohm, nearly; or, in other words, the resistance of a column of mercury 72.3 inches high, having a sectional area of 1 square inch, is .002686 ohm. From this result calculate the resistance of the column when its sectional area is .001257 square inch, by using formula **2**, $r_2 = \frac{r_1 a_1}{a_2}$, where r_1 is the original resistance, r_2 is the resistance after the sectional area has been changed, a_1 is the original sectional area, and a_2 is the changed sectional area. At this stage of the example, $r_1 = .002686$ ohm, $a_1 = 1$ square inch, and $a_2 = .001257$ square inch. Hence,

$$r_2 = \frac{r_1 a_1}{a_2} = \frac{.002686 \times 1}{.001257} = 2.1368 \text{ ohms. Ans.}$$

(22) By formula **6**, $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is tho total E. M. F. in volts generated, and R is the total resistance in ohms of the circuit. Since the total resistance of a closed circuit is the sum of the external and internal circuits, $R = 33 + 30 = 63$ ohms, and $E = 45$ volts; hence, $C = \frac{E}{R} = \frac{45}{63} = .7143$ ampere. Ans.

(23) By formula **19**, $W = CE$, where W is the power in watts, E is the E. M. F., or difference of potential in volts, and C is the current in amperes. In this example, $E = 510$ volts and $C = 24.3$ amperes; hence, $W = 510 \times 24.3 = 12,393$ watts. Ans.

(24) Referring to Art. **56**, the total E. M. F. de by connecting several cells in series is equal to the E of one cell multiplied by the number of cells; he E. M. F. of one of the groups of 6 cells is $6 \times 1.5 =$ In the same article it is stated that connecting multiple, or parallel, does not change the E. M. F. the main conductors. In this case each group o can be considered as one large cell developing an of 9 volts, and, consequently, the E. M. F. of groups connected in multiple, or parallel, is 9 volt would be the E. M. F. indicated by a voltmeter, co to the main conductors c and c'. Ans.

(25) By formula **22**, H. P. $= \frac{W}{746}$; by form $W = CE$; therefore (see Art. **81**), H. P. $= \frac{EC}{746}$, whe is the horsepower, E is the E. M. F. in volts, and current in amperes. In this example, $E = 250$ v $C = 65.7$ amperes; hence, H. P. $= \frac{EC}{746} = \frac{250 \times 65.7}{746}$ $= 22.0174$ horsepower. Ans.

(26) End a; since, Art. **29**, in looking at the fa end a, the current circulates around the core in t direction as the movement of the hands of a watch.

(27) From Art. **21**, iron and its alloys, nickel, manganese, oxygen, cerium, and chromium.

(28) Toward the south pole, since, from Art. **2** poles attract one another.

(29) Toward the north pole, since, from Art. **2** poles attract one another.

(30) The current should enter the wire at th since, Art. **29**, in looking at the face of the south the magnet, the current should circulate around in the direction of motion of the hands of a watch.

(31) By formula **1**, $r_2 = \frac{r_1 l_2}{l_1}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its length has been changed, l_1 is the original length, and l_2 is its changed length. In this example, $r_1 = 100.8$ ohms, $l_1 = (112 \times 12) + 6 = 1{,}350$ inches, $l_2 = 11.7$ inches. Hence,

$$r_2 = \frac{r_1 l_2}{l_1} = \frac{100.8 \times 11.7}{1{,}350} = .8736 \text{ ohm.} \quad \text{Ans.}$$

(32) By formula **4**, $r_2 = r_1 (1 + t\,k)$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its temperature has risen, k is the temperature coefficient, and t is the number of degrees rise Fahrenheit. In this example, $r_1 = 91.8$ ohms, $t = 72 - 45 = 27°$, and $k = .000244$, from Table I. Hence, $r_2 = r_1 (1 + t\,k) = 91.8 (1 + 27 \times .000244) = 91.8 \times 1.006588 = 92.4048$ ohms. Ans.

(33) By formula **5**, $r_2 = \frac{r_1}{1 + t\,k}$, where r_1 is the original resistance of a conductor, r_2 is the resistance after its temperature has fallen, t is the number of degrees fall Fahrenheit, and k is the temperature coefficient of the material. In this example, $r_1 = .144$ ohm, $t = 87 - 41 = 46°$ F., and $k = .002155$, from Table I. Hence,

$$r_2 = \frac{r_1}{1 + t\,k} = \frac{.144}{1 + 46 \times .002155} = \frac{.144}{1.09913} = .131 \text{ ohm.}$$

Ans.

(34) From Art. **53**, the fundamental equation of the Wheatstone's bridge is $X = \frac{M}{N} \times P$, where X is the unknown resistance, M is the resistance of the upper balance arm, N is the resistance of the lower balance arm, and P is the resistance of the adjustable arm. It will be seen from the connections of the battery and galvanometer circuits in the diagram that the coils lying between c and a form the upper balance arm of the bridge, and, hence, in this example, $M = 1$ ohm; the coils between a and d form the lower balance arm, and, hence, $N = 100$ ohms; the coils between

a and *b* form the adjustable arm, and, hence, $P = 500 + 200 + 20 + 2 + 1 = 723$ ohms. Substituting these values in
he fundamental equation gives

$$X = \frac{M}{N} \times P = \frac{1}{100} \times 723 = 7.23 \text{ ohms.} \quad \text{Ans.}$$

(35) By formula **6**, $C = \frac{E}{R}$, where C is the current in
nperes flowing in a closed circuit, E is the total E. M. F.
volts developed in the circuit, and R is the total resist-
ce in ohms of the circuit. In this example, $E = 36$ volts
d $R = 24 + 18 = 42$ ohms; since, according to Art. **60**,
e total resistance of a closed circuit is the sum of the
ternal and external resistances. Hence, $C = \frac{E}{R} = \frac{36}{42}$
$= .8571$ ampere. Ans.

(36) By formula **7**, $R = \frac{E}{C}$, where R is the total resist-
e in ohms of a closed circuit E is the total E. M. F. in
ts developed in the circuit, and C is the current in
amperes flowing through the circuit. In this example, $E = 12.6$ volts and $C = 2.7$ amperes; hence, $R = \frac{E}{C} = \frac{12.6}{2.7}$ $= 4.6667$ ohms. Ans.

(37) By formula **8**, $E = C R$, where E is the total E.M.F. in volts developed in a closed circuit, C is the current in amperes flowing through the circuit, and R is the total resistance of the circuit. In this example, $C = .8$ ampere and $R = 31.5 + 11 = 42.5$ ohms, since, Art. **60**, the total resistance of a closed circuit is the sum of the internal and external resistances. Hence, $E = C R = .8 \times 42.5 = 34$ volts. Ans.

(38) (*a*) By formula **7**, $R = \frac{E}{C}$, where R is the total resistance in ohms between two points in a circuit, E is the drop or loss of potential in volts between the two points, and C is the current in amperes flowing in the circuit. In

this example, the two conductors leading to and from the receptive device can be considered as in series, forming one single conductor 1,200 feet in length, in which the drop or loss of potential is 10 per cent. of 250 volts, or $.10 \times 250 = 25$ volts; that is, $E = 25$ volts. Since $C = 80$ amperes, then, $R = \frac{E}{C} = \frac{25}{80} = .3125$ ohm; or, in other words, the sum of the resistances of two conductors that transmit a current of 80 amperes to and from the receptive device with a loss of 25 volts is .3125 ohm. Ans.

(*b*) The resistance per foot of any conductor is found by dividing its total resistance by its length in feet. Assume the two conductors leading to and from the receptive device to be one single conductor 1,200 feet in length and offering a resistance of .3125 ohm; hence, its resistance per foot is

$$\frac{.3125}{1,200} = .00026 \text{ ohm.} \quad \text{Ans.}$$

(39) By formula **6**, $C = \frac{E}{R}$, where C is the current in amperes flowing in a closed circuit, E is the total E. M. F. in volts developed in the circuit, and R is the total resistance in ohms of the circuit. In this example, $E = 24$ volts and $R = 8.1 + 15.9 = 24$ ohms, since, Art. **60**, the total resistance of a closed circuit is the sum of the internal and external resistances. Hence,

$$C = \frac{E}{R} = \frac{24}{24} = 1 \text{ ampere.}$$

By formula **9**, $E' = E - C r_i$, where E' is the available, or external, E. M. F. in volts of a battery or other electric source in a closed circuit, E is the total E. M. F. in volts developed in the source, C is the current in amperes flowing through the circuit, and r_i is the internal resistance of the battery or electric source. In this example, $E = 24$ volts, $C = 1$ ampere, and $r_i = 8.1$ ohms. Hence, $E' = E - C r_i = 24 - (8.1 \times 1) = 15.9$ volts. Ans.

[illegible] the second; [illegible] amperes.

[illegible] by substi-tuting [illegible]

[illegible] amperes. Ans.

[illegible] branches is [illegible] therefore, the differ-ence [illegible] branch A, or [illegible] amperes. Ans.

[illegible] horse-power.

[illegible] watts. [illegible] 22,

[illegible] horsepower. Ans.

[illegible] watts. [illegible] transposing for-mula 19, [illegible]

[illegible] amperes. Ans.

[illegible] work and [illegible] kilowatt [illegible]

[illegible] watts. [illegible] = 4.4 kilo-watts [illegible] watt-hours = 4.4 [illegible] Ans. See Art. 83.

DYNAMOS AND MOTORS.

(1) By formula **1,** $E = \frac{2\,N\,S\,n}{10^8}$. In this example, $N = 6{,}250{,}000$ lines of force, $S = 100$ outside, or face, wires in series, for if 200 turns were wound around the core, there would be 200 outside, or face, wires and from Art. **23** one-half would be connected in series, and $n = \frac{1{,}200}{60}$ revolutions per second. Substituting these values in above formula gives $E = \frac{2\,N\,S\,n}{10^8} = \frac{2 \times 6{,}250{,}000 \times 100 \times 1{,}200}{100{,}000{,}000 \times 60}$ $= 250$ volts. Ans.

(2) From Art. **36,** it will be seen that the current in the shunt field of a dynamo is equal to the difference of potential between the brushes divided by the resistance of the shunt field circuit, or $C_s = \frac{E}{R_s}$. In this example, $E = 220$ volts and $R_s = 440$ ohms; hence, $C_s = \frac{E}{R_s} = \frac{220}{440} = .5$ ampere. Ans.

(3) An alternating current is one that is continually reversing in direction in a regular periodic manner, flowing first in one direction in a circuit and then in the other, while a continuous current is one that flows, without stopping, in one direction. See Arts. **13** and **14.**

For notice of copyright, see page immediately following the title page.

[illegible] cur- [illegible] so [illegible] the circuit [illegible] the [illegible] or converter, to transform a con- [illegible] current [illegible] flows in an external circuit into an alternating current [illegible] passes into the winding of a motor [illegible] 11 and 19.

(8) Use formula **4.** In this example, the input $= \frac{100 \times 65,000}{90.5} = 71,823.2$ watts. Since 1 horsepower equals 746 watts, then 71,823.2 watts equal $\frac{71,823.2}{746} = 96.28$ H. P. Ans.

(9) If the current circulates in the direction indicated by the arrowheads, *neither* pole piece will be a north pole; for it will be seen that the north pole of one field coil is opposite the south pole of the other, and the lines of force circulate around the magnets without passing through the armature. If the winding of the right-hand coil were reversed, its top would be a north pole, and the top of the left-hand coil being also north, the pole piece P would become a north consequent pole and P' a south consequent pole.

(10) In this example, it is first necessary to change the input from horsepower to watts. Since 1 horsepower is equivalent to 746 watts, then 10 horsepower are equivalent to 10 × 746 = 7,460 watts. Then, by formula **2,** the efficiency $\mathbf{E} = \frac{100 \times 6,341}{7,460} = 85$ per cent. Ans.

(11) From b to a through the conductor; for, by applying the thumb-and-finger rule given in Art. **8** as indicated, it will be seen that the middle finger is pointing toward a from b.

(12) In this example, it is first necessary to find the input in watts. In this example, the output = 11,900 watts and the per cent. efficiency = 85. Then, by formula **4,** the input $= \frac{100 \times 11,900}{85} = 14,000$ watts input. As in Art. **64,** the watts lost are found by multiplying the input by the percentage of loss and dividing by 100. Hence, $\frac{14,000 \times 1.8}{100}$ = 252 watts lost in core by eddy currents and hysteresis. Ans.

(13) (*a*) When a current is being generated in the armature of a dynamo, the reaction between this current and the

lines of force produced by the field oppose the rotation of the armature conductors; in fact, it tends to rotate the armature in the opposite direction. This opposing effect of the current in the armature of a dynamo is known as the counter torque of a dynamo. See Art. **25**.

(13) If a conductor is forcibly moved, as in a dynamo armature, across a magnetic field in a given direction, say in the direction of the arrows *a*, *a*, Fig. 33, a current will flow in the conductor from *c* to *c'*. If this same conductor is not forcibly moved, but lies in this same field, and a current is sent through it in the same direction as before, that is from *c* to *c'*, then the conductor will move across the field in the direction of the arrows *b*, *b*, if there is nothing to oppose its motion. The force causing this latter motion evidently will oppose the force producing the motion of the conductor in the dynamo armature. These two opposing forces are evidently present in a dynamo armature, the counter torque being the smaller, of course, else the armature would not revolve. See Art. **25**.

(14) $W = C^2 r$. In this example, $C = 120$ amperes and $r = .04$ ohm; hence, $W = 120^2 \times .04 = 576$ watts. Ans.

(15) Under "Field Losses," in Art. **66**, the watts lost in the series coils is found by using the formula $W = C^2 R$. In this example, $C = 40$ amperes and $R = .04$ ohm; hence, $W = 40^2 \times .04 = 40 \times 40 \times .04 = 64$ watts, which represents the loss in the series coils. The watts lost in the shunt coil is given by the formula $W = \frac{E^2}{R}$. In this case, $E = 550$ volts and $R = 550$ ohms; hence, $W = \frac{E^2}{R} = \frac{550 \times 550}{550} = 550$ watts, which is the loss in the shunt field. The total loss in the fields of a compound-wound dynamo is equal to the sum of the losses in the series and shunt coils. Hence, the total loss in this case is $64 + 550 = 614$ watts. Ans.

(16) When current flows in the armature conductors, the armature core is magnetized by this current in such a

direction as will produce lines of force exactly at right angles to the lines of force produced by the field, if the two magnetizing forces could act independently of each other. But lines of force cannot cross each other, hence the two systems of lines of force rearrange themselves and produce, as a result, neutral points that are shifted forwards, in the case of a dynamo, in the direction of rotation when current flows or increases in strength in the armature conductors. See Art. **29.**

(17) From Art. **63,** the total loss in a dynamo is the sum of the separate losses; hence, in this example, the total loss in watts is $356 + 178 + 263 + 423 + 50 = 1{,}270$ watts. From Art. **59,** the input to the dynamo in this case is $15{,}000 + 1{,}270 = 16{,}270$ watts. By formula **2,** the efficiency $\mathsf{E} = \frac{100 \times 15{,}000}{16{,}270}$, or 92.19 per cent. at this output. Ans.

(18) From example 17, the loss in mechanical friction is 356 watts, and the input is 16,270 watts; hence (see Art. **72**), the percentage loss is $\frac{356 \times 100}{16{,}270}$, or 2.1881 per cent. Ans.

From example 17, the loss in the core by eddy currents and hysteresis is 178 watts, and the input is 16,270 watts; hence, the percentage of loss is $\frac{178 \times 100}{16{,}270}$, or 1.094 per cent. Ans.

From example 17, the loss in the field coils is 263 watts, and the input is 16,270 watts; hence, the percentage of loss is $\frac{263 \times 100}{16{,}270}$, or 1.6165 per cent. Ans.

From example 17, the loss in the armature $(C^2 r) = 423$ watts, and the input is 16,270 watts; hence, the percentage of loss is $\frac{423 \times 100}{16{,}270}$, or 2.5999 per cent. Ans.

From example 17, the sum of the separate losses is 1,270 watts, and this is the difference between the input and the output; the input is 16,270 watts; then, by formula **3,** the total percentage of loss $L = \frac{100 \times 1{,}270}{16{,}270}$, or 7.8058 per cent. Ans.

(19) From Art. **22,** it will be seen that the electromotive force generated in an armature is proportional to the speed, other conditions and quantities remaining unchanged. Hence, in this example, if E represents the electromotive force that is generated when the armature is driven at 1,400 revolutions per minute, then, by proportion, $440 : E :: 1{,}200 : 1{,}400$, or $E \times 1{,}200 = 440 \times 1{,}400$; therefore, $E = \frac{440 \times 1{,}400}{1{,}200} = 513\frac{1}{3}$ volts. Ans.

(20) They spark less than copper brushes, especially when the load is a variable one. See Art. **79.**

(21) Yes; because, Art. **1,** a change takes place in number of lines that pass through the coil. From the rule given in Art. **7,** it will be seen that the current will circulate around the ring in the same direction as the movements of the hands of a watch; for the effect of the motion is to diminish the number of lines of force that pass through the coil, and the observer is looking along the magnetic field in the direction of the lines of force.

(22) See Art. **42.**

(23) The *rate of cutting* lines of force is found by dividing the number cut by the time required to cut them; hence, in this case, the rate of cutting is $\frac{8{,}000{,}000}{.25} = 32{,}000{,}000$ lines of force per second.

(24) Because the solid iron core would act as a large conductor cutting lines of force, thereby producing *local*, or *eddy*, currents in the core, heating it badly, and uselessly dissipating a large amount of energy. See Art. **16.**

(25) From Art. **22,** it will be seen that the electromotive force generated in an armature is proportional to the number of lines of force passing through the core. Let E represent the electromotive force that is generated when 1,250,000 lines of force are passing through the core; then, by proportion, $200 : E :: 750{,}000 : 1{,}250{,}000$, or

$E \times 750{,}000 = 200 \times 1{,}250{,}000$; therefore, $E = \frac{200 \times 1{,}250{,}000}{750{,}000}$ $= 333\frac{1}{3}$ volts. Ans.

(26) (*a*) See Art. **65.**
(*b*) See Art. **70.**
(*c*) See Art. **66.**

(27) Toward the side *a*; for, by applying the thumb-and-finger rule given in Art. **26,** and making the forefinger point in the direction of the lines of force and the middle finger in the direction of the current, the thumb will point toward the side *a*.

(28) Use the formula given under "Field Losses" in Art. **67,** $C_s = \frac{E_e}{r_s}$, which is a modification of the formula $C = \frac{E}{R}$. In this example, $E_e = 525$ volts and $r_s = 650$ ohms; hence, $C_s = \frac{E_e}{r_s} = \frac{525}{650} = .8076$ ampere. Ans.

(29) The increase in voltage from no load to full load is $124.2 - 115 = 9.2$ volts, which is $\frac{9.2 \times 100}{115} = 8$ per cent. of the normal voltage. Therefore, the over-compounding is 8 per cent. Ans.

(30) It is radiated from the surface into the surrounding air, and some of it may be conducted away through the shaft, bearings, and base.

(31) See Arts. **34, 36,** and **39.**

(32) From Art. **72,** the percentage of loss in the core is found by dividing the number of watts lost in the core by the input and multiplying by 100. Reducing 64 horsepower to watts gives $64 \times 746 = 47{,}744$ watts. Consequently, the loss in the core is $\frac{800 \times 100}{47{,}744} = 1.6756$ per cent. Ans.

(33) See Art. **76.**

(34) See Art. **43.**

INDEX.

NOTE.—All items in this index refer first to the section (see the Preface) and then to the page of the section. Thus, "Acceleration 4 7" means that acceleration will be found on page 7 of section 4.

A.	*Sec.*	*Page.*
Abstract number	1	1
Accelerating force	4	15
" force, Constant	4	16
Acceleration	4	7
" due to the force of gravity	4	16
" Work of	4	24
Addendum	5	31
" circle	5	31
" line	5	31
Addition	1	4
" of decimals	1	39
" of denominate numbers	2	16
" of fractions	1	28
Adjacent angle	3	14
Adjustable arms of Wheatstone bridge	8	41
Aeriform bodies	4	2
Aggregation, Signs of	1	52
Allowable pressures on bearings	4	37
Alternating current	9	16
" current dynamos	9	55
Altitude, Definition of	3	17
" of cone	3	36
" of triangle	3	23
Amber	8	1
Ammeter	8	30
" Weston	8	30
Amount in percentage	2	8
" of magnetism	8	19
Ampere	8	26
" International	8	27

	Sec.	*Page.*
Ampere meter	8	30
" turns	9	44
Angle	3	14
" inscribed in circle	3	27
" Vertex of	3	14
Angles or arcs, Measure of	2	12
Application of Ohm's law to derived circuits	8	55
" of Ohm's law to voltaic cells	8	53
Applications of inclined plane	5	50
" of Ohm's law	8	50
" of Ohm's law to closed circuits	8	50
" of Ohm's law to drop or loss of potential	8	52
Arc of contact of belts, To find the	5	14
Area	3	18
Arithmetic, Definition of	1	1
Armature	8	14
" losses of dynamo	9	78
" of dynamo	9	33
" of dynamo	9	63
" reaction	9	34
" reaction	9	39
Artificial magnets	8	14
Atom	4	1
Average velocity	4	7
Avoirdupois weight	2	11
Axis of magnetism	8	15
Axle	5	1
" and wheel	5	5

B.	Sec.	Page.
Backlash	5	31
Balance arms of Wheatstone bridge	8	41
" Running	5	10
" Standing	5	9
Balancing pulleys	5	9
Base, Definition of	3	17
" in percentage	2	3
Battery, Voltaic or galvanic	8	9
Bearing, Projected area of	4	38
Bearings, Allowable pressures on	4	37
" or journals of dynamo	9	66
Belt, To find the horsepower of a	5	16
" To find the length of a	5	13
Belts	5	12
" Calculations for	5	13
" Care and use of	5	19
" Cementing	5	23
" Cotton	5	13
" Double	5	18
" Effective pull of	5	14
" Flapping of	5	20
" Joining the ends of	5	21
" Lacing	5	21
" Leather	5	13
" Rubber	5	13
" Rubber, Care of	5	24
" Speed of	5	16
" Stretch of	5	24
" To find the arc of contact of	5	14
" To find the width of	5	14
Bevel gear	5	28
Bipolar dynamo	9	54
" field magnets, Types of	9	54
Block	5	42
" Weston differential pulley	5	46
Blow, Force of a	4	25
Body, Change in motion of a	4	17
Bodies	4	2
" Aeriform	4	2
" Gaseous	4	2
" Liquid	4	2
" Solid	4	2
Bolts	5	57
" Machine	5	57
Box, Resistance	8	39
Brace	1	52
"	3	2
Brackets	1	52
"	3	2
Bridge, Wheatstone	8	41
Brittleness	4	5
Broken circuit	8	12
Brushes	9	17
" and brush holders	9	64
Building up of a dynamo	9	47

C.	Sec.	Page.
Calculations for belts	5	13
" involving percentage	2	4
" relating to gears	5	34
Cancelation	1	20
Capacity, Measures of	2	12
Capscrews	5	57
Cardew voltmeter	8	46
Care and use of belts	5	19
" of rubber belts	5	24
Cell, Internal resistance of voltaic	8	38
" Poles of a	8	9
" Simple voltaic or galvanic	8	9
Cells, Connecting, in multiple series	8	12
" Connecting, in parallel or multiple arc	8	12
" Connecting, in series	8	11
Cementing belts	5	23
Center	3	14
" Definition of	3	27
" of gravity	4	15
" of gravity	4	39
" of gravity of a solid	4	42
Centrifugal force	4	43
Centripetal force	4	44
Chain hoist	5	46
" hoist, Differential	5	47
Change in motion of a body	4	17
Charge, Electric	8	3
" Negative	8	4
" Positive	8	4
" Static	8	3
Circle	3	14
" Addendum	5	31
" Definition of	3	27
" Pitch	5	29
" Root	5	31
Circuit, Broken	8	12
" Closed	8	12
" Complete	8	12
" Derived	8	13
" Earth	8	13
" External	8	13
" Grounded	8	13
" Internal	8	13
" Magnetic	8	17

	Sec.	*Page.*
Circuit, Open	8	12
Circular pitch	5	30
" pitch system	5	34
Circumference	3	14
Classes of motors	9	88
Closed circuit	8	12
" circuits, Applications of Ohm's law to	8	50
" coil winding	9	28
Coach screws	5	59
Coefficient of friction	4	33
" Temperature	8	35
Coefficients of friction	4	35
Coil, Exciting	9	6
" Magnetizing	8	23
" Primary	9	6
" Secondary	9	6
" Standard ohm	8	38
Collector of dynamo	9	33
" ring	9	14
Combination of pulleys	5	43
" of pulleys, Law of	5	44
Common denominator	1	26
Commutator	9	17
"	9	64
" Neutral points of	9	29
" of dynamo	9	33
Comparison of forces	4	10
Compass	8	15
Components of a force	4	29
Composition of forces	4	26
Compound denominate number	2	10
" dynamo	9	51
" lever	5	3
" lever, Law of	5	5
" numbers	2	9
" wound motors	9	90
Compressibility	4	4
Concrete number	1	1
Conductivity of conductor	8	55
Conductor, Direction of lines of force around a	8	20
Conductors	8	5
" in series	8	13
" Joint conductivity of	8	56
Cone	3	36
" Frustum of	3	37
Connecting cells in multiple series	8	12
" cells in parallel or multiple arc	8	12
" cells in series	8	11
" voltmeters and ammeters	8	48

	Sec.	*Page.*
Consequent poles	8	18
" poles	9	55
Conservation of energy	4	24
Constant accelerating force	4	16
" current dynamos	9	55
" potential dynamos	9	55
" potential dynamos, Efficiency of	9	69
" potential dynamos, Output of	9	81
" potential generators	9	56
" retarding force	4	16
Construction of dynamo, Mechanical	9	61
Continuous current	8	9
" current	9	16
Convex surface	3	33
Core, Drum	9	22
" Laminated	9	22
" Losses of dynamo	9	73
" of electromagnet	8	23
" Ring	9	22
Cotton belts	5	13
Coulomb	8	60
Counter torque of dynamo	9	38
Couple, Voltaic	8	9
Crowning pulley faces	5	8
Cube	3	32
" root	2	35
Cubic measure	2	11
Current	8	3
" Alternating	9	16
" Continuous	8	9
" Continuous or direct	9	16
" Pulsating	9	18
" strength	8	11
" Unit strength of	8	27
Currents, Foucault	9	80
Curve, Epicycloidal	5	33
Curve, Hypocycloidal	5	33
" Involute	5	33
Curved line	3	13
Cycloidal tooth	5	32
Cylinder	3	32
Cylinders, Pitch	5	29
Cylindrical ring	3	40
" ring, Convex area of	3	40
" ring, Volume of	3	41

D.	*Sec.*	*Page.*
Decimal point	1	37
" Reduction of, to a fraction	1	50
Decimals, Addition of	1	39
" Definition of	1	37
" Division of	1	44

B. *Sec. Page.*

Backlash 5 31
Balance arms of Wheatstone bridge 8 41
" Running 5 10
" Standing 5 9
Balancing pulleys 5 9
Base, Definition of 3 17
" in percentage 2 3
Battery, Voltaic or galvanic 8 9
Bearing, Projected area of 4 38
Bearings, Allowable pressures on 4 37
" or journals of dynamo 9 66
Belt, To find the horsepower of a 5 16
" To find the length of a 5 13
Belts 5 12
" Calculations for 5 13
" Care and use of 5 19
" Cementing 5 23
" Cotton 5 13
" Double 5 18
" Effective pull of 5 14
" Flapping of 5 20
" Joining the ends of 5 21
" Lacing 5 21
" Leather 5 13
" Rubber 5 13
" Rubber, Care of 5 24
" Speed of 5 16
" Stretch of 5 24
" To find the arc of contact of 5 14
" To find the width of 5 14
Bevel gear 5 28
Bipolar dynamo 9 54
" field magnets, Types of 9 54
Block 5 42
" Weston differential pulley 5 46
Blow, Force of a 4 25
Body, Change in motion of a 4 17
Bodies 4 2
" Aeriform 4 2
" Gaseous 4 2
" Liquid 4 2
" Solid 4 2
Bolts 5 57
" Machine 5 57
Box, Resistance 8 39
Brace 1 52
" 3 2
Brackets 1 52
" 3 2
Bridge, Wheatstone 8 41

Sec. Page.

Brittleness 4 5
Broken circuit 8 12
Brushes 9 17
" and brush holders 9 64
Building up of a dynamo 9 47

C. *Sec. Page.*

Calculations for belts 5 13
" involving percentage 2 4
" relating to gears 5 34
Cancelation 1 20
Capacity, Measures of 2 12
Capscrews 5 57
Cardew voltmeter 8 46
Care and use of belts 5 19
" of rubber belts 5 24
Cell, Internal resistance of voltaic 8 38
" Poles of a 8 9
" Simple voltaic or galvanic 8 9
Cells, Connecting, in multiple series 8 12
" Connecting, in parallel or multiple arc 8 12
" Connecting, in series 8 11
Cementing belts 5 23
Center 3 14
" Definition of 3 27
" of gravity 4 15
" of gravity 4 39
" of gravity of a solid 4 42
Centrifugal force 4 43
Centripetal force 4 44
Chain hoist 5 46
" hoist, Differential 5 47
Change in motion of a body 4 17
Charge, Electric 8 3
" Negative 8 4
" Positive 8 4
" Static 8 3
Circle 3 14
" Addendum 5 31
" Definition of 3 27
" Pitch 5 29
" Root 5 31
Circuit, Broken 8 12
" Closed 8 12
" Complete 8 12
" Derived 8 13
" Earth 8 13
" External 8 13
" Grounded 8 13
" Internal 8 13
" Magnetic 8 17

Sec. Page.
Circuit, Open.................... 8 12
Circular pitch.................. 5 30
" pitch system.......... 5 34
Circumference..................... 3 14
Classes of motors.............. 9 88
Closed circuit.......................... 8 12
" circuits, Applications of Ohm's law to......... 8 50
" coil winding........... 9 28
Coach screws................... 5 59
Coefficient of friction............ 4 33
" Temperature....... 8 35
Coefficients of friction.......... 4 35
Coil, Exciting.................... 9 6
" Magnetizing............. 8 23
" Primary.................... 9 6
" Secondary................ 9 6
" Standard ohm........... 8 38
Collector of dynamo.............. 9 33
" ring.... .... 9 14
Combination of pulleys........ 5 43
" of pulleys, Law of............... 5 44
Common denominator.......... 1 26
Commutator.................... 9 17
" 9 64
" Neutral points of. 9 29
" of dynamo........ 9 33
Comparison of forces.......... 4 10
Compass........................ 8 15
Components of a force......... 4 29
Composition of forces.......... 4 26
Compound denominate number.............. 2 10
" dynamo........... 9 51
" lever................ 5 3
" lever, Law of..... 5 5
" numbers.......... 2 9
" wound motors.... 9 90
Compressibility................ 4 4
Concrete number.............. 1 1
Conductivity of conductor..... 8 55
Conductor, Direction of lines of force around a............ 8 20
Conductors......................... 8 5
" in series............ 8 13
" Joint conductivity of............... 8 56
Cone............................. 3 36
" Frustum of............... 3 37
Connecting cells in multiple series............ 8 12
" cells in parallel or multiple arc.... 8 12
" cells in series..... 8 11
" voltmeters and ammeters....... 8 48

Sec. Page.
Consequent poles............... 8 18
" poles............... 9 55
Conservation of energy........ 4 24
Constant accelerating force.... 4 16
" current dynamos.... 9 55
" potential dynamos.. 9 55
" potential dynamos, Efficiency of........ 9 69
" potential dynamos, Output of.......... 9 81
" potential generators. 9 56
" retarding force....... 4 16
Construction of dynamo, Mechanical...................... 9 61
Continuous current............ 8 9
" current............ 9 16
Convex surface................ 3 33
Core, Drum...................... 9 22
" Laminated............... 9 22
" Losses of dynamo....... 9 73
" of electromagnet......... 8 23
" Ring.......................... 9 22
Cotton belts..................... 5 13
Coulomb......................... 8 60
Counter torque of dynamo..... 9 38
Couple, Voltaic.................. 8 9
Crowning pulley faces......... 5 8
Cube............................ 3 32
" root........................ 2 35
Cubic measure.................. 2 11
Current.......................... 8 3
" Alternating........... 9 16
" Continuous........... 8 9
" Continuous or direct.. 9 16
" Pulsating............. 9 18
" strength.............. 8 11
" Unit strength of...... 8 27
Currents, Foucault............ 9 80
Curve, Epicycloidal............ 5 33
Curve, Hypocycloidal.......... 5 33
" Involute............... 5 33
Curved line..................... 3 13
Cycloidal tooth................. 5 32
Cylinder........................ 3 32
Cylinders, Pitch................ 5 29
Cylindrical ring.............. 3 40
" ring, Convex area of................ 3 40
" ring, Volume of.... 3 41

D. *Sec. Page.*
Decimal point.................. 1 37
" Reduction of, to a fraction............. 1 50
Decimals, Addition of.......... 1 39
" Definition of........ 1 37
" Division of.......... 1 44

Sec. Page.
Decimals, Multiplication of .. 1 42
" Reading of 1 37
" Subtraction of 1 41
Definition of mechanics 4 1
Degrees 3 15
Denominate numbers 2 9
" numbers, Addition of 2 16
" numbers, Division of 2 21
" numbers, Multiplication of 2 20
" numbers, Reduction of 2 13
" numbers, Subtraction of 2 18
Denominator 1 23
Density, Magnetic 8 19
Derived circuit 8 13
" circuits, Application of Ohm's law to 8 55
Diameter of circle 3 27
" Pitch 5 29
Diametral pitch 5 31
" pitch system 5 35
Difference in percentage 2 3
Differential chain hoist 5 47
" pulley block, Weston 5 46
Direct current 9 16
" current motors 9 87
" proportion 2 44
Direction of flow of electric current 8 7
" of lines of force 8 17
" of lines of force around a conductor 8 20
Dividend 1 16
Divisibility 4 3
Division 1 16
" of decimals 1 44
" of denominate numbers 2 21
" of fractions 1 33
Divisor 1 16
Double belts 5 18
Driven wheel 5 25
Driver 5 10
" 5 25
" and follower 5 25
Driving mechanism of dynamos 9 67
Drop or loss of potential, Application of Ohm's law to .. 8 52
Drum core 9 22
Dry measure 2 10

Sec. Page.
Ductility 4 5
Dynamics 4 15
Dynamo 9 33
" Armature of 9 33
" Armature of 9 63
" Armature losses of .. 9 78
" Bipolar 9 54
" Collector of 9 33
" Commutator of 9 33
" Compound 9 51
" Compounding of 9 54
" Core losses of .. 9 73
" Counter torque of 9 38
" Field losses of 9 75
" Frame of 9 63
" Friction losses of 9 73
" Internal-pole 9 61
" Load of a 9 48
" Magnetic field of 9 33
" Mechanical construction of 9 61
" Over-compounding of 9 54
" Physical theory of 9 9
" Separately excited 9 43
" Series 9 49
" Shunt and series 9 51
" Terminals of 9 54
Dynamos, Alternating-current 9 55
" Constant-current 9 55
" Constant-potential 9 55
" Driving mechanism of 9 67
" Efficiency of constant-potential 9 69
" Heating of 9 81
" Multipolar 9 56
" Sparking of 9 81
" Types of 9 55

E. *Sec. Page.*
Earth circuit 8 13
Edges of a solid 3 32
Effective pull of belts 5 14
Efficiency 5 60
" of constant-potential dynamos 9 69
Elasticity 4 4
Electric charge 8 3
" current, Direction of flow of 8 7
" series 8 5
Electrical power 8 63
" quantity 8 60
" unit 8 26
" work 8 61
Electricity 8 1
Electrodes 8 9

	Sec.	Page.
Electrodynamics	8	3
"	8	6
Electrolyte	8	9
Electromagnet	8	23
" Core of	8	23
" Horseshoe	8	24
" Iron-clad	8	25
" Yoke of	8	24
Electromagnetic induction	9	1
" induction	9	5
Electromagnetism	8	19
Electromotive force	8	26
" force, Practical unit of	8	45
" series	8	10
Electrostatics	8	8
Elements, Galvanic	8	9
" Voltaic	8	9
Energy	4	22
" Conservation of	4	24
" Kinetic	4	22
" losses	9	70
" Potential	4	23
Epicycloid	5	32
Epicycloidal curve	5	33
" tooth	5	32
Equilateral triangle, Definition of	3	20
Equilibrium	4	45
" Neutral.	4	46
" Stable	4	45
" Unstable	4	46
Ether	8	1
Evolution	2	27
Exciting coil	9	6
Expansibility	4	4
Extension	4	3
External circuit	8	13

F.	Sec.	Page.
Faces of a solid	3	32
Factors of a number	1	20
Fastenings, Screws used as	5	56
Field losses of dynamo	9	75
" Magnetic	8	16
" magnets	9	42
" magnets, Types of bipolar	9	54
" magnets, Types of multipolar	9	61
Fixed and movable pulleys	5	42
" pulleys	5	42
Flapping of belts	5	20
Flow of an electric current, Direction of	8	7
Follower and driver	5	25
Foot-pound	4	20

	Sec.	Page.
Foot-pound	8	61
Force.	4	10
" Accelerating	4	15
" arm	5	2
" Centrifugal	4	43
" Centripetal	4	44
" Components of a	4	29
" Constant accelerating	4	16
" Constant retarding	4	16
" Electromotive	8	26
" General principles of	4	10
" Magnetizing	9	44
" of a blow	4	25
" of gravity	4	15
" of gravity, Acceleration due to	4	16
" Retarding	4	15
" Time effect of a	4	19
Forces, Comparison of	4	10
" Composition of	4	26
" Names of.	4	10
" Reduction of	4	11
" Resolution of	4	29
" Triangle of.	4	26
Forms of gear-teeth	5	32
" of keys	5	52
Formula, Definition of	3	1
Foucault currents	9	80
Fraction, Reduction of a, to a decimal	1	48
Fractions	1	23
" Addition of	1	28
" Division of	1	33
" Multiplication of	1	31
" Reduction of	1	25
" Subtraction of	1	29
Frame of dynamo	9	63
Friction	4	33
" Coefficient of	4	33
" Coefficients of	4	35
" Laws of	4	33
" losses of dynamo	9	73
" of a screw	5	53
Frictional losses with pulleys.	5	44
Frustum of cone	3	37
" of pyramid	3	37

G.	Sec.	Page.
Galvanic cell	8	9
" elements	8	9
Galvanometer	8	28
Gas, Permanent	4	2
Gaseous bodies	4	2
Gear	5	27
" Bevel	5	28
" calculations	5	34
" Miter	5	28

	Sec.	*Page.*
Gear, Spur	5	27
" teeth, Forms of	5	32
" teeth, Proportions of	5	31
" wheels	5	27
" wheels, Teeth of	5	29
Gears, Number and speed of teeth of	5	38
General properties of matter	4	3
Generators, Constant-potential	9	56
Globe	3	29
Gravitation	4	15
" Law of	4	17
Gravity, Acceleration due to the force of	4	16
" Center of	4	15
" Center of	4	39
" Force of	4	15
Grounded circuit	8	13

H.	*Sec.*	*Page.*
Hardness	4	5
Heating of dynamos	9	81
Heptagon	3	24
Hexagon	3	24
Horizontal line	3	13
Horsepower	4	21
"	8	63
" of a belt	5	16
Horseshoe electromagnet	8	24
Hypocycloid	5	32
Hypocycloidal curve	5	33
Hypotenuse	3	20
Hysteresis	9	74

I.	*Sec.*	*Page.*
Impenetrability	4	3
Improper fraction	1	24
Inclined plane	5	47
" plane, Applications of the	5	50
Indestructibility	4	4
Induction, Electromagnetic	9	1
" Electromagnetic	9	5
" Magnetic	8	18
" Mutual	9	6
Inertia	4	3
"	4	12
" Law of	4	12
" Resistance due to	4	12
" Resistance of	4	12
Input	9	69
Insulators	8	5
Internal circuit	8	13
" resistance of voltaic cells	8	38
International ampere	8	27
" ohm	8	31

	Sec.	*Page.*
Inverse proportion	2	44
" proportion	2	46
" ratio	2	41
Involute	5	32
" curve	5	33
" teeth	5	33
Involution	2	25
Iron-clad electromagnet	8	25
Isosceles triangle, Definition of	3	20

J.	*Sec.*	*Page*
Joining the ends of belts	5	21
Joint conductivity of conductors	8	56
Joule	8	62
Journals or bearings of dynamo	9	66

K.	*Sec.*	*Page*
Keeper	8	14
Keys, Forms of	5	52
" Taper of	5	50
Kilowatt-hour	8	66
Kinetic energy	4	22
Kinetics	4	15

L.	*Sec.*	*Page*
Lacing belts	5	21
Lagscrews	5	50
Laminated core	9	22
Lamp, Pilot	9	68
Law of combination of pulleys	5	44
" of compound lever	5	5
" of gravitation	4	17
" of inertia	4	12
" of the lever	5	2
Laws of friction	4	33
" of motion, Newton's	4	11
" of static charges	8	4
" of the screw	5	54
" of weight	4	17
Leads, Voltmeter	8	47
Least common denominator	1	26
Leather belts	5	13
Length of a belt, To find the	5	13
Lever, Compound	5	3
" Compound, Law of	5	5
" Law of the	5	2
" wheel, and axle	5	5
Like numbers	1	1
Line, Addendum	5	31
" Horizontal	3	13
" Pitch	5	29
" Root	5	31
" Vertical	3	13
Lines of force	8	16
" of force, Direction of	8	17

	Sec.	*Page.*
Lines of force, Direction of, around a conductor	8	20
Linear measure	2	10
Liquid bodies	4	2
" measure	2	10
" measure	2	12
Load of a dynamo	9	48
Location of the resultant	4	28
Lodestone	8	14
Long ton table	2	11

M.	*Sec.*	*Page.*
Machine bolts	5	57
" elements	5	1
" screws	5	57
Magnet, Neutral line of a	8	14
" North pole of a	8	15
" Poles of a	8	14
" South pole of a	8	15
Magnets	8	14
" Artificial	8	14
" Field	9	42
" Permanent	8	14
Magnetic circuit	8	17
" density	8	19
" field	8	16
" field of dynamo	9	33
" force, Lines of	8	16
" induction	8	18
" permeability	8	23
" saturation	9	45
" substances	8	16
Magnetism	8	14
" Amount of	8	19
" Axis of	8	15
" Quantity of	8	19
Magnetizing coil	8	23
" force	9	44
Magneto-machine	9	42
Malleability	4	5
Mass	4	17
Matter	4	1
" and its properties	4	1
" General properties of	4	3
" Special properties of	4	3
Mean velocity	4	7
Measure, Cubic	2	11
" Dry	2	12
" Linear	2	10
" Liquid	2	12
" of angles or arcs	2	12
" of money	2	13
" of time	2	12
" Square	2	10
Measures	2	10
" of capacity	2	12
" of extension	2	10

	Sec.	*Page.*
Measures of weight	2	11
Mechanical construction of dynamo	9	61
Mechanics, Definition of	4	1
" Principles of	4	1
Megohm	8	34
Mensuration, Definition of	3	13
Metal screws	5	56
Meter, Ampere	8	30
Microhm	8	34
Minutes	3	15
Miter gear	5	28
Mixed number	1	24
Mobility	4	3
Molecule	4	1
Momentum	4	19
Money, Measure of	2	13
Motion	4	5
" Change in the, of a body	4	17
" Definition of	4	5
" Newton's laws of	4	11
" Path of a body in	4	6
Motors, Classes of	9	88
" Compound-wound	9	90
" Direct-current	9	87
" Series-wound	9	89
" Shunt-wound	9	89
Movable and fixed pulleys	5	42
" pulley	5	43
Multiple-arc circuit	8	18
" series, Connecting cells in	8	12
" winding	9	58
Multiplicand	1	11
Multiplication	1	11
" of decimals	1	42
" of denominate numbers	2	20
" of fractions	1	31
Multiplier	1	11
Multipolar dynamos	9	56
" field magnets, Types of	9	61
Mutual induction	9	6

N.	*Sec.*	*Page.*
Names of forces	4	10
Negative charge	8	4
Neutral equilibrium	4	46
" line of a magnet	8	14
" points of commutator	9	29
" spaces	9	29
Newton's laws of motion	4	11
Non conductors	8	5
Non-magnetic substances	8	16
North pole of a magnet	8	15
Notation	1	1

	Sec.	Page.
Number, Definition of	1	1
" of teeth and speed of gears	5	38
Numbers, Denominate	2	9
Numeration	1	1
Numerator	1	23
O.	*Sec.*	*Page.*
Obtuse angle	3	14
Ohm	8	26
"	8	31
" coil, Standard	8	38
" International	8	31
Ohm's law	8	27
" law, Applications of	8	50
" law, Application of, to drop or loss of potential	8	52
Open circuit	8	12
" coil winding	9	28
Output	9	60
" of constant-potential dynamos	9	61
Over-compounding	9	54
P.	*Sec.*	*Page.*
Parallel circuit	8	13
" lines	3	13
" winding	9	58
Parallelogram, Definition of	3	17
Parallelopipedon	3	32
Parenthesis	1	52
"	3	2
Path of a body in motion	4	6
Pentagon	3	24
Percentage, Calculations involving	2	4
" Definition of	2	1
Permanent gas	4	2
" magnets	8	14
Permeability, Magnetic	8	23
Perpendicular	3	13
Physical theory of the dynamo	9	9
Picking-up of a dynamo	9	47
Pilot lamp	9	68
Pinion	5	28
Pitch circle	5	29
" Circular	5	30
" Circular, system	5	34
" cylinders	5	29
" diameter	5	29
" Diametral	5	31
" Diametral, system	5	35
" line	5	29
" of a screw	5	53
Plane, Applications of the inclined	5	50
Plane figure, Definition of a	3	17
" Inclined	5	47
Plug rheostat	8	40
Polarity of a solenoid	8	22
Poles, Consequent	8	18
" Consequent	9	55
" of a cell	8	9
" of a magnet	8	14
" Salient	9	55
Polygon, Definition of a	3	24
" Regular	3	24
Polygons	3	24
Porosity	4	3
Positive charge	8	4
Potential	8	6
" energy	4	23
Pound	4	10
Power	4	21
"	8	62
" Electrical	8	63
Practical unit of electromotive force	8	45
Pressure, Tangential	4	30
Pressures, Allowable, on bearings	4	37
Primary coil	9	6
Prime factor	1	20
" number	1	20
Principles of mechanics	4	1
Prism	3	32
" and cylinder	3	32
Projected area of bearing	4	38
Proper fraction	1	24
Properties of matter	4	1
Proportion	2	43
" Direct	2	44
" Inverse	2	44
Proportions of gear-teeth	5	31
Pull of belts, Effective	5	14
Pulley block, Weston differential	5	46
" Crowning faces of	5	8
" Fixed	5	42
" Movable	5	43
" Split	5	7
Pulleys	5	7
" Balancing	5	9
" Combination of	5	43
" Fixed and movable	5	42
" Frictional losses with	5	44
" Law of combination of	5	44
" Rules for speeds of	5	10
" Solid	5	7
Pulsating current	9	18
Pyramid	3	36
" Frustum of	3	37

Q. *Sec. Page.*
Quadrilateral, Definition of.... 3 17
Quadrilaterals 3 17
Quantity........................ 3 1
“ Electrical............. 8 60
“ of magnetism......... 8 19
Quotient.. .... 1 16

R. *Sec. Page.*
Rack............................ 5 29
Radius of circle.... .... 3 27
Rate per cent........ .... 2 3
Ratio........................... 2 40
“ Inverse.................. 2 41
“ Reciprocal............... 2 41
Reciprocal ratio........ .... 2 41
Rectangle, Definition of........ 3 17
Reducing a decimal to a fraction............ ... 1 50
“ a fraction to a decimal 1 48
“ inches to a decimal part of a foot...... 1 49
Reduction of denominate numbers..... .... 2 13
“ of forces 4 11
“ of fractions 1 25
Regular polygon........... ... 3 24
Resistance box.................. 8 30
“ due to inertia...... 4 12
“ of inertia........... 4 12
“ Specific electric.... 8 36
“ Unit of electrical... 8 31
Resolution of forces 4 29
Resultant. 4 27
“ Location of the...... 4 28
Retarding force............ .. 4 15
“ force, Constant . . 4 16
Retardation............ .... 4 7
“ Work of........... 4 24
Rheostat........................ 8 39
“ Plug............. .... 8 40
Rhomboid, Definition of....... 3 17
Rhombus, Definition of........ 3 17
Right angle..................... 3 14
“ angled triangle, Definition of................. 3 20
“ triangle 3 20
Ring core....................... 9 22
Root............................ 5 31
“ circle..................... 5 31
“ Cube 2 35
“ line....................... 5 31
“ Square.................. 2 28
Rubber belts.................... 5 13
“ belts, Care of 5 24
Rules for speeds of pulleys 5 10
“ for velocity problems... 4 7

Sec. Page.
Rules for wheel work. 5 25
Running balance............ .. 5 10

S. *Sec. Page.*
Salient poles........... .... 9 55
Saturation, Magnetic........... 9 45
Scalene triangle, Definition of. 3 20
Screw 5 53
“ Friction of a............. 5 53
“ Laws of the............ 5 54
“ Pitch of a............... 5 53
Screws, Coach.................. 5 59
“ Machine 5 57
“ Metal 5 56
“ used as fastenings 5 56
“ Wood......... .... 5 56
“ Wood.................. 5 59
Secondary coil.................. 9 6
Seconds......................... 3 15
Self-exciting shunt dynamo.... 9 46
“ induction...... .... 9 5
Semicircle 3 28
Semi-circumference 3 28
Separately excited dynamo.... 9 43
Series dynamo................ . 9 49
“ Electric................. 8 5
“ winding................. 9 61
“ wound motors........... 9 89
Short methods of abstracting square root.................... 2 34
Shunt and series dynamo...... 9 51
“ circuit................... 8 13
“ dynamo.................. 9 46
“ wound motors........... 9 89
Side clearance.................. 5 31
Signs of aggregation........... 1 52
Simple denominate numbers.. 2 9
“ voltaic or galvanic cell. 8 9
Slant height of cone or pyramid............................ 3 36
Sliding contact rheostat........ 8 39
Solenoid........................ 8 22
“ Polarity of... .... 8 22
Solid bodies 4 2
“ Center of gravity of a.... 4 42
“ Edges of a.. .... 3 32
“ Faces of a 3 32
“ pulley.................... 5 7
South pole of a magnet........ 8 15
Sparking of dynamos........... 9 81
Special properties of matter... 4 3
Specific electric resistance..... 8 36
Speed and number of teeth of gears............... ... 5 28
“ of belts 5 16
Speeds of pulleys, Rules for... 5 10
Sphere........ .... 3 39

OCT 10 1929